Chapter V: Probability Models - Exercises

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GROUP 89 - COMPUTER ENGINEERING

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Exercise

Let X be a *Bernoulli* random variable that takes value 1 with probability 0.01. We take a sample made of n experiment realizations. Compute the minimum value of n such that the probability to get at least once the value 1 is greater or equal to 0.95.

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SOLUTION:

$$n \geq \frac{\ln 0.05}{\ln 0.99} = 298.073$$

Exercise

A parking has 2 entrances.
Cars arrive to the entrance I according to a *Poisson* random variable with parameter 3 cars per *hour* and to the entrance II with parameter 2 cars per *half a hour*.
If the number of cars that arrive at each entrance is independent of each other,
what is the probability that in a hour 3 cars arrive to the parking?

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SOLUTION:

$$\frac{7^3}{3!}e^{-3} = 0.05$$

Exercise

In a university we know that 75% of the graduate students get a job during the first 12 months after their graduation. We randomly choose 8 graduate students. Find:

a) The probability that at least 6 got a job during the first 12 months after graduating.

b) The probability that at most 6 of them got a job during the first 12 months after graduating.

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Solution:

X={Number of graduate students that have a job in the first 12 months};

$$X \sim \mathrm{B}(8, .75).$$

a) $\Pr\{X \geq 6\} = 67.85\%$.

b) $\Pr\{X \leq 6\} = 1 - \Pr\{X \geq 6\} + \Pr\{X = 6\} = 63.29\%$.

Exercise

Assume that on average a person gets 3 colds during winter and their number is distributed according to a Poisson.

a) Find the probability that a person in a winter gets *at least* 1 cold.

b) Find the probability that in a set of 5 randomly chosen people, only 4 of them have got 2 colds during the winter.

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SOLUTION:

a) $\Pr(X \geq 1) = 0.95$

b) $\Pr(Y = 4) = 0.009$

Exercise

The phone calls received in a house arrive according to a *Poisson process* with parameter $\lambda = 2\text{hour}^{-1}$.

a) What is the probability that the phone rings at least once during 1 hour?

b) How much time can we take a shower having the probability to interrupt it to receive a phone call at most equal to 50%?

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SOLUTION:

a) 0.864;

b) $t \approx 21$ minutes.

Exercise

Two male friends of same age speak about their respective heights.
The shorter of the two is 160 cm tall and he says that he fills shorter than most of people of his generation and that only 1 over 10 boys is shorter than him.
The taller of the two is 175 cm tall and he says that he does not fill higher than the average since he finds that the number of people taller than him is equal to the ones shorter than him.
If we assume that the height of people of same age of theirs is distributed according to a *Normal* distribution, find:

a) The mean μ and variance σ^2 starting from the information given by the two friends.

b) How many boys will be taller than 190 cm?

c) If we randomly choose 5 boys, what is the probability that most of them are shorter than 160 cm?

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Solution:

a) $\mu = 175$cm, $\sigma^2 = 137$ cm^2;

b) 10%

c) 0.85%

Exercise

A test is made of 50 question, each having 4 different choices as answer.
A student is able to identify and eliminate one of them as incorrect and choose uniformly one of the remaining three.
S/he passes the exam if s/he is able to correctly answer at least 26 questions.

a) What is the probability that the student passes the exam?

b) What is the probability that the student passes the test if s/he is able to correctly eliminate 2 over the 4 choices?

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SOLUTION:

a) $\sum_{k=26}^{50} \binom{50}{k} \left(\frac{1}{3}\right)^k \left(\frac{2}{3}\right)^{50-k} = 0.0040 = \Pr(Z \geq 2.65)$

b) $\left(\frac{1}{2}\right)^{50} \sum_{k=26}^{50} \binom{50}{k} = 0.4439 \approx 0.4443 = \Pr(Z \geq 0.14)$

Exercise

The lifetime of an electronic component is exponentially distributed with mean 10000 hours.

a) Compute the probability that a component already working for 20000 hours, has a lifetime of at least 21000 hours.

b) Compare last result with the probability that the lifetime of new device is between 0 and 1000 hours. Justify the result.

c) After installing a series of 4 components, compute the probability that the whole devise is still working after 10000 hours.

