Chapter VI: Introduction to Statistical Inference - Exercises

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GROUP 89 - COMPUTER ENGINEERING

2010-2011

Exercise

The lifetime of a system up to a failure can be modeled by an exponential random variable $T \sim Exp(\lambda)$. During some time we take notes of the duration of the system between failures and we get the following data measured in hours: $18, 94, 22, 143, 114$. Estimate the parameter λ of the exponential distribution using the Moments' Method.

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Solution:

Since T is an exponential random variable we have $\lambda = \frac{1}{\mathbb{E}[T]}$.

Therefore the Moments' Method we get $\hat{\lambda} = \frac{1}{\bar{T}}$. We can compute $\bar{T} = \frac{18+94+22+143+114}{5} = 78.2$ hours and therefore an estimation for λ is given by $\hat{\lambda} = 178.2 = 0.013$ fault/hour.

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Say if the following sentences are true or false

a) For the Central Limit Theorem we know that μ is a Normal random variable.

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g) To lower the variance of $\bar{X}$ to the half of it, we have to take at least 100 samples.

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The random variable X_1 is distributed according to a Normal $N(\mu, \sigma^2)$, and the random variable X_2, independent form the previous one, is distributed as a $N(2\mu, 3\sigma^2)$. We take a sample of size n_1 of the former variable and a sample of size n_2 of the latter.

To estimate the value of the parameter μ we use the estimator $\hat{\mu} = a\,\bar{x}_1 + b\,\bar{x}_2$ where a and b are constants and $\bar{x}_1$ and $\bar{x}_2$ are the sample means of the two samples.

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SOLUTION:

The distributions for the sample means are $\bar{x}_1 \sim N(\mu, \frac{\sigma^2}{n_1})$ and $\bar{x}_2 \sim N(2\mu, \frac{3\sigma^2}{n_1})$.
$\mathbb{E}[\hat{\mu}] = \mathbb{E}[a\,\bar{x}_1 + b\,\bar{x}_2] = a\mathbb{E}[\bar{x}_1] + b\mathbb{E}[\bar{x}_2] = (a + 2b)\mu$ and to have a null bias the two constants have to satisfy the relation $a + 2b = 1$.