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New evidences using a Dagum Parametric Income Distribution Model

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1. Introduction

The last decade witnessed the deterioration of equality in the distribution of income and the increase of unemployment in most of the developed as well as developing countries. For policy-makers, these parallel developments were interpreted as bringing enough evidence in support of the idea that the rise in unemployment was behind the deteriorating conditions of equality in society. Then, to abate unemployment became a challenge in most countries, and the fight against unemployment soon became the first goal of income policy. Previously, income policy covered a wider range of aspects but in the last decade it narrowed, became more focused and increasingly related with the practice of implementing policies and programs specifically designed to abate unemployment. So far, results around the world, perhaps with the exception of the U.S. economy, are not very encouraging. After several years of practicing this approach to income policy, unemployment has remained at the high level it reached during the previous decade. Therefore, it is time to assess whether this practice was an effective way for governments to apply income policies.

The personal distribution of income may indeed deteriorate or improve as a result of a number of factors. For example, a deterioration is said to exist when the relative distance between the first and the last decile of individuals in the labor force widens, irrespective of movements in the unemployment rate. Similarly, when the distribution moves as a whole to the left, the mean income of each decile decreases (as does the total distribution mean) and this type of movement is also interpreted as a deterioration, that might not be necessarily associated with changes in the level of unemployment.

The paper seeks to contribute to the study of the effect of unemployment on the distribution of personal income. A parametric model of the Dagum type will be applied to this purpose. The model is particularly interesting because it shows the association between the probability density function (PDF) of a population of individuals in the labor force, with certain parameters representing the level of unemployment and other economic conditions. The analysis will be based on a simulation that allows the study of partial and combined effects of the parameters in the PDF of individuals in the labor force.

The proposed model exhibits several interesting features that would help in exploring alternative explanations. First, the model is quite strong in reproducing the probability density function of economic units among the population. Empirical applications based on

income data from several countries: Canada, the United States, Italy and Argentina (Dagum 1983, 1990, Botargues and Petrecolli 1999) provided strong evidence of the power of the Dagum model in producing superior descriptions of the whole range of income. Second, it does so with two parameters “beta” and “delta” that represent conditions of equality for different subgroups in the population while a third parameter “alpha” captures the effect of variations in the rate of unemployment. Third, the distinct effects associated with each of the parameters have specific economic meaning and intervene in the calculation of the parametric Gini ratio. Fourth, the Dagum function has also a fourth parameter “lambda”, that accounts for the monetary scale, henceforth it is a tool to adjust for inflation and to facilitate cross-country comparisons of income distributions that are expressed in different monetary units. However, the equity parameters are scale free in the Dagum model, thus, the scale parameter does not affect the measurement of the parametric Gini ratio.

The Dagum model is applied to data representing individuals in the labor force in Greater Cordoba. First, the cumulative and the probability density functions are calculated from the May Household Survey (EPH- Encuesta Permanente de Hogares from 1992 to 2000) applying equation (2). Then, the 1992 estimates of the position parameters in the distribution (the modal, median and mean income) are taken as a benchmark, a modification in the original parameters of the equation is simulated and the relative and joint effects on the mode, the median and the mean income and on the Gini ratio of altering the original alpha, beta and delta values of the PDF are evaluated. Third, two scenarios will be constructed: (a) the data for 2000 would help to set up the less desirable economic scenario while (b) the 1992 data will be used to simulate the more desirable scenario.

Three questions will be explored here: (i) what percentage of the change in the value of the Gini ratio can be accounted for by changes in the average rate of unemployment?, (ii) what sort of changes can be predicted for the welfare of economic units at the lower tail of the distribution when the measures of location in the distribution (the mean, the median and the modal income) moves up or down by a certain amount? and, (iii) what would be the impact of these movements on middle and upper-middle individuals in the labor force? The next section reviews the model. The following, develops a parametric simulation exercise that will help to examine how movements in the parameters of the model (alpha, beta and delta) will affect the modal, median and mean income and the Gini ratio. The paper concludes with an application of the simulation exercise to assess the relative contribution of alpha, beta and delta in explaining the observed change in the distribution of income in Greater Cordoba between 1992 and 2000.

