Unas distribuciones útiles

Distribuciones discretas

1. **Binomial**

   $X \sim \mathcal{BI}(n, \theta)$ si

   $$P(X = x) = \left(\begin{array}{c} n \\ x \end{array} \right) \theta^x (1-\theta)^{n-x} \quad x = 0, 1, \ldots, n.$$

   $E[X] = n\theta$ y $V[X] = n\theta(1-\theta)$.

2. **Binomial Negativa**

   $X \sim \mathcal{BN}(r, \theta)$ si

   $$P(X = x) = \left(\begin{array}{c} x + r - 1 \\ x \end{array} \right) \theta^r (1-\theta)^x \quad x = 0, 1, 2, \ldots$$

   $E[X] = r\frac{1-\theta}{\theta}$ y $V[X] = r\frac{(1-\theta)}{\theta^2}$.

3. **Geométrica**

   $X \sim \mathcal{GE}(\theta)$ si

   $$P(X = x) = \theta(1-\theta)^x \quad x = 0, 1, 2, \ldots$$

   $E[X] = \frac{1-\theta}{\theta}$ y $V[X] = \frac{1-\theta}{\theta^2}$.

4. **Hipergeométrica**

   $X \sim \mathcal{H}(N, R, n)$ si

   $$P(X = x) = \frac{\left(\begin{array}{c} R \\ x \end{array} \right) \left(\begin{array}{c} N - R \\ n - x \end{array} \right)}{\left(\begin{array}{c} N \\ n \end{array} \right)} \quad x = 0, 1, \ldots, n$$

   $E[X] = n\frac{R}{N}$ y $V[X] = n\frac{R}{N}\left(1 - \frac{R}{N}\right)\frac{N-n}{N-1}$.

5. **Poisson**

   $X \sim \mathcal{P}(\theta)$ si

   $$P(X = x) = \frac{\theta^x e^{-\theta}}{x!} \quad x = 0, 1, 2, \ldots$$

   $E[X] = \theta = V[X]$.

6. **Uniforme Discreta**

   $X \sim \mathcal{UD}[\theta_1, \theta_2]$ si

$$P(X = x) = \frac{1}{\theta_2 - \theta_1 + 1} \quad x = \theta_1, \theta_1 + 1, \ldots, \theta_2.$$

   $E[X] = \frac{\theta_1+\theta_2}{2}$ y $V[X] = \frac{(\theta_2-\theta_1)(\theta_2-\theta_1+2)}{12}$.

Distribuciones continuas

1. **Behrens Fisher**

   $X \sim \mathcal{BF}(\nu_1, \nu_2, \theta)$ si $X = T_1 \cos\theta - T_2 \sin\theta$ y $T_i \sim \mathcal{T}(\nu_i, 0, 1)$.

   $E[X] = 0$ y $V[X] = \frac{\nu_1 \sin^2\theta}{\nu_1 - 2} + \frac{\nu_2 \cos^2\theta}{\nu_2 - 2}$ para $\nu_1, \nu_2 > 2$.

2. **Beta**

   $X| \sim \mathcal{B}(\alpha, \beta)$ si

$$f(x) = \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1} \quad 0 < x < 1,$$

   donde $B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$.
   $E[X] = \frac{\alpha}{\alpha+\beta}$ y $V[X] = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$.

3. **Cauchy**

   $X \sim \mathcal{C}(\mu, \sigma^2)$ si

$$f(x) = \frac{1}{\pi} \frac{\sigma}{\sigma^2 + (x-\mu)^2} \quad \forall\, x.$$

   No existen los momentos.

4. **Chi-cuadrada**

   $X \sim \chi^2_\nu$ si $X \sim \mathcal{G}(\nu/2, 1/2)$.

$$f(x) = \frac{2^{-\nu/2}}{\Gamma(\nu/2)} x^{\nu/2-1} e^{-x/2} \quad x > 0.$$

   $E[X] = \nu$ y $V[X] = 2\nu$.

5. **Chi-cuadrada invertida**

   $X \sim \mathcal{I}\chi^2_\nu$ si $1/X \sim \chi^2_\nu$. $E[X] = \frac{1}{\nu-2}$ si $\nu > 2$ y $V[X] = \frac{2}{(\nu-2)^2(\nu-4)}$ si $\nu > 4$.

6. **Exponencial**

   $X \sim \mathcal{E}(\theta)$ si

$$f(x) = \theta e^{-\theta x} \quad x > 0.$$

   $E[X] = \frac{1}{\theta}$ y $V[X] = \frac{1}{\theta^2}$.

7. **F de Fisher**
   $X \sim \mathcal{F}(\alpha, \beta)$ si

$$f(x) = \frac{1}{B(\alpha/2, \beta/2)} \left(\frac{\alpha}{\beta}\right)^{\alpha/2} x^{\alpha/2-1} \left(1 + \frac{\alpha}{\beta}x\right)^{-\frac{\alpha+\beta}{2}} \quad \forall\, x.$$

   $E[X] = \frac{\alpha}{\beta-2}$ si $\beta > 2$ y $V[X] = \frac{2\beta^2(\alpha+\beta-2)}{\alpha(\beta-2)^2(\beta-4)}$ si $\beta > 4$.