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SOLUTION:

a) $\Pr(T > 21000 | T > 20000) = \frac{e^{-2.1}}{e^{-2}} = e^{-0.1} = 0.905 = \Pr(T > 1000)$;

b) $\Pr(T > 21000 | T > 20000) = \Pr(T > 1000)$ for the memoryless property. Then $\Pr(0 \leq T \leq 1000) = 1 - \Pr(T > 1000) = 0.095$

c) $\Pr(\text{It works}) = \Pr(T > 10000)^4 = 0.018$.

Exercise

Let X be a random variable (r.v.) that models the "*number of hours employed to complete some activity*" and assume it is distributed according to the following density function

$$f(x) = \begin{cases} \frac{1}{4}(x+1) & 0 < x < 2; \\ 0 & \text{otherwise.} \end{cases}$$

a) Compute the probability that the time spent is greater than 1.5 hours.

b) If 10 tasks distributed as the r.v. X are executed, compute the probability that exactly 3 of them require more than 1.5 hours to be completed.

Exercise Ex. FEB 2004 Ing. Tel. (C1a)

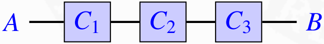
We want to analyze a communication channel. When the transmitter send a digit 1, the receiver receives a random variable that is Normal distributed with mean 1 and variance 0.5. When the transmitter send a digit 0, the receiver receives a random variable that is Normal distributed with mean 2 and variance 0.5. Define $P(1)$ as the probability for the transmitter to send the digit 1.

a) If $P(1) = 0.75$, and assuming that the receiver has got a signal above the level 2, what is the probability that it was sent the digit 1?

Exercise $1/2$

A system is made of 3 components connected in series and it fails to work when at least one of its components fails.
The components C_1 and C_2 have lifetime respectively T_1 and T_2 that are independent and identically distributed according to an Exponential with mean 28000 hours.

The lifetime of the component C_3, say T_3, is distributed as $N(3000, 200)$, and it is independent of the r.v. T_1 and T_2.



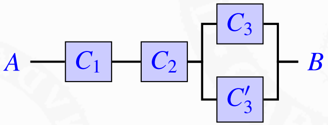
a) Compute the probability that the component C_1 works at least 3000 hours.

b) Compute the probability that T_1 is greater than 6000 hours, knowing that the component $C1$ is already working for 3000 hours.

c) Compute the probability that the whole system will work for more than 3000 hours.

Exercise 2/2

d) To make the component C_3 more reliable a new identical component, C_3', is installed in parallel. Compute the probability that the whole system will work for more than 3000 hours.



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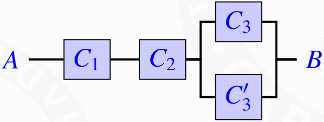


SOLUTION:

a) $\Pr(T_1 > 3000) = e^{-3/28} = 0.898$

b) $\Pr(T_1 > 6000 | T > 3000) = e^{-3/28} = 0.898$

c) $\Pr(T_S > 3000) = \frac{1}{2} e^{-6/28} = 0.4036$

d) $\Pr(T_S' > 3000) = \frac{3}{4} e^{-6/28} = 0.6053$

Exercise Exam Feb'05 - $1/2$

The electronic microchips are built starting from a silicon wafer and they are very sensible to surface failures. We define as *fatal failure* any defective that implies the lost of a working chip.
The number of *fatal failures* per $100\,mm^2$ of silicon wafer is a random variable with a mean equal to 0.1.

a) What is the probability that in a chip with a $20 \times 20\,mm^2$ surface there is *more than one fatal failure*?

b) If we randomly take 25 different chips with a $10 \times 10\,mm^2$ surface, what is the probability that *more than* 22 of them have *no fatal failures*?

Exercise Exam Feb'05 - $2/2$

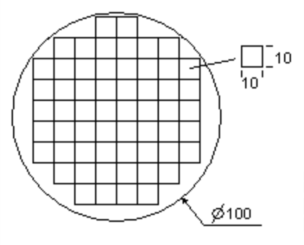
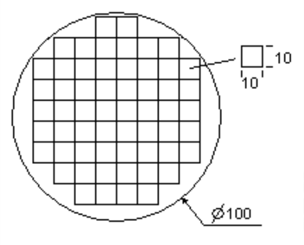


Figure: 58 chips de $10 \times 10\, mm^2$

c) If we want to create chips with a $20 \times 20\, mm^2$ surface using a wafer of diameter $100\, mm$,

What is the probability to find more than 12 *fatal failures* on the total surface of 4 wafers?

Exercise Exam Feb'05 - $2/2$



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SOLUTION:

a) 0.0615

b) 0.537

c) ≈ 0.9898