2. The model

Theoretical representations of income distribution have traditionally relied on two-parameter cumulative density functions because these are relatively easy to estimate. However, the two-parameter models cannot deal with the existence of null and negative incomes or an unknown positive but small minimum value of income without the introduction of assumptions that limit their goodness-of-fit (Dagum, 1980). Instead, Dagum (1977, 1980) proposed a superior theoretical description based on the observed characteristic of regularity of income-elasticity in observed income distributions. In general, Income-Elasticity of the Cumulative Distribution Function (CDF) with respect to the origin α of $F(x)$, is represented as a monotonically decreasing function of $F(x)$. The mathematical representation of this argument is a differential equation, with three or four parameters where:

$$(1) \quad \frac{d \ln[F(x)-\alpha]}{d \ln x} = \beta \delta \left[1 - \left(\frac{F(x)-\alpha}{1-\alpha} \right)^{\frac{1}{\beta}} \right], x > 0$$

with solution, for the CDF

$$(2) \quad F(x) = \alpha + \frac{(1-\alpha)}{(1 + \lambda x^{-\delta})^{\beta}}, (\beta, \delta, \lambda) > 0$$

with derivative $F'(x) = f(x)$, the PDF.

The CDF equation (2) is well defined for all δ greater than zero. However, for income distributions, the existence of finite income mean is assumed; hence, the existence of the corresponding Lorenz curves and Gini ratios is subject to the condition δ greater than one (Dagum, 1980).

The equality (shape) parameters for the lower and upper-middle tail of the distribution are β and δ ; and, $\lambda = \exp.$ of the constant of integration, is a scale parameter (Dagum, 1977).

The Gini ratio¹ associated with the Dagum model, tends to zero, perfect equality, with increasing values of β and/or δ ; and tends to 1, perfect inequality, with decreasing values of β and/or δ . This means that there is an improvement of the income distribution when the values of β and/or δ are increased.

This theoretical model will adjust empirical distributions with null, negative or positive starting income represented by the value taken by α in equation (2); which, in general, is unimodal and positively skewed; with income range in the (X_0, ∞) , where $X_0 > 0$ for sample distribution of employed individual with starting positive income.

There are three types of the Dagum family of models:

- a) Dagum Type I of a CDF corresponds to origin $\alpha = 0$, then it is said to best represent the behavior of (employed) wage earners.

$$(3) \quad F(x) = \frac{1}{(1 + \lambda x^{-\delta})^\beta}$$

- b) Dagum Type II corresponds to $0 < \alpha < 1$, then

$$(4) \quad F(x) = \alpha + \frac{(1-\alpha)}{(1 + \lambda x^{-\delta})^\beta}$$

is a four parameter CDF.

Here, $0 < \alpha < 1$ measures income units with zero or negative income; in Dagum (1977) it has been interpreted as a proxy for the rate of unemployment (because it indicates the percentage of population in the labor force having no positive earnings). If unemployment is high, α becomes critical to understand inequality in the labor force (Dagum, 1983 a). Thus, α is an inequality parameter, while λ is a scale parameter that would allow time or space comparison between distributions when income data is expressed in different monetary units.

- c) Dagum Type III implies $\alpha < 0$, and:

$$(5) \quad F(x) = \alpha + \frac{(1-\alpha)}{(1 + \lambda x^{-\delta})^\beta}$$

is a four parameter CDF.

Type III is an appropriate representation of sample income distribution of total income receivers (wage earners plus those with income from property) because it starts accumulating income of a population with initial positive earnings (Dagum, 1983a). The interpretation of parameters α , β , δ and λ are as in model II above.

Dagum (1977) proved that the parametric Gini ratio can be obtained from the corresponding Lorenz curve of the CDF and it is an increasing function of α and a decreasing function of β and δ^2 . The corresponding formula is:□

$$(6) \quad G = (2\alpha - 1) + (1 - \alpha) \frac{\Gamma(\beta)\Gamma(2\beta + \frac{1}{\delta})}{\Gamma(2\beta)\Gamma(\beta + \frac{1}{\delta})}$$

where $\Gamma(\cdot)$ is the complete Gamma function specified in Dagum (1977).

The Dagum Type II Model, that corresponds to $0 < \alpha < 1$ has been estimated here.

3. Data and Methodology

The basic information has been taken from the May EPH (The Household Survey run by the INDEC) for the Greater Córdoba from 1992 to 2000. Table 1 shows the

corresponding parametric Gini ratio, calculated from (6), and the estimated values for alpha, beta and delta that resulted from application of equation (2) to a sample of individuals in the labor force belonging to Greater Córdoba, from 1992 to 2000.

