Chapter IV: Random Variables - Exercises

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GROUP 89 - COMPUTER ENGINEERING

October 7th, 2010

Exercise

Let X be a random variable with density function

$$f(x) = a\,x^{-\frac{1}{2}},$$

defined in the interval $0 < x \leq 1$.

a) Find the value of a such that $f(x)$ is a density function.

Let Y be an additional random variable with mean $\mathbb{E}[Y] = 2/3$.

b) Compute the expectation of the random variable $Z = X + Y$. (*June '05*)

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Let Y be an additional random variable with mean $\mathbb{E}[Y] = 2/3$.

b) Compute the expectation of the random variable $Z = X + Y$. (*June '05*)

SOLUTION:

a) $a = 1/2$;

b) $\mathbb{E}[Z] = 1$.

Exercise

Let X be a discrete random variable that assumes the values $x = 1, 2, 3, 4, 5$ and let c be a deterministic constant.

Which of the following functions can be considered as probability function of the variable X?

a) $p(x) = \frac{c}{x-2}$
b) $p(x) = c(x+1)$
c) $p(x) = x^2 - 3$
d) $p(x) = c - x$

Exercise

A telecommunication system is made of a transmitter that sends to a receiver the symbols 0, 1 and 2 throw a communication channel. Due to some noise, the channel can change the transmitted symbol.

The probability that the receiver receives the correct symbol is 0.8.

In case the received symbol is different from the transmitted one, it will equiprobably be equal to any of the other two symbols.

The transmission probabilities of the symbols 0, 1 and 2 are 0.5, 0.3 and 0.2 respectively.

If we call X the random variable that assumes the value of the received symbol, find:

a) $\mathbb{E}[X]$

b) $\mathbb{V}\mathrm{ar}[X]$

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b) $\mathbb{V}\text{ar}[X]$

SOLUTION:

a) $\mathbb{E}[X] = 0.79$

b) $\mathbb{V}\text{ar}[X] = 0.6459$

Exercise

An industrial process is completed by doing two successive operations, both done in an automatic way by two industrial computers with different CPUs, in the way that is depicted in Figure (1). Define T_1 as the time the **CPU-1** takes to complete the *Task 1* and T_2 as the time the **CPU-2** takes to complete the *Task 2*.

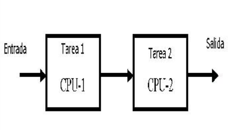

Figure: (1)

The probability that the **CPU-1** takes more than $t > 0$ seconds to complete the *Task 1* is

$$\Pr(T_1 > t) = e^{-t/\alpha};$$

being $\alpha > 0$ a constant, while the probability that the **CPU-2** takes more than $t > 0$ seconds to complete the *Task 2* is

$$\Pr(T_2 > t) = e^{-t/\beta};$$

with $\beta > 0$ a constant that verifies the relation $\beta > \alpha$. These probabilities are based on exponential functions with negative exponents and they are frequently used to model real phenomena.

Figure (2) shows these probabilities where one can note that the probability that one task takes more than t seconds fades off *exponentially* as a function of t.

a) Compute the distribution function of T_1.

b) Compute the *average time* that the **CPU-2** takes to execute the *Task 2*.

c) Compute the *average time* that the whole process will take to finish (*Task 1* + *Task 2*).

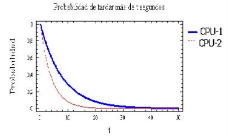

Figure: (2)

SOLUTION:

a)

$$F_{T_1}(t) = 1 - e^{-t/\alpha};$$

b)

$$\mathbb{E}[T_2] = \beta;$$

c)

$$\mathbb{E}[T_1 + T_2] = \alpha + \beta.$$