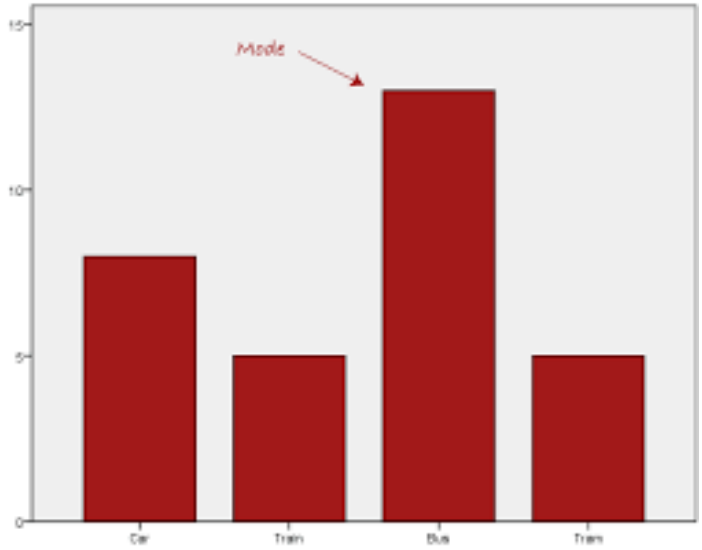
Class 4: Analysis of univariate data: measures of location of a data set



Recommended reading:

To really confuse yourself, look at how the Wikipedia defines percentiles.

Measures of location

There are three standard measures of location of a sample data set.

- Mode
- Median
- Mean

These measures are used to represent typical, central or average values.

Other points of location are also of interest to represent more extreme values.

- Minimum
- Maximum
- Quartiles and Percentiles

The mode

Times voted	Absolute frequency (n)
0	4
1	10
2	12
3	15
4	11
5	5
6	1
7	1
8	1
>8	0
Total	60

The mode is the most frequent value

A data set could be unimodal, bimodal, trimodal, …

Could we calculate the mode with qualitative data?

Does the definition make sense with continuous data?