Table 1: Parameters of Dagum Type II Model and Gini ratio for Greater Córdoba 1992-2000

Year	Gini Ratio	Alpha	Beta	Lambda*	Delta	R ²
1992	0.46066	0.0388	2.7895	0.0334	2.0289	0.9951
1993	0.44733	0.0439	1.5024	0.0793	2.2261	0.9956
1994	0.45919	0.0419	1.7911	0.0877	2.1182	0.9955
1995	0.45291	0.0732	0.8774	0.1714	2.5067	0.9934
1996	0.45552	0.0808	0.6926	0.1557	2.6686	0.9922
1997	0.46021	0.0817	0.5385	0.1935	2.8440	0.9923
1998	0.44175	0.0654	0.6266	0.1511	2.7819	0.9951
1999	0.44912	0.0640	0.6611	0.1663	2.6760	0.9943
2000	0.48057	0.0599	0.6144	0.1914	2.4940	0.9949

Note: * Lambda stands for an income of \$1,000.

Source: own estimates, based on May EPH-INDEC.

As it was previously explained, the parameters alpha, beta and delta are scaleless and help in interpreting equality. Also a growing alpha reflects the increased number of individuals having no income in the PEA. The values of alpha in Table 1 col.2 are consistent with the real unemployment rate for each year. For example, the observed unemployment rate for 2000 of about 13 per cent of the PEA is equivalent to approximately 6 per cent of the Total Population, the value of alpha for that year. More generally, alpha increased from 1992 to 1997, had a sharp decrease in 1998 and again in 2000. In spite of the decrease, the value for 2000 exceeded that of 1992 by almost 60 per cent. A rise in beta values reflects the welfare improvements in the groups of population located at the lower tail of the distribution, those with the lower income levels, while delta increments imply improvement in the welfare situation of population groups at the middle and upper-middle tail of the income distribution. Table 1, col.3, shows a strong decrease in beta values, from 2.8 to 0.6 in 1992 and 2000 respectively. This would suggest a deteriorating trend in the welfare situation of PEA members located at the left tail of the income distribution. The increase in unemployment and the multiplication of low paid jobs in the informal job market would help to explain this. The value of delta increased from 1992 to 1997 but decreased from 1998 to 2000.

Table 2 provides information to additional assessment of the goodness-of-fit of the model and test whether there have been significant change in the mean value of the parameters from 1992 to 2000. For example, one intuitive way to check whether the parameters have followed a definite trend over time would be to check if the lower bound of the confidence interval in year t lies above the upper bound of the confidence interval in year $t+n$ (a decreasing trend between year t and $t+n$ would be confirmed).

Table 2: Sample Estimates of the 95 per cent Confidence Intervals for the Parameters of the Dagum Type II Model. PEA - Greater Cordoba - 1992-2000

Year	Alpha		Beta		Lambda		Delta	
	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
1992	0.0347	0.0429	2.3796	3.1995	0.0289	0.0379	1.9996	2.0582
1993	0.0396	0.0482	1.3781	1.6267	0.0734	0.0852	2.1939	2.2583
1994	0.0382	0.0455	1.6384	1.9437	0.0806	0.0949	2.0877	2.1487
1995	0.0678	0.0786	0.8083	0.9466	0.1587	0.1841	2.4554	2.5580
1996	0.0752	0.0864	0.6388	0.7463	0.1453	0.1661	2.6052	2.7320
1997	0.0762	0.0872	0.5025	0.5746	0.1827	0.2042	2.7735	2.9146
1998	0.0599	0.0709	0.5865	0.6667	0.1434	0.1588	2.7203	2.8434
1999	0.0579	0.0700	0.6125	0.7097	0.1562	0.1763	2.6104	2.7416
2000	0.0554	0.0645	0.5851	0.6437	0.1834	0.1994	2.4471	2.5409

Note: The Bootstrap technique included in the SPSS package was used to estimate the 95 per cent confidence intervals.

Source: own estimates, based on May EPH-INDEC.

Let's observe first the parameter alpha. The upper limit is 0.04 in 1992 and the lower limit in 2000 is 0.05; therefore, there is strong evidence that the estimated parameters belong to different populations: the one in 1992 having less unemployment than the respective population in 2000. Then, similar interpretations can be drawn for beta and delta with respect to structural changes in equality.

