Chapter 2

Model fitting and checking

Based on Peña et al. (2000), Chapters 4 and 5.

CHAPTER 2. CONTENTS.

CHAPTER 2. MODEL FITTING AND CHECKING

2.1. Prediction error and the estimation criterion.

PREDICTION ERROR

- The estimation of the parameters of the time series models could be considered to be just a technical matter carried out by computers.
- The aim of this section is to explain the criteria and methods by which parameter estimates are obtained.
- Should enable you to interpret and use the results of estimation intelligently.

Prediction error

- It is true that the more important tasks to be carried by the modeler, which require understanding of models, are:
  - Model selection (identification)
  - Checking

PREDICTION ERROR

- It is, however, also important to understand:
  - The model estimation criterion
  - What features of the data it captures
  - Whether the fitted model has those properties considered important in the identification stage.

Prediction error

- Moreover, the estimation method is effectively one of nonlinear least squares requiring iterative steps.
  - As with all such methods parameter estimation may fail to provide good estimates, even though the model is appropiate for the data.
  - It can usually be avoided by providing initial estimates determined by some simple and realiable scheme.

PREDICTION ERROR

Model estimation:

- is efficient in the statistical sense of making best use of the information of the data.
- is based on assumptions about the distributional properties of the data.
- makes use of standard statistical inference procedures (Bayes and likelihood inference)

Prediction error

- Practical results are similar (with Bayes or likelihood inference) and lead to the following scheme:
  - Apply the model to predicting succesive values of the recorded time series data.
  - Choose the parameters that minimize the sum of squares of the resulting one-step-ahead prediction errors.

Prediction error

- The models we consider are all members of the class of general ARMA(p,q) models:

$$\phi(B)z_t = \theta(B)a_t$$

- The prediction errors we use in the sum of squares would then be the innovations a_t except that not all past values are known because of the finite length of the observed time series

Prediction error

- Example: consider a AR(1):

$$z_t = \phi z_{t-1} + a_t$$

- The innovation at t=1 will be unknown since z_0 is not available.

PREDICTION ERROR

- This "end effect" is generally handled in one of two ways:
  - Estimation of series values previous to the observed data (exact estimation).
  - Use of predictions errors made using only previous observed data (conditional estimation)

Prediction error

- When properly computed, that is, without further approximations, the likelihoods calculated from these two approaches are identical, althoug there will be a transient discrepancy between the estimated errors for the early part of the data.

Prediction error

Assumptions

- 1. The series being modeled is Gaussian
  - That is, the joint distribution of any sample is multivariate normal.
  - Equivalently, the errors from the linear prediction of each term on previous terms are independent normal.

PREDICTION ERROR

- 2. The observed series is stationary (any transformation needed has been carried out)
- 3. The observed sample is assumed to be from a multivariate normal distribution whose covariance structure is specified by the autocovariances implied by the model.

Prediction error

- Placing the observations in a column vector z, the covariance structure is described by the symmetric nxn matrix V with elements

$$V_{i,j} = \text{cov}(z_i, z_j) = \gamma(i-j)$$

Prediction error

- The likelihood of the observations is then derived from the joint pdf (probability density function:

$$f(z_1, z_2, \ldots, z_n) = |V|^{-(1/2)} \exp\left\{-(\tfrac{1}{2}) z' V^{-1} z\right\}$$

- where $|V|$ is the determinant of V.

PREDICTION ERROR

- For ARMA models the innovation variance is a natural scale parameter for V; thus we can write.

$$V = \sigma_a^2 M$$

- Where M depends only on the ARMA model parameters

$$\beta = (\phi_1, \phi_2, ..., \phi_p, \theta_1, \theta_2, ..., \theta_q)$$

- Then the log-likelihood is

$$-\tfrac{1}{2}\left\{\log\left|M\right| + n\log\sigma_a^2 + \frac{S}{\sigma_a^2}\right\}$$

- Where we have replaced the cuadratic form $z'M^{-1}z$ by S in recognition of the fact that, it can be expressed as a sum of squares of prediction errors.

