Chapter VII: Large-Sample Inference - Exercises

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GROUP 89 - COMPUTER ENGINEERING

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Exercise

I3 Y SEPT1997

The city of León is considering the possibility of erecting a statue in memory of a famous philanthropist in the city.
A random sample of 610 Leoń's taxpayers reveals that the 50.7% of the people is against this idea.
Find a confidence interval of 99% for the proportion of the population that disagrees with erecting the statue.

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SOLUTION:

$$\frac{\alpha}{2} = 0.005; \quad z_{0.005} = 2.5758$$

$$\hat{p} = 0.507; \quad \hat{q} = 1 - \hat{p} = 0.493;$$

$$IC99\% : p \in \left[\hat{p} \pm z_{0.005}\sqrt{\frac{\hat{p}\,\hat{q}}{n}}\right] = \left[0.507 \pm 2.5758\sqrt{\frac{0.507 \cdot 0.493}{610}}\right] = [0.507 \pm 0.052]$$

$$IC99\% : p \in [0.455, 0.559]$$

Exercise

I5 Y JUNE2002

Let X be the unit consumption of certain material in a production process (milligrams per unit of product obtained).
We known that X is normal with mean μ and standard deviation $\sigma = 20$ *mg*.
We take a random sample of 25 observations and we compute a sample mean consumption of $\bar{x} = 120$ *mg*.

a) From this sample information, estimate a 95% confidence interval for the mean consumption of this product.

b) What sample size would be necessary to reduce the range of the 95% confidence interval to 10*mg*:
(interval range = difference between the ends)

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SOLUTION:

a) $[112.16, 127.84]$;

b) $n \approx 62$.

Exercise

I11

The file *Resistencias1KO.sf3* contains data from a sample of resistors with nominal value 1000*Ohms*. The measurements belong to two types of resistance: the resistances of *golden* band and the ones of *brown* band. The brown band resistances must have a value nearest to the nominal one than the gold band resistances. Compute:

a) A statistical test with significance 5% that the brown band resistances come from a population with nominal value of 1000*Ohms*.

b) A 95% confidence interval for their mean.

c) Repeat the analyses above for the golden band resistances and compare the two results.

In the following it is shown the summary statistics for both kind of resistances produced by STATGRAPHICS.

	Golden Band	Brown Band
Count	50	50
Average	989.02	999.14
Median	988.5	999.0
Variance	267.612	48.8576
Standard deviation	16.3588	6.98982
Standard error	2.31349	0.988509
Minimum	946.0	983.0
Maximum	1025.0	1016.0

SOLUTION:

a) We cannot reject with significance $\alpha = 0.05$ the Null Hypothesis that the population has mean$\mu = 1000$;

b) $(997.2, 1001.08)$;

c) $(984.49, 993.55)$.

Exercise

I7

A store chain plans to open a new store in a central pedestrian zone. The final decision would depend on the pedestrian traffic moving through these streets. It is known that the store to be successful requires that a pedestrian flow of at least 2000 pedestrians per day goes through the street during business hours.

To check whether this condition holds an experiment is performed on two streets in the area. The experiment consists of counting the number of pedestrians who, during business hours, passes in these two main streets. The experiment lasted for *one week*. In the *street-1* 12600 pedestrians passed during that week, while in the *street-2* 12880 pedestrians did.

Assuming that the number of people that passes every day a street is a Poisson random variable, compute:

a) A 95% confidence interval for the parameter of the Poisson distribution "number of daily pedestrian" traveling through *street-1* and *street-2* [June '02].

b) test, with $\alpha = 0.01$, if both streets are adequate to open a new store. That is, say if these streets have an average pedestrian flow of at least 2000 pedestrians per day.

SOLUTION:

a) $\lambda_1 \in (1768, 1831)$; $\lambda_2 \in (1808, 1872)$;

b) There is no evidence, with the chosen significance, that both streets have the required flow.

Exercise

I4 Y JUN1998

In a survey to 10000 High School students the question about their weekly consumption of soft drinks revealed a mean of 5 bottles, with a standard deviation of 2. Find the 95% confidence interval for the average consumption of the entire population of high school students.

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I4 Y JUN1998

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SOLUTION:

$$[4.96, 5.04]$$

Exercise

I2 Y SEPT1999

In order to determine the proportion of people who own a car in a particular province was performed a simple random sampling, so that among the 100 respondents, 30 of them had a car.

a) Find a confidence interval for the proportion of the population to own a car ($\alpha = 0.05$).

b) If you wished to estimate the proportion with a precision of 0.02 and 95% of confidence, how many people should be surveyed?

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SOLUTION:

a) $[0.2101, 0.3898]$;

b) $n \approx 2017$.

Exercise

I6

A web server of a computer network receives on average 5 virus attack attempts per 1000 connection requests. The organization that owns the server makes an advertising campaign to spread its website address. Next day of the campaign, the server receives 830 requests for connection, which 10 are virus attacks. There is enough evidence to say that the advertising campaign has increased the virus? (Use $\alpha = 0.05$)

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SOLUTION:

It seems that the advertising campaign has increased the number of virus attacks.

Exercise

I10

The length in cm, X, of the parts manufactured by a machine is a random variable with density function

$$f(x) = \lambda^2 x e^{-\lambda\, x}, \quad x \geq 0\,.$$

Compute:

a) What is the estimator for λ using the *method of moments*?

Taken a data sample of size 100 we get that the average length is $\bar{x} = 52$ cm.

b) Construct a 95% confidence interval for the parameter λ.

c) Contrast with a significance level $\alpha = 0.05$ that $\lambda = 0.05$.

Use the function

$$\Gamma(p) = \int_0^\infty e^{-x}\, x^{p-1}\, dx, \quad p > 0$$

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Solution:

a) $\hat{\lambda} = 2/\bar{x}$,

b) $[0.03267, 0.04333]$,

c) We reject the hypothesis with $\alpha = 0.05$.

Exercise

I12

An electric company is interested in evaluating a site for installing a wind farm. Given the technology and costs to manage this company, it has been established that there is interest in a location only if the speed of wind blowing at that position exceeds 6 *m/s* for more than 30% of the times.
An anemometer is installed at that location to record wind speeds. After 1000 *hours* of recording for 320*hours* the wind speeds resulted more than 6 *m/s*.
Analyze the suitability of the site by a hypothesis test ($\alpha = 0.05$).
Note: In order to protect the interests of the company, the test will be done in such a way that the location is chosen only if there is enough evidence in favor of the installation.

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SOLUTION:

With a significance level of 5%, in that location is not worth a wind farm.