The parametric Gini exhibits, in Table 1, a relative stability, with variations in the interval (0.46, 0.44) from 1992 to 1999, it then increased to a peak of 0.48 in year 2000. The value of alpha represents a total of 3.9 per cent unemployed, on the total sampled population of may 1992 but 6.0 per cent in may 2000³. Beta shows a decreasing trend over the period assuming a value of 2.8 in 1992, and 0.6 in 2000. Delta was 2.0 in 1992 and 2.5 in 2000. The estimated R^2 in Table 1 can be interpreted as a measure of the goodness of fit because it is the result of computing the square sum of the distances between the theoretical and the observed data. The high R^2 (higher than 0.99) confirms the good adjustment of the Dagum model.

4. The alpha, beta and delta effects on income distribution equality

From equation (6) it is clearly seen that any increase in alpha, a proxy measure for unemployment, will cause the Gini ratio to rise, holding beta and delta constant. This finding might help to explain the actual support most policy-makers are now giving to the intuitive idea that increased inequality in income distribution resulted from increased unemployment (Gasparini et.al., 2000). Beta and delta summarize information about the density, or frequency distribution of individuals in the labor force at different income levels. It can easily be seen from (2) that beta and delta have a joint effect on the shape of the distribution. Because the joint effect expresses itself through the expanded Gamma function, the independent, partial contribution to equality of beta or delta may not be

easily assessed through the standard practice of setting a linear function. Instead, by introducing a numerical simulation, partial effects can be examined with certain detail. In fact, using (2) and moving beta and delta, holding the other constant, it was found: (i) any increase (decrease) in beta with delta constant would represent an improvement (deterioration) in equality. Similarly, any increase (decrease) in delta would indicate improvements (deterioration) in equality; (ii) turning back to formula (2), beta and delta have opposite signs; therefore, the joint beta-delta effect summarizes the partial improvement (deterioration) of equality associated with a rise (decrease) in beta together with the partial deterioration (improvement) due to a decrease (rise) in delta; (iii) the delta effect outweighs the beta effect in our simulations.

Beta summarizes information that represents more strongly the effect of events on the density of individuals in the labor force at the left of or located near the modal income; thus, it has the visual effect of either sharpening or flattening the peak of the distribution. It is in this rather limited sense that we will attach a localized effect to beta movements; and because the distribution of income is skewed to the right, the beta effect is predicted to be stronger in accounting for local conditions at the lower tail of the distribution (for example, having stronger impact on individuals located below and close to the modal income level than individuals in any other frequency range in the distribution). The parameter delta captures effects of a more generalized nature. An increase in delta will sweep the income distribution to the right. However, although the mean, the median and the modal income will all move to the right, the distance between the mean and the mode will become smaller, suggesting that equality in income distribution improves with a rise in delta.

Thus, the parametric Gini depends on the behavior of parameters having an important economic impact on equality. While the parameters beta and delta have a joint effect on the shape of the distribution, alpha has an immediate association with the unemployment rate and its independent contribution to equality can be numerically explored through (6). Be $\alpha = 0.06$, a value representing the ratio of the number of unemployed over the total population at time t and having a Gini ratio of 0.48, as it was in year 2000, using (6) the resulting value for the combined beta and delta effect is 1.447 ($[0.48 - (2\alpha - 1)] / (1 - \alpha) = 1.447$). Next, by holding the beta-delta effect constant at 1.447 and moving alpha, a table can be constructed to estimate the sensitivity of the Gini ratio to changes in alpha. Table 2 shows the resulting Gini ratio for hypothetical values of alpha, holding the beta-delta effect constant at 1.447

Table 3: Parametric Gini ratio estimates

Alpha*	Gini ratio
0.06	0.48
0.054	0.476
0.0486	0.474
....
0.0300	0.463

Note: *given a beta-delta combined effect of 1.447

Source: own estimates, based EPH-INDEC.

The table 3 illustrates the fact that a big cutting unemployment (i.e. cutting unemployment by half) will have a small impact on the Gini ratio since it improved only a 3.5 per cent, from 0.48 to 0.463.

Similarly, it would be possible to evaluate the response of the Gini ratio to changes in beta and delta. A more specific exercise will help assessing the partial effect of a change in alpha, beta and delta respectively. Simulations will be carried out using 1992 and 2000 data on individuals in the labor force in Greater Cordoba. The Dagum PDF will be used to estimate the parameters and the modal, median and mean income that will be the benchmark in our comparison. For this exercise alpha will be cut in half and beta and delta will be doubled holding the others parameters constant. Figure 1(a) shows the cutting alpha by half increases the mean income by 2 per cent, the median by 1.7 per cent and has no effect on the mode. Overall it contributes to decrease the Gini ratio (slightly improving equality) by 2.4 per cent.