Prediction error

- This is important because we can “concentrate out” the scale parameter σ_a^2 and maximize the log likelihood with respect to σ_a^2.
- This is done by setting $\hat{\sigma}_a^2 = S/n$

PREDICTION ERROR

- Omitting additive constants, we obtain the conventional criterion, minus twice the concentrated likelihood.

$$-2L(\beta) = n \log\left\{ |M|^{1/n} S \right\}$$

PREDICTION ERROR

- Maximizing the likelihood with respect to the remaining parameters β is therefore equivalent to minimizing either this quantity or, more simply,

$$|M|^{1/n} S$$

PREDICTION ERROR

- The factor $|M|^{1/n}$ is associated with the end effect of estimating series values previous to the observed data.

- (could be omitted in large samples).

$$|M|^{1/n} \to 1 \quad n \to \infty$$

PREDICTION ERROR

- After substitution of the parameter estimates, the criterion $-2L(\hat{\beta})$ is a useful tool for comparing different methods.

- The inverse Hessian of $-L(\hat{\beta})$ provides the standard errors of $\hat{\beta}$

PREDICTION ERROR

- For a pair of nested models the difference in –2L may be used as a statistic to test the null hypothesis that the smaller model is adequate.

- The statistic is referred to its null chisquared distribution with degrees of freedom equal to the difference in the number of parameters

CHAPTER 2. MODEL FITTING AND CHECKING

2.2. The likelihood of ARIMA models.

THE LIKELIHOOD OF ARIMA MODELS

- Examples to illustrate the various aspects of estimation.

- The emphasis is on the calculations of S and the determinant with a brief outline of how the criterion may be minimized.

The likelihood of ARIMA models

- AR(1) model (stationary)

$$w_t = \phi w_{t-1} + a_t$$

- In this case we can calculate the prediction errors a_t for $t = 2,3, \dots, n$ from the data w_t as

$$a_t = w_t - \phi w_{t-1}$$

- Because a_t, for $t = 2,3, \dots, n$ are independent of each other, and of w_1, we can use this relationship to obtain the pdf of the data as

The likelihood of ARIMA models

$$f(w_1, \ldots, w_n) = f(w_1) f(a_2) f(a_3) \ldots f(a_n)$$

$$\propto f(w_1) \sigma_a^{-(n-1)} exp - \frac{1}{2\sigma_a^2} \left\{ \sum_{t=2}^{n} a_t^2 \right\}$$

3. Two ways of proceed

- 3.1. It is possible to consider w_1 as a fixed quantity that, considered alone, does not contribute to the information need to estimate ϕ. This is to condition on the initial value

Then, we obtain the concentrated likelihood as $-2L(\phi) = (n-1)\log(S)$ where

$$S = \sum_{t=2}^{n} a_t^2$$

THE LIKELIHOOD OF ARIMA MODELS

- Minimizing S is then the standard least-squares problem of regressing $w_2, w_3, \dots, w_n$ on $w_1, w_2, \dots, w_{n-1}$.

- This lagged regression is a rather obvious way to estimate autoregressive models of all orders.

THE LIKELIHOOD OF ARIMA MODELS

3.2. In order to obtain the likelihood exactly, we need to take into account the information from w_1, which has the variance $\sigma_a^2 / (1-\phi^2)$ of the stationary series.

Then, including the term

$$f(w_1) \propto \left\{ \frac{(1-\phi^2)}{\sigma_a^2} \right\}^{1/2} exp - \frac{(1-\phi^2)}{2\sigma_a^2} w_1^2$$

and writing

$$a_t = w_t - \phi w_{t-1}$$

$$|M_n|^{1/n} S = (1-\phi^2)^{-(1/n)} \left\{ (1-\phi^2) w_1^2 + \sum_{t=2}^{n} (w_t - \phi w_{t-1}) \right\}$$

THE LIKELIHOOD OF ARIMA MODELS

- This expression requires minimization by a nonlinear least-squares procedure.
  - But the departure from linear squares is small and convergence is usually rapid.

- This method provides an estimate that necessarily satisfies the stationarity condition.

- The method readily generalizes to the AR(p) model.

THE LIKELIHOOD OF ARIMA MODELS

- The MA(1) model.

$$w_t = a_t - \theta a_{t-1}$$

1. To calculate the prediction errors from the data use recursively

$$a_t = w_t + \theta a_{t-1}$$

THE LIKELIHOOD OF ARIMA MODELS

The pdf of the data together with the assumed value of a_1 is

$$f(a_1, w_2, w_3, \dots, w_n) = f(a_1)f(a_2)\dots f(a_n)$$

$$\propto \sigma_a^{-n} exp - \frac{1}{2\sigma_a^2} S$$

where

$$S = \left\{ \sum_{t=1}^{n} a_t^2 \right\}$$

The likelihood of ARIMA models

Strategies for dealing with the Unknown a_1

a) Assume that $a_1 = 0$.

b) Backforecasting

$$\hat{a}_1 = \widehat{w}_1 = -(\theta w_2 + \theta^2 w_3 + \theta^3 w_4 + \cdots)$$

c) construct $\hat{a}_1$ as a least-squares estimate, by minimizing S above with respect to a_1.

