Chapter IV: Random Variables - Exercises

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GROUP 89 - COMPUTER ENGINEERING

2010-2011

Exercise

The time to repair a machine expressed in hour is a random variable with distribution function given by:

$$F(x) = \begin{cases} 0, & \text{if } \quad x \leq 0; \\ x/2, & \text{if } 0 \leq x \leq 1; \\ 1/2, & \text{if } 1 \leq x \leq 2; \\ x/4, & \text{if } 2 \leq x \leq 4; \\ 1, & \text{if } 4 \leq x. \end{cases}$$

a) Draw the distribution function.
b) Compute the density function.
c) If the repairing time is *more than* 1 *hour*, what is the probability that it is *greater than* 3.5 *hours*?

Exercise

An experiment consists of tossing 4 fair coins. Compute the probability and distribution functions for the following random variables:

1. Number of heads before the first tail.
2. Number of heads after the first tail
(in the case of no tail the number of heads is considered to be 0)
3. Number of heads less the number of tails.

Exercise

A random variable X that assumes values in the interval $[0, 1]$ has density function

$$f(x) = a + b\,x,$$

where a and b are to constant to be determined.

a) Compute a and b such that $f(x)$ a density function with probability density in $X = 1$ being the double of the probability density in $X = 0$.

b) Compute the quartiles of the random variable X

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SOLUTION:

a) $a = 2/3$, $b = 2/3$;

b) $Q_1 = \frac{1}{2}\left(\sqrt{7} - 2\right) = 0.3229$,

$Q_2 = M = \frac{1}{2}\left(\sqrt{10} - 2\right) 0.5811$ and $Q_3 = \frac{1}{2}\left(\sqrt{13} - 2\right) = 0.8028$.

Exercise

Let X be a random variable with density function

$$f(x) = \begin{cases} k(1-x^2), & \text{if } 0 \le x \le 1; \\ 0, & \text{otherwise.} \end{cases}$$

Find k, together with the expectation and the variance of the random variable Y defined as $Y = 3X - 1$.

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SOLUTION:

- $k = 3/2$
- $\mathbb{E}[Y] = 1/8$
- $\mathbb{V}\mathrm{ar}[Y] = 171/320$

Exercise

Let Z be a *Gamma* random variable with parameters λ and $b > 0$ (we normally use the notation $Gamma(\lambda, b)$ to refer to this probability model) whose density function $f(x)$ is defined as

$$f(x) = \begin{cases} \frac{\lambda^b}{\Gamma(b)} e^{-\lambda x} x^{b-1}, & x > 0; \\ 0, & \text{otherwise.} \end{cases}$$

being $\Gamma(p) = \int_0^\infty e^{-x} x^{p-1}\, dx$, with $p > 0$.

a) Compute $\mathbb{E}[Z]$

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Solution:

a) b/λ;

b) b/λ^2.

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We want to insure a car of 12000 euro. The probability that a car is involved in an accident during a year is 0.15 in which case the amount of damage is

- 20% of its value with probability 0.8
- 60% of its value with probability 0.12
- 100% of its value with probability 0.08

Find the first annual premium the insurer must charge to have the expected cost of the company equal to 0.

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SOLUTION:

561.6 euro