Figure 1(b) indicates the effect of a hundred per cent increase in beta. The mean income is increased by 41 per cent, the median by 45 per cent and the mode by 48. Its impact on the Gini ratio is a slight improvement of 3.3 per cent.

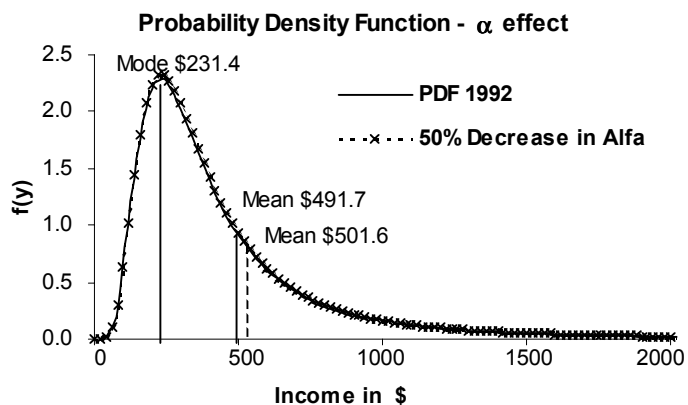
The partial effect of delta in Figure 1(c) is shown to rise the mean income by 30 per cent, the median by 72 per cent and the mode by 123 per cent. The Gini ratio is improved by 48 per cent.

A preliminary interpretation of these results would be that the delta effect is stronger than the beta effect for a wider range of individuals in the labor force, because of its property of sweeping the distribution to the right towards normality with any rise of its original value, meaning that a greater number of individuals will concentrate around the mean income. The effect of beta is stronger on the left side of the distribution; with any rise in beta positively affecting low individuals in the labor force, which explains the rise in the mean income of the distribution. Figure 2 shows similar finding for year 2000.

Figure 1

The effects of α , β and δ movements on the probability density function – Year 1992

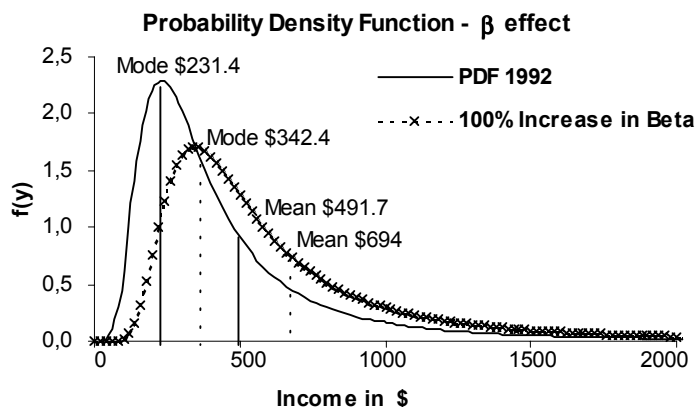
	1992	$\Delta\alpha = -50\%$
Mean	491.74	501.66
Median	338.13	343.73
Mode	231.42	231.42
Gini ratio	0.4606	0.4497



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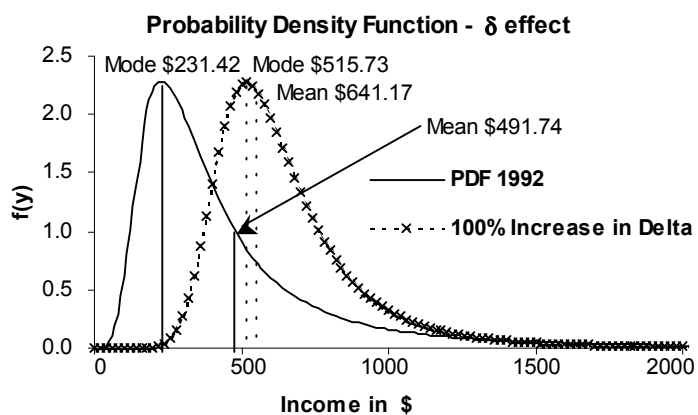
(a)

	1992	$\Delta\beta = 100\%$
Mean	491.74	694.07
Median	338.13	492.05
Mode	231.42	342.45
Gini ratio	0.4606	0.4455



(b)