8. **Gamma**
   $X| \sim \mathcal{G}(\alpha, \beta)$ si

$$f(x) = \frac{\beta}{\Gamma(\alpha)}(\beta x)^{\alpha-1} e^{-\beta x} \quad x > 0.$$

   $E[X] = \frac{\alpha}{\beta}$ y $V[X] = \frac{\alpha}{\beta^2}$.

9. **Gamma Invertida**
   $X \sim \mathcal{GI}(\alpha, \beta)$ si $X^{-1} \sim \mathcal{G}(\alpha, \beta)$.

$$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{-(\alpha+1)} e^{-\beta/x} \quad x > 0.$$

   $E[X] = \frac{\beta}{\alpha-1}$ si $\alpha > 1$ y $V[X] = \frac{\beta^2}{(\alpha-1)^2(\alpha-2)}$ si $\alpha > 2$.

10. **Normal**
    $X \sim N(\mu, \sigma^2)$ si

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2} \quad \forall x.$$

    $E[X] = \mu$ y $V[X] = \sigma^2$.

11. **Pareto**
    $X \sim \mathcal{PA}(\alpha, \beta)$ si

$$f(x) = \beta\alpha^\beta x^{-\beta-1} \quad x > \alpha.$$

    $E[X] = \frac{\alpha\beta}{\beta-1}$ si $\beta > 1$ y $V[X] = \frac{\alpha^2\beta}{(\beta-1)(\beta-2)}$ si $\beta > 2$.

12. **T de Student**
    $X \sim \mathcal{T}(\nu, \mu, \sigma^2)$ si

$$f(x) = \frac{\Gamma((\nu+1)/2)}{\Gamma(\nu/2)\sqrt{\nu\pi\sigma}} \left(1 + \frac{1}{\nu}\left(\frac{x-\mu}{\sigma}\right)^2\right)^{-(\nu+1)/2} \quad \forall\, x.$$

    $E[X] = \mu$ si $\nu > 1$ y $V[X] = \frac{\nu}{\nu-2}\sigma^2$ si $\nu > 2$.
    La distribución $\mathcal{T}(\nu, 0, 1)$ es la distribucin t de Student estandar.

13. **Uniforme**
$X \sim \mathcal{U}(0,\theta)$ si
$$f(x) = \frac{1}{\theta} \quad 0 < x < \theta.$$
$E[X] = \frac{\theta}{2}$ y $V[X] = \frac{\theta^2}{12}$.

Distribuciones multivariables

1. **Dirichlet**
$\mathbf{X} = (X_1, \ldots, X_p)^T \sim \mathcal{D}(\boldsymbol{\theta})$ si
$$f(\mathbf{x}) = \frac{\Gamma\left(\sum_{i=1}^p \theta_i\right)}{\Gamma(\theta_1)\cdots\Gamma(\theta_p)} \prod_{i=1}^p x_i^{\theta_1 - 1} \quad \text{para } 0 < x_1 < 1,\ \sum_{i=1}^p x_i = 1$$
$E[X_j] = \frac{\theta_j}{\sum_{i=1}^p \theta_i}$ y $V[X_j] = \frac{\theta_j(1-\theta_j)}{(\sum_{i=1}^p \theta_i)^2(\sum_{i=1}^p \theta_i + 1)}$.

2. **Normal multivariante**
$\mathbf{X} = (X_1, \ldots, X_p)^T \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ si
$$f(\mathbf{x}) = \frac{1}{(2\pi)^{p/2}|\boldsymbol{\Sigma}|^{1/2}} \exp\left(-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^T\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\right)$$
$E[\mathbf{X}] = \boldsymbol{\mu}$ y $V[\mathbf{X}] = \boldsymbol{\Sigma}$.

3. **Multinomial**
$\mathbf{X} = (X_1, \ldots, X_p)^T \sim \mathcal{MN}(n, \boldsymbol{\theta})$ si
$$P(\mathbf{X} = \mathbf{x}) = \frac{n!}{\prod_{i=1}^p x_i!} \prod_{i=1}^p \theta_i^{x_i} \quad \text{donde } \sum_{i=1}^p x_i = n \text{ y } 0 < \theta_i < 1\ \forall i$$

4. **Wishart**
$\mathbf{V} \sim \mathcal{W}(\nu, \boldsymbol{\Sigma})$ si
$$f(\mathbf{V}) = \left(2^{\nu k/2}\pi^{k(k-1)/4}\prod_{i=1}^k \Gamma\left(\frac{\nu+1-i}{2}\right)\right)^{-1} |V|^{-\nu/2}|\boldsymbol{\Sigma}|^{(\nu-k-1)/2} \exp\left(-\frac{1}{2}tr(\boldsymbol{\Sigma}^{-1}\mathbf{V})\right)$$
donde $k = \dim\mathbf{V}$.
$E[\mathbf{V}] = \nu\boldsymbol{\Sigma}$.

5. **Wishart invertida**
$\mathbf{V} \sim \mathcal{WI}(\nu, \boldsymbol{\Sigma}^{-1})$ si $\mathbf{V} \sim \mathcal{W}(\nu, \boldsymbol{\Sigma}^{-1})$.
$E[\mathbf{V}] = (\nu - k - 1)^{-1}\boldsymbol{\Sigma}$.