THE LIKELIHOOD OF ARIMA MODELS

- The a'terms that contribute to S do not depend linearly on θ , so iterative nonlinear least-squares methods must be used to obtain the maximum likelihood estimates.

Chapter 2. Model fitting and checking

2.3. Properties of estimates and problems in estimation.

Consider first the estimation of the coefficient ϕ in the stationary AR(1) model

$$w_t = \phi w_{t-1} + a_t$$

by simply lagged regression of $w_2, w_3, \dots, w_n$ on $w_1, w_2, \dots, w_{n-1}$.

The results given by this regression are generally valid; the estimates and the standard errors provided by the ordinary least-squares procedure provide reliable and efficient inference on ϕ.

PROPERTIES OF ESTIMATES

Properties for AR(p) model: Anderson (1971)

A problem would be indicated if the usual 95% confidence interval for ϕ included unity.

The estimate is

$$\hat{\phi} = \frac{\sum_{t=2}^{n} w_t w_{t-1}}{\sum_{t=2}^{n} w_{t-1}^2}$$

Substituting for $w_t = \phi w_{t-1} + a_t$ gives

$$\hat{\phi} = \phi + \frac{\sum_{t=2}^{n} a_t w_{t-1}}{\sum_{t=2}^{n} w_{t-1}^2}$$

Properties of estimates

If this were standard linear regression, we would treat the values w_{t-1} of the regression as fixed quantities

$$\hat{\phi} \sim normal\left(\phi, \frac{\sigma_a^2}{\sum_{t=2}^{n} w_{t-1}^2}\right)$$

This argument cannot be applied in the context of time series regression because fixing the values of w_{t-1} would also fix the values of a_t.

$$\frac{\sum_{t=2}^{n} a_t w_{t-1}}{\sum_{t=2}^{n} w_{t-1}^2}$$

The mean and variance of the numerator are zero and $(n-1)\sigma_a^2\sigma_w^2$

Properties of estimates

The denominator in large samples may be replaced by $(n-1)\sigma_w^2$ with a small relative error.

PROPERTIES OF ESTIMATES

Using the fact that $\sigma_a^2 = (1-\phi^2)\sigma_w^2$ then

$$\hat{\phi} \sim normal\left(\phi, \frac{(1-\phi^2)}{n-1}\right)$$

Properties of estimates

For most practical purposes the two approaches of inference are very close to each other.

An important exception arise when the model is not stationary: $\phi = 1$. In this case, inference can no longer be made as if the lagged regression had the properties of simple linear regression.

In particular, the distribution of the estimate is no longer normal and distributional results developed by Dickey and Fuller (1979) must be used.

Properties of estimates

The estimation of the parameter θ in the MA(1) model

$$w_t = a_t - \theta a_{t-1}$$

is a nonlinear regression problem

The sum of squares to be minimized is obtained by the recursive regeneration of

$$a_t = w_t + \theta a_{t-1}$$

for $t = 2,3,\ldots,n$

Properties of estimates

We assume for simplicity that a_1 is set to some fixed value

The derivatives a_t^θ of the "residuals" a_t with respect to the parameter θ may also be recursively generated by differentiating the expression in the previous slide obtaining:

$$a_t^\theta = a_{t-1} + \theta a_{t-1}^\theta$$

Properties of estimates

The fact that this derivative depends also on the value of θ demostrates that the derivative is a nonlinear function of θ

We may write

$$a_t^{\theta} = b_{t-1}$$

where

$$b_t = a_t + \theta b_{t-1}$$

Properties of estimates

Taking an initial parameter estimate to be θ_0 with the corresponding residuals $a_{t,0}$ and derivatives $b_{t-1,0}$ we can produce a linear approximation

$$a_t \approx a_{t,0} + (\theta - \theta_0) b_{t-1,0}$$

which we write so as to appear like a linear regression for estimating the parameter correction $\delta\theta = (\theta - \theta_0)$

$$a_{t,0} = -\delta\theta\, b_{t-1,0} + a_t$$

Properties of estimates

giving

$$\delta\hat{\theta} = \frac{\sum_{t=2}^{n} a_{t,0} b_{t-1,0}}{\sum_{t=2}^{n} b_{t-1,0}^{2}}$$

The old parameter is then corrected by this estimate to give the new parameter $\theta_1 = \theta_0 + \delta\hat{\theta}$ and the process repeated to convergence.