	1992	$\Delta\delta = 100\%$
Mean	491.74	641.17
Median	338.13	581.49
Mode	231.42	515.73
Gini ratio	0.4606	0.2377



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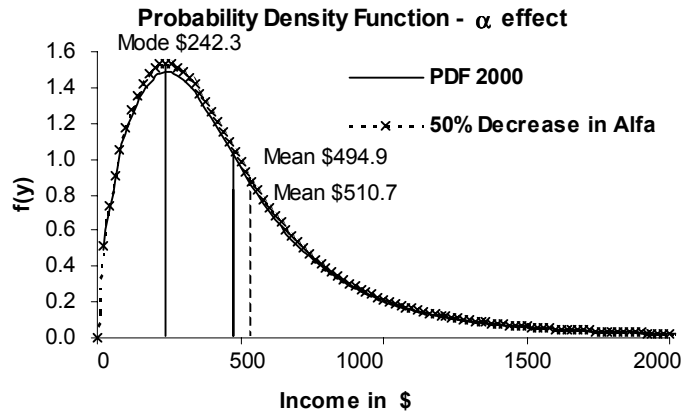
(c)

Note: figures were obtained from estimation of PDF.

Figure 2

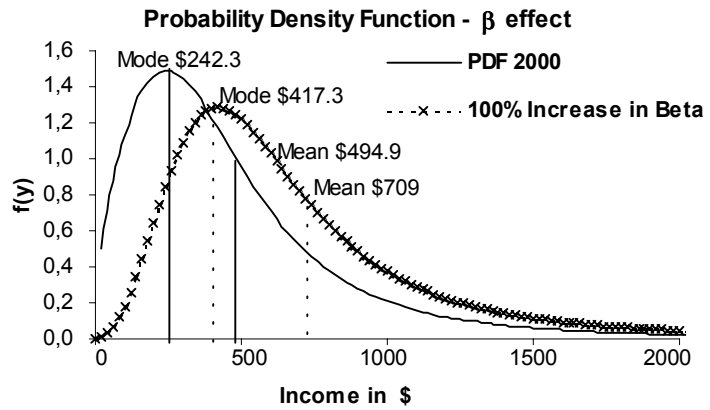
The effects of α , β and δ movements on the probability density function – Year 2000

	2000	$\Delta\alpha = -50\%$
Mean	494.92	510.70
Median	360.38	372.14
Mode	242.33	242.33
Gini ratio	0.4806	0.4640



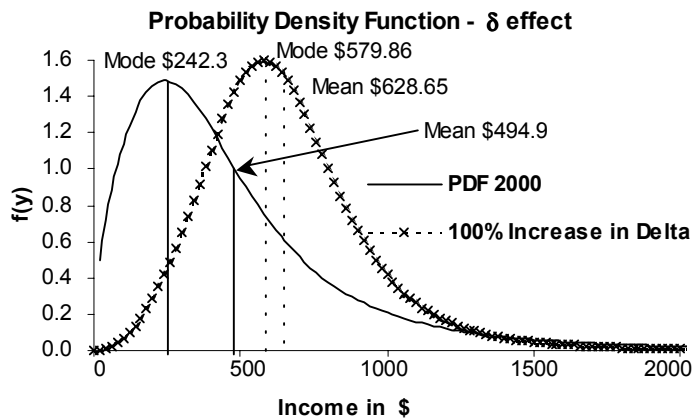
(a)

	2000	$\Delta\beta = 100\%$
Mean	494.92	709.09
Median	360.38	548.81
Mode	242.33	417.31
Gini ratio	0.4806	0.4230



(b)

	2000	$\Delta\delta = 100\%$
Mean	494.92	628.64
Median	360.38	600.31
Mode	242.33	579.85
Gini ratio	0.4806	0.2804



(c)

Note: figures were obtained from estimation of PDF.

The fourth section has produced results that should help to illuminate some of the most plausible implications for income policy design. For example, if unemployment is cut in half (i.e. from a rate of 18 to 9 per cent) how would the Gini ratio register this significant improvement in employment conditions? The answer is that no significant improvement in the Gini ratio can be expected from acting solely on unemployment. Similarly, beta and delta summarize the effect of movements in the mean, the median and the mode, the measures of location of the distribution, henceforth, they help to simulate impacts on the Gini ratio of stylized movements in the relative position of individuals in the labor force at the lower, the middle and upper-middle percentiles in the distribution. Through equation (2) it is seen that beta and delta move simultaneously in opposite directions. However, the net effect would follow the direction of the delta effect, that resulted from a wider, rather than a narrow interpretation of what an income policy should eventually be. Thus, the exercise has an immediate economic meaning.

