Chapter VIII: Statistical Process Control - Exercises

Bernardo D'Auria

Statistics Department

Universidad Carlos III de Madrid

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Exercise

An industrial process produces a speed sensor for an ABS electronic control. The sensor specifications require that the variable X = "impedance" is 30 kohms ± 10 kohms. To perform the statistical control of this process 6 sample sensors are collected every half hour. With the collected data, a total of 30 samples, the following information were obtained: $\bar{x} = 30.11$ and $\hat{s}_{\bar{x}} = 2.00$.

a) Construct control charts to monitor that the population mean and variance remain constant.

b) When there is a change in the mean of $+3$ kohms, what is the probability of detecting it in the next sample after the mismatch?

Solution: a)

The control chart for the mean is

$$\begin{array}{rcl|rcl} UCL & = & \mu + 3\frac{\sigma}{\sqrt{n}} & UCL & = & \bar{\bar{x}} + 3\frac{\hat{s}_{\bar{x}}}{c_4\sqrt{n}} = 30.11 + 3\frac{2.00}{0.9515\sqrt{6}} = 32.69 \\ CL & = & \mu & CL & = & \bar{\bar{x}} = 30.11 \\ LCL & = & \mu - 3\frac{\sigma}{\sqrt{n}} & LCL & = & \bar{\bar{x}} + 3\frac{\hat{s}_{\bar{x}}}{c_4\sqrt{n}} = 30.11 - 3\frac{2.00}{0.9515\sqrt{6}} = 27.54 \end{array}$$

where μ has been estimated by the total sample mean $\bar{x} = \bar{\bar{x}} = 30.11$ and σ has been estimated by the mean of the sample standard deviations$\hat{s}_{\bar{x}}$ as $\hat{\sigma} = \hat{s}_{\bar{x}}/c_4$ where for $n = 6$ we have that $c_4 = 0.9515$.

The control chart for the standard deviations is

$$UCL = B_4\hat{s}_{\bar{x}} = 1.970 \times 2.00 = 3.94$$
$$CL = \hat{s}_{\bar{x}} = 2.00$$
$$LCL = B_3\hat{s}_{\bar{x}} = 0.030 \times 2.00 = 0.06$$

where we used the fact that, when $n = 6$, $B_3 = 0.030$ and $B_4 = 1.970$.

SOLUTION: b)

The data suggest that the sample means are Normal distributed with the following parameters

$$\bar{x} \sim N\left(30.11, \left(\frac{2.00}{\sqrt{6}}\right)^2\right)$$

If we have a change in the mean of $+3$ kohms than the new sample means ($\bar{y}$) will have (estimated) distribution

$$\bar{y} \sim N\left(33.11, 0.857^2\right).$$

However the control limits do not change and stay $UCL = 32.69$ and $LCL = 27.54$. We will detect the change in the sample mean if its graph will move out of the limit region, i.e.

$$\begin{aligned}
\Pr(\text{alarm}) &= 1 - \Pr(27.54 \leq \bar{y} \leq 32.69) \\
&= 1 - \Pr\left(\frac{27.54 - 33.11}{0.857} \leq z \leq \frac{32.69 - 33.11}{0.857}\right) \\
&= 1 - \Pr(-6.50 \leq z \leq -0.49) = 1 - 0.312 = 68.8\%
\end{aligned}$$

Exercise

In a foundry a statistical process control takes place by measuring the hardness (X) of ingots. It is assumed that X is normally distributed with mean μ and standard deviation σ, both known. If samples of size n are taken.

a) Establish the limits and the centerline of the control chart for average hardness.

b) Calculate how many ingots (n) we must use if we want to detect changes in the average hardness of magnitude $+2\sigma$, with probability 0.8, assuming constant the variance.

SOLUTION:

a) Since $\bar{x} \sim N(\mu, \sigma/\sqrt{n})$ the control limits are $\mu \pm 3\sigma/\sqrt{n}$ and the center line is the mean μ

b) If the mean has changed by an amount $+2\sigma$ the sample means ($\bar{y}$) will be distributed as

$$\bar{y} \sim N(\mu + 2\sigma, \sigma/\sqrt{n})$$

and so we will get an alarm if its graph will go out of the controlled region $\mu \pm 3\sigma/\sqrt{n}$ taht stays unchanged. So we can compute the alarm probability as

$$\begin{aligned}
\Pr(\text{alarm}) &= 1 - \Pr\left(\mu - 3\sigma/\sqrt{n} \leq \bar{y} \leq \mu + 3\sigma/\sqrt{n}\right) \\
&= 1 - \Pr\left(\frac{\mu - 3\sigma/\sqrt{n} - (\mu + 2\sigma)}{\sigma/\sqrt{n}} \leq z \leq \frac{\mu + 3\sigma/\sqrt{n} - (\mu + 2\sigma)}{\sigma/\sqrt{n}}\right) \\
&= 1 - \Pr\left(-3 - 2\sqrt{n} \leq z \leq -3 + 2\sqrt{n}\right)
\end{aligned}$$

Since we want $\Pr(\text{alarm}) = 80\%$ we get that

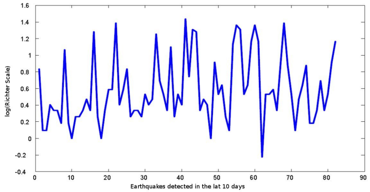
$$\Pr\left(-3 - 2\sqrt{n} \leq z \leq -3 + 2\sqrt{n}\right) \approx \Pr\left(z \leq -3 + 2\sqrt{n}\right) = 20\%$$

and hence $-3 + 2\sqrt{n} = -z_{0.2} = -0.84$ that gives

$$n > \left(\frac{3 - 0.84}{2}\right)^2 = 3.60 \Rightarrow n = 4$$

Exercise

The following graph shows the time series of the magnitudes of the last 75 earthquakes detected in the peninsula, measured by the Richter scale. We work in logarithms to provide a variable more similar to normal than the one obtained by looking directly at the values in the original scale.



It has been thought to make a control chart in this series $X = log(Magnitude)$ to detect if at any time seismic conditions are going through some period away from stationarity, and to help predict increases in seismic activity. It has been thought to make a graph of averages with subgroups of size $n = 3$ of the variable $X = log(Magnitude)$. The sample mean is $\bar{x} = 0.587$ and the sample standard deviation is $\hat{s}_{\bar{x}} = 0.331$. Compute the control limits of mean graph would be obtained with the values provided in the preceding paragraph.

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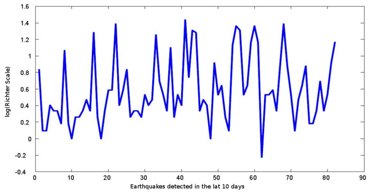

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Solution:

The control chart for the mean is: $UCL = 1.2339$; $CenterLine = 0.587$; $LCL = -5.9929 \cdot 10^2$.