Properties of estimates

The method is possible for

- noninvertible models

- MA(q) with high order q

Properties of estimates

Properties of the parameter estimates

$$b_t = a_t + \theta b_{t-1} \ (1)$$

$$a_{t,0} = -\delta\theta b_{t-1,0} + a_t \ \ (2)$$

Adding $\theta_0 b_{t-1,0}$ to both sides of (2) and using (1) we get

$$b_{t,0} = (\theta_0 - \delta\theta) b_{t-1,0} + a_t$$

Properties of estimates

This is now an autoregressive equation with parameter $\theta_0 - \delta\theta = 2\theta_0 - \theta$.

Given any value of θ_0 sufficiently close to the true value θ, this tells us that the sampling properties of $2\theta_0 - \hat{\theta}$ are the same as those of an autoregression with the same parameter.

Properties of estimates

In particular, considering θ_0 to be true

$$\hat{\theta} \sim normal\left(\theta, \frac{(1-\theta^2)}{n-1}\right)$$

PROPERTIES OF ESTIMATES

- A similar approach may be applied in the case of ARMA models.
- However, for ARMA models convergence may not take place if the initial parameter values are not close to the global minimum.

PROPERTIES OF ESTIMATES

Hannan and Rissanen (1982) method

- Useful to obtain preliminary parameter estimates in an ARMA(p,q) model.
- Uses two steps of linear regression.

PROPERTIES OF ESTIMATES

1) A relative high-order AR model fitted to the series using simple lagged regression. An authomatic order selection criterion could be used.

2) Next, the regression of w_t on $w_{t-1}, w_{t-2}, \ldots, w_{t-p}$ and $\hat{a}_{t-1}, \hat{a}_{t-2}, \ldots, \hat{a}_{t-p}$ is fitted to obtain estimates of the coefficients $\phi_1, \phi_2, \ldots, \phi_p$ and $\theta_1, \theta_2, \ldots, \theta_q$.

Properties of estimates

This is a very useful procedure but still there are problems when:

Autoregressive and moving-average parts of the model can have near-canceling factors with roots close to the boundary of stationarity and invertibility.

Chapter 2. Model fitting and checking

2.4. Checking the fitted model.

CHECKING THE FITTED MODEL

- An estimated model needs to be checked to discern whether it provides a good fit to the data.

- The estimated model may not fit the data
  - because it was not well chosen and cannot provide a good fit to the data
  - because it was poorly estimated, even though it is capalble of a good fit.

Checking the fitted model

- We will consider several aspects of model checking:
- 1. The residuals show <u>no evidence of autocorrelation</u>
  - this check requires that we look at the residuals and their statistical properties. Correlograms.

Checking the fitted model

- A formal test of whether the series is white noise uses the statistic

$$X = n\sum_{j=1}^{K} r_j^2$$

- this is based on the large sample property

$$r_k \sim \text{normal}(0, 1/n)$$

CHECKING THE FITTED MODEL

- Under the assumption that the model fits the data the large sample distribution of X is chi-squared with degrees of freedom.

- A modification to this statistic improves its properties in small samples (Ljung-Box, 1978).

CHECKING THE FITTED MODEL

- Box-Ljung statistic

$$Q_K = n(n+2)\sum_{j=1}^{K} r_j^{\,2}/(n-j)$$

- A choice must be made regarding the number K of autocorrelations included.
- Evidence of lack of fit generally comes from patterns of larger values of low lag correlations.
- Other aspect is validation by forecasting out-of-sample values.