5. A parametric estimation of income inequality in Greater Cordoba, from 1992 to 2000

Table 4 shows the mean income in 1992 and 2000 of individuals in Greater Cordoba and the relative impact of changes in alpha, beta and delta on the Gini ratio and on the mean income of individual by income level.

Table 4: Mean Income and Gini ratio for changes in the value of the parameters α , β and δ by income level

			α , β and δ by income level						
			Mean Income ⁽¹⁾		Total Effect	Decomposition of Effects			
						Partial Effects ⁽²⁾			Cross Effect
						α (+54%)	β (-78%)	δ (+23%)	
			1992	2000					
Mean Income Level	Lower (1 - 2 deciles)	(\$)	151.16	90.85	-60.31	-5.30	-126.00	62.99	8.00
		(%)			-39.90	-3.50	-83.40	41.70	5.30
	Middle (3 to 6 deciles)	(\$)	291.11	291.65	0.45	-6.64	-193.42	74.25	126.26
		(%)			0.20	-2.30	-66.40	25.50	43.40
	Upper-middle (7 - 8 deciles)	(\$)	495.42	552.48	57.06	-6.98	-287.31	68.65	282.70
		(%)			11.50	-1.40	-58.00	13.90	57.10
Gini Ratio ⁽³⁾			G	0.46066	0.48057	0.02	0.0118	0.0966	-0.0861
			(%)			4.30	2.60	21.00	-18.60

Note: (1) Mean Income in pesos. (2) Partial effects shown as mean income differences in pesos and in percentage change relative to 1992. (3) Gini ratio varies from 1 (perfect inequality) to 0 (perfect equality)

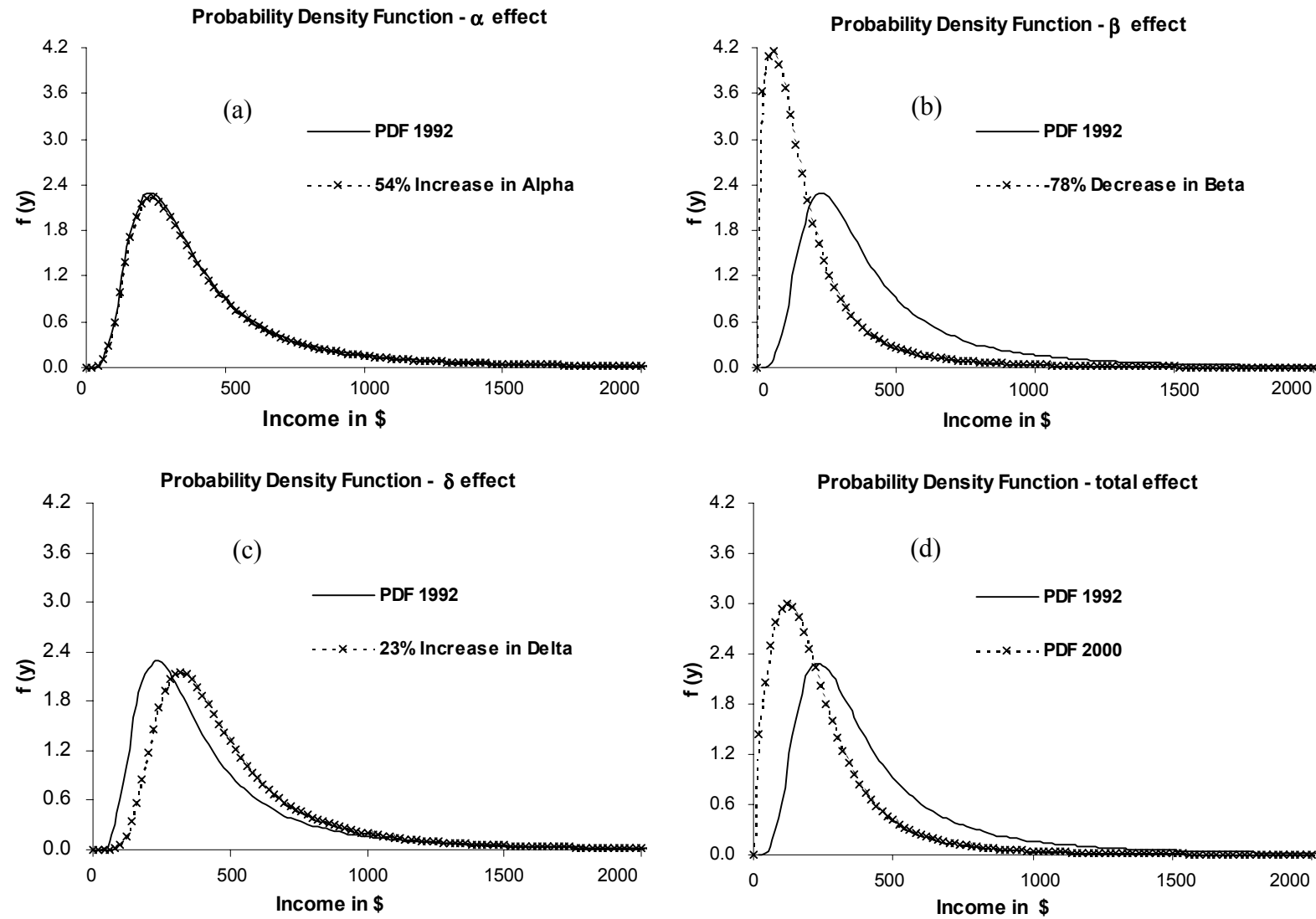
Source: own estimates, based on EPH-INDEC.