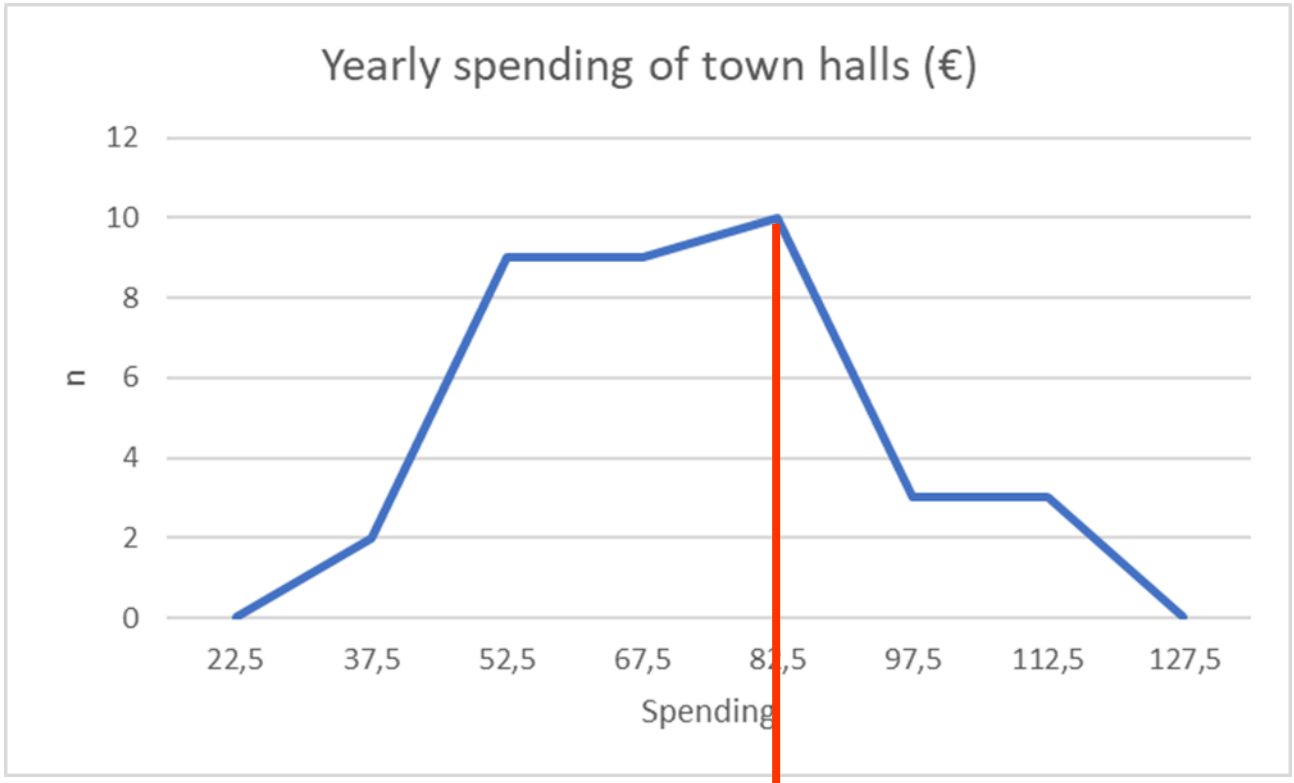
The mode with continuous data

We have a modal Interval

Yearly spending (€)	Absolute frequency
≤ 30	0
(30,45]	2
(45,60]	9
(60,75]	9
(75,90]	10
(90,105]	3
(105,120]	3
> 120	0
Total	36

What would we do if the classes were of different widths?

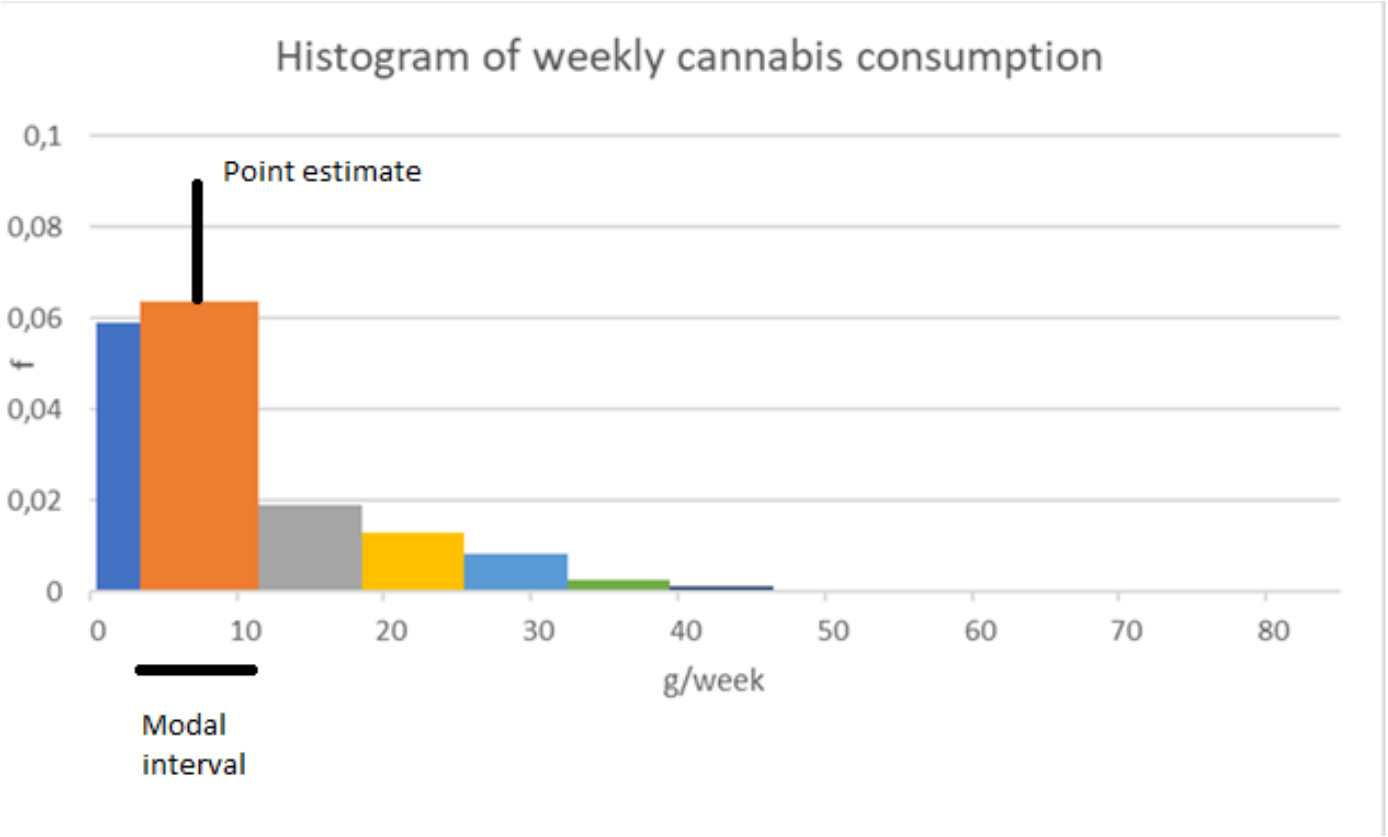
An exact value for the mode with grouped data



The mode is at the centre of the modal Interval.

What about different width intervals?

The mode with intervals of different width



For an alternative point estimate of the mode, see this page.

The median

The median is the most central observation:

5 3 11 21 7 5 2 1 3

What is the median?

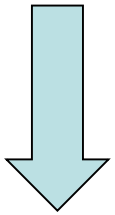
Would it make a difference if N was odd or even?

Could we calculate the median with qualitative data?

Times voted

3 3 3 4 1 2 4 5 2 3 1 1 3 8 4 1 3 4 2 5 0 0 5 4 2 1 2 3 3 2

1 4 3 2 3 5 0 6 3 1 3 5 4 1 4 1 2 4 4 3 3 0 7 2 2 1 3 4 2 2



0 0 0 0 1 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 3 3 3 3

3 3 3 3 3 3 3 3 3 3 3 4 4 4 4 4 4 4 4 4 4 5 5 5 5 5 6 7 8

The median is ½*(3+3)=3

Calculating the median from the frequency table (discrete data)

Times voted →	Absolute frequency (n)	Cumulative frequency (N)	Relative frequency (f)	Cumulative relative frequency (F)	
0	4	4	4/60 = 0,0667	0,0667	
1	10	4+10 = 14	0,1667	14/60 = 0,2333	
2	12	4+10+12 = 26	0,2000	0,4333	< 0,5
3	15	41	0,2500	0,6833	≥ 0,5
4	11	52	0,1833	0,8667	
5	5	57	0,0833	0,9500	
6	1	58	0,0167	0,9667	
7	1	59	0,0167	0,9833	
8	1	60	0,0167	1,0000	
>8	0	60	0,0000	1,0000	
Total	60		1,0000		

Median

The median with grouped data

Median interval →

Spending	n_i	N_i	f_i	F_i
≤ 30	0	0	0	0
(30,45]	2	2	0,05555556	0,05555556
(45,60]	9	11	0,25	0,30555556
(60,75]	9	20	0,25	0,55555556
(75,90]	10	30	0,27777778	0,83333333
(90,105]	3	33	0,08333333	0,91666667
(105,120]	3	36	0,08333333	1
> 120	0	36	0	1
Total	36		1	

For a point estimate of the median, see this page.

The mean

The mean (or arithmetic mean) is the average of the data.

3 3 3 4 1 2 4 5 2 3 1 1 3 8 4 1 3 4 2 5 0 0 5 4 2 1 2 3 3 2

1 4 3 2 3 5 0 6 3 1 3 5 4 1 4 1 2 4 4 3 3 0 7 2 2 1 3 4 2 2

For the case of the number of times voted, the mean is:

$(3 + 3 + \ldots + 2 + 2)/60 = 2.817$

Can we calculate the mean for qualitatve data?

Calculating the mean from the frequency table (discrete data)

Times voted (x)	Absolute frequency (n)	Relative frequency (f)	n x	f x
0	4	0,07	0	0,00
1	10	0,17	10	0,17
2	12	0,20	24	0,40
3	15	0,25	45	0,75
4	11	0,18	44	0,73
5	5	0,08	25	0,42
6	1	0,02	6	0,10
7	1	0,02	7	0,12
8	1	0,02	8	0,13
>8	0	0,00	0	0,00
Total	60	1,00	169	2,82
		169/60 =	2,82	

The mean with grouped data

Yearly spending (€)	Centre (x)	Absolute frequency (n)	Relative frequency (f)	x f
≤ 30	22,5	0	0,00	0,00
(30,45]	37,5	2	0,06	2,08
(45,60]	52,5	9	0,25	13,13
(60,75]	67,5	9	0,25	16,88
(75,90]	82,5	10	0,28	22,92
(90,105]	97,5	3	0,08	8,13
(105,120]	112,5	3	0,08	9,38
> 120	127,5	0	0,00	0,00
Total		36	1,00	72,5

This is an approximation using the centre of each Interval as the *marca de clase*.

Sensitivity of the mean to outlying data

1 2 2 3 3 3 3 4 4 5

Mode = median = mean = 3

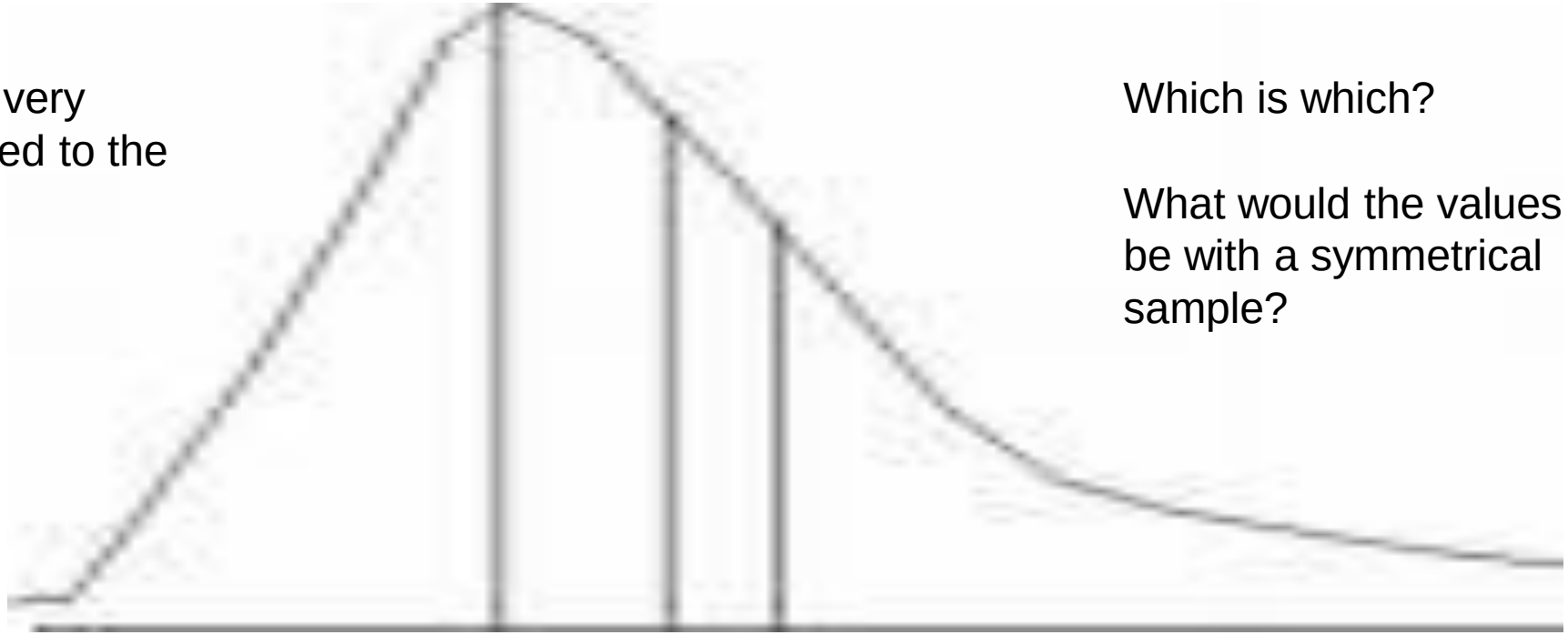
1 2 2 3 3 3 3 4 4 500

Mode = median = 3, mean = 52.5

The mean is very sensitive to extreme or outlying data!

The mode, median and mean with skewed data

Data very skewed to the right.



Which is which?

What would the values be with a symmetrical sample?

Other points of the distribution: minimum, maximum.

We are not always interested in just measures of central location.
Often it is more interesting to look at the most extreme data.

0 0 0 0 1 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 3 3 3 3

3 3 3 3 3 3 3 3 3 3 4 4 4 4 4 4 4 4 4 4 5 5 5 5 5 6 7 8

Minimum = 0
Maximum = 8

Next week, we'll see how to check if these data are "strange" or outlying.

Other points of the distribution: percentiles.

The idea of the median is to find a point to separate the data into two halves. We could generalise this by trying to separate into parts of size p and 1-p.

The point that does this is called the *p x 100% percentile*.

How can we calculate this?

5 3 11 21 7 5 2 1 3

Other points of the distribution: percentiles.

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How can we calculate this?

5 3 11 21 7 5 2 1 3

- Before we do anything, we have to order the data.

1 2 3 3 5 5 7 11 21

Other points of the distribution: percentiles.

- Now calculate r = p x (N + 1) where N is the sample size.

1 2 3 3 5 5 7 11 21

Suppose we want the 80% percentile.

r= 0.8 x (9 + 1) = 8.

What if we want the 66.67% percentile?

r= 2/3 x (9+1) = 6.67.

Other points of the distribution: percentiles.

- If r is a whole number, then the percentile is the r'th point of the ordered data set.

1 2 3 3 5 5 7 11 21

Suppose we want the 80% percentile.

r = 0.8 x (9 + 1) = 8.

The 80% percentile is equal to 11.

Other points of the distribution: percentiles.

- If r is a not a whole number, say r = s + a/b, then the percentile is:

$$x_s + \frac{a}{b}(x_{s+1} - x_s)$$

1 2 3 3 5 5 7 11 21

Suppose we want the 66.67% percentile.

r= 2/3 x (9+1) = 6.67 = 6 + 2/3.

The 66.67% percentile is equal to 5 + 2/3(7-5) = 6.33.

Other points of the distribution: quartiles.

The 25% and 75% percentiles are called the first and third quartiles.

1 2 3 3 5 5 7 11 21

r = ¼ x (9 + 1) = 2.5

The first quartile is 2 + ½ (3-2) = 2.5.

What is the third quartile?

The minimum is the 0th quartile, the median the second quartile and the maximum the 4th quartile.

Calculating percentiles from the frequency table (discrete data)

Times voted →	Absolute frequency (n)	Cumulative frequency (N)	Relative frequency (f)	Cumulative relative frequency (F)
0	4	4	4/60 = 0,0667	0,0667
1	10	4+10 = 14	0,1667	14/60 = 0,2333
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6	1	58	0,0167	0,9667
7	1	59	0,0167	0,9833
8	1	60	0,0167	1,0000
>8	0	60	0,0000	1,0000
Total	60		1,0000	

What is the 80% percentile?

Exercise

The following table represents the ages of 10 PP mayors in the Community of Madrid.

40	40	35	50	50	40	40	60	50	35

Mark the correct response among the following:

a) The mode and median are 40 and the mean is 44.
b) The mode and mean are 40 and the median 44.
c) The mean and median are 40 and the mode is 44.
d) None of the above.

Ejercicio (Test 1 2008-2009)

Certain politicians are well known for letting their speeches go on a long time. The following chart records the lengths of some of the last political speeches (in minutes) of a very well known politician (FC).

Estimate the average speech length.

Time	n_i	f_i
(0-30]	6	0.18
(30-60]	13	0.38
(60-90]	13	0.38
(90-120]	2	0.06
Total	**34**	**1**

Exercise

The table below shows the gender and age of various Ministers in the Zapatero government.

Name	Gender	Ministry	Age
Bibiana Aído	F	Igualdad	33
Carme Chacón	F	Defensa	38
Ángeles González-Sinde	F	Cultura	44
Cristina Garmendia	F	Ciencia e innovación	47
Trinidad Jiménez	F	Sanidad y Política Social	47
José Blanco	M	Fomento	48
Ángel Gabilondo	M	Educación	60
Elena Salgado	F	Economía y Hacienda	60

- What is the modal class of the gender of the ministers?
- Calculate the mode, median and mean age of the ministers.
- Estimate the third quartile of the ages of the ministers.