CHECKING THE FITTED MODEL

Date: 05/17/11 Time: 12:19
Sample: 1 1000
Included observations: 1000

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.057	-0.057	3.2366	0.072
		2	-0.030	-0.033	4.1459	0.126
		3	-0.004	-0.007	4.1601	0.245
		4	0.006	0.005	4.2015	0.379
		5	-0.019	-0.019	4.5750	0.470
		6	0.020	0.018	4.9716	0.547
		7	-0.007	-0.006	5.0238	0.657
		8	-0.036	-0.036	6.3273	0.611
		9	-0.006	-0.010	6.3616	0.703
		10	0.038	0.034	7.7874	0.650
		11	-0.037	-0.033	9.1735	0.606
		12	-0.073	-0.076	14.550	0.267
		13	-0.001	-0.012	14.550	0.336
		14	0.108	0.104	26.331	0.023
		15	0.013	0.026	26.494	0.033
		16	0.004	0.008	26.509	0.047
		17	0.013	0.015	26.676	0.063
		18	-0.045	-0.040	28.776	0.051
		19	-0.023	-0.028	29.310	0.061
		20	-0.028	-0.044	30.095	0.068
		21	-0.013	-0.017	30.263	0.087
		22	-0.005	0.004	30.288	0.112
		23	0.008	0.003	30.355	0.139
		24	0.020	0.011	30.774	0.160
		25	0.025	0.035	31.425	0.175
		26	0.021	0.041	31.859	0.198
		27	0.004	0.011	31.878	0.237
		28	0.032	0.025	32.945	0.238
		29	0.026	0.025	33.631	0.253
		30	-0.038	-0.043	35.119	0.238
		31	0.026	0.013	35.792	0.254
		32	0.042	0.046	37.575	0.229
		33	0.024	0.010	38.181	0.246

CHECKING THE FITTED MODEL

- 2. The residuals show no evidence of nonlinearity. Maravall(1983).
  - If z_t is linear $$\rho_k\left(z_t^2\right)=\left[\rho_k\left(z_t\right)\right]^2$$
  - If we take the square residuals and calculate their autocorrelations, these (under normality) must be less or equal to those of the residuals.

CHECKING THE FITTED MODEL

- The test consists on looking for significative values in the correlogram of the square residuals.

CHECKING THE FITTED MODEL

- 3. The residuals have zero mean. The estimated residuals of an ARMA model are subject to the restriction

$$\sum \hat{a}_t = 0$$

- (note: the restriction apply if we estimate an AR(p) conditionally)

- The statistic to contrast the null hypothesis of zero mean, if we have no observations and $p + q$ parameters is:

$$Z = \frac{\bar{a}}{\hat{S}_a/\sqrt{n}} \sim N(0, 1)$$

where

$$\bar{a} = \frac{\sum_{t=1}^{n} a_t}{n},$$

$$\hat{S}_a = \frac{\sum_{t=1}^{n} (a_t - \bar{a})^2}{(n - p - q)}$$

CHECKING THE FITTED MODEL

- The test must be applied once that the no-autocorrelation property has been verified to ensure that $\hat{S}_a$ is a reasonable estimate of the variance .

Checking the fitted model

- 4. Constant variance The stability of the variance can be checked by graphical inspection of the residuals over time.

- If any doubt, the sample can be subdivided into 3 or 4 parts and apply a likelihood ratio test.

- Likelihood ratio test.

  - 1. Divide the n residuals into k groups $\left(\sum n_i = n\right)$
  - 2. Lets σ_i^2 the estimation of the group i variance and the MV estimator of the variance for all residuals σ^2
  - 3. Then

$$H_0 : \hat{a}_t \sim N(0, \sigma^2)$$

$$H_1 : \hat{a}_t \sim N(0, \sigma_i^2)$$

4. The logarithm of the likelihood ratio is then:

$$2L(.) = n log \sigma^2 - \sum_{n_i} n_i log \sigma_i^2 \sim \chi^2_{n-p-q}$$

CHECKING THE FITTED MODEL

- 5. Normality.

$$JB = n\frac{sk^2}{6} + \frac{(k-3)^2}{24} \sim \chi^2_2$$

Where s is the asymmetry and k kurtosis.

- 6. Search for outliers: chapter 4.

Respecification of the fitted model.

- In the diagnosis of an estimated ARMA model, it is important to consider the residuals as a new time series and study its dynamic structure.

Overfitting

- Suppose two ARMA models that explain the data equally well:
- model 1: $\phi(B)z_t = \theta(B)a_t$
- model 2: $\phi^*(B)z_t = \theta^*(B)a_t$
- where,

$$\phi^*(B) = \phi(B)(1 - \phi B)$$

$$\theta^*(B) = \theta(B)(1 - \theta B)$$

Checking the fitted model

- If model 1 explains the data correctly but we estimate the overfitted model 2, all the estimated parameters will be significative.

- The overfitting can only be detected if the AR and MA polynomials are factorized.

CHECKING THE FITTED MODEL

- It is always convenient to obtain the roots of the AR and MA polynomials in mixed models and check that there are not common factors.

- Special case. Cancelation of unit roots. For instance, in a MA (1) model.

CHECKING THE FITTED MODEL

Analysis of the degree of differencing.

- In small samples, it is often the case that the order of differencing to achieve stationarity it is not clear.

- We can have two models, with different d that explain the data equally well.

- Suppose two models:
- Model 1:

$$(1-0.8B)z_t = a_t$$

- Model 2 :

$$(1-B)z_t = (1-0.2B)a_t$$

Checking the fitted model

- These models are very difficult to distinguish with samples of less than 200 observations.
- If we do not take into account terms less than 0.01, model 2 can be rewritten as,

$$(1-0.2B)^{-1}\nabla z_t \approx (1+0.2B+0.04B^2)\nabla z_t$$
$$= (1-0.8B-0.16B^2)z_t = a_t$$

which is very similar to model 1.

CHECKING THE FITTED MODEL

- Still, the distinction between models 1 and 2 is very important for interpretation of results and prediction of future values.
  - Model 1: the series is stationary and tends to go back to the mean value. The prediction is therefore, the mean.
  - Model 2: the series is non stationary and, therefore, does not have a fixed mean. The prediction is then the last observation.

CHECKING THE FITTED MODEL

- <u>Overdifferencing</u>
  - small loose in efficiency in the estimation. Still the parameters are unbiases and consistent
  - the variance of the prediction errors are greater.
- <u>Subdifferencing</u>
  - the model is not robust and cannot adapt to future values.
  - The prediction errors grow with the horizon and the variances are underestimated.

CHECKING THE FITTED MODEL

- Augmented Dickey-Fuller test.
- Suppose we have differenced our data d times and want to know whether it is neccesary to take another difference.

CHECKING THE FITTED MODEL

- The test consist on estimating the regression

$$\nabla z_t = c + \delta z_{t-1} + \sum_{i=1}^{p} \beta_i \nabla z_{t-i} + a_t$$

and checking for the significance of δ using the statistic

$$t = \frac{\hat{\delta}}{std(\hat{\delta})} \sim DF$$

CHECKING THE FITTED MODEL

- For a significance level of 0.05,

$$DF = \begin{cases} -3 & n = 25 \\ -2.89 & n = 100 \\ -2.87 & n = 500 \end{cases}$$

- Not robust to the presence of outliers or breaking trends.

CHECKING THE FITTED MODEL

Other integration tests

- Phillips-Perron test: more robust than DF.
- Use of AIC and BIC criteria (like TRAMO)

Seasonal unit root tests

In many cases we have to decide on the presence of regular and seasonal roots for a given series. The Osborn test allows us to make that decision:

$$\nabla\nabla_{12} z_t = c + \sum_{s=1}^{11} \delta_s D_{st} + \beta_1 \nabla_{12} z_{t-1} + \beta_2 \nabla z_{t-12} + \sum_{i=1}^{l} \phi_i \nabla\nabla_{12} \mathrm{z}_{\mathrm{t-i}} + \varepsilon_{\mathrm{t}}$$

CHECKING THE FITTED MODEL

The I(1,1) null hypothesis, $\beta_1 = \beta_2 = 0$, tested by an F-type statistic.

- Alternative 1) stationarity is obtained after first differences: $\beta_1 = 0$ and $\beta_2 < 0$.

- Alternative 2) the process requires annual differencing to be stationary: $\beta_1 < 0$ and $\beta_2 = 0$.

CHECKING THE FITTED MODEL

Following OCSB, separate t-type statistics for $\beta_1 = 0$ and $\beta_2 = 0$ can be used to distinguish between the two possible alternatives.

5% critical values for $\beta_1 = 0$, $\beta_2 = 0$ and the joint F test for $\beta_1 = \beta_2 = 0$ are - 2.10, -5.67 y 18.34 respectively

1% critical values -2.78 -6.37 y 22.93.

AUTOMATIC VERSUS MANUAL ANALYSIS.

- Increased analyst's productivity.
  - For accomplished analysts, allows them to invest time on troublesome data.
  - For non-experts, allows them to use a powerful methodology that could not use otherwise.
- Objective procedure.
- More appropiate when many series have to been analyzed.