Between 1992 and 2000, alpha experimented a rise of 54.0% that had a small and decreasing effect by income level: the mean income of the first and second deciles had a decrease of (-)3.5%, while the decrease for the mean of the 3-6 decile was (-)2.3% and for the 7-8 decile the decrease was (-)1.4%. The beta coefficient had a variation of (-) 78.0% with an important but rapidly decreasing impact by income level, of (-)83.4%, (-)

66.4% and (-)58.0%, respectively. The delta variation (23.0%) had a moderate but rapidly decreasing impact by income level, of 41.7%, 25.5% and 13.9%, respectively.

In sum, in Greater Cordoba, between 1992 and 2000, the individual impact of alpha, and beta is a deterioration in income distribution because the Gini increased 2.6% and 21.0%, respectively; but delta is associated with an improvement in income distribution because the Gini diminished 18.6%; the cross effect had a relatively minor impact. The combined effect on the Gini is a net deterioration of 4.3%. Consequently, the alpha effect accounts for 60% of the combined effect, while beta accounts for 488%, delta accounts for (-) 433% and the cross effect is 14% of the total effect.

Figure 3
The effects of α , β and δ movements on the probability density function – Year 1992 to 2000



Note: figures were obtained from estimation of PDF.

6. Final Remarks

The paper introduced a four parameter Dagum type of income distribution function of the labor force to generate a numerical calculation of the parametric Gini ratio in Greater Cordoba from 1992 to 2000. The model helped in assessing the relative contribution to equality of proportionate changes in alpha (a proxy for unemployment), and the beta and delta, two other equality parameters, beta and delta. Firstly, a simple exercise in simulation was introduced to assess the sensibility of the Gini ratio to changes in the value of the parameters. As part of the exercise, changes in the parameters were simulated to measure the impact on the estimated values of the position parameters in the income distribution equation (the mean, the median and the modal income), the benchmark, or control values obtained from equation (2). Then, the relative impact on the lower, middle and upper-middle tail in the distribution was also investigated. The findings tend to confirm that cutting unemployment is insufficient to improve the Gini ratio because of its weak relative impact. A positive change in beta and delta shows a greater impact over a wider range in the distribution; however, while beta seems to improve relatively the left tail of the distribution, delta has the effect of sweeping the modal and mean income to the right, and improving the modal value at a higher speed. Hence, if delta is positive, equality is likely to improve because the mass of individuals around the mean income increases. In the last section, the Gini ratio was recalculated with the 1992 and 2000 mean income of individuals in the labor force in Greater Cordoba. The alpha parameter increased by 54%, the beta parameter decreased by 78% but the delta parameter increased by 23% in the 1992-2000 period. The result was a net rise in the Gini ratio of 4.3 % where the alpha effect accounted for 60% of the combined effect, while beta accounted for 48.8% and delta accounted for (-) 43.3%. The beta and delta effects operating in the opposite direction.

This numerical approach helped to improve our understanding about the partial contribution of each of the parameters to the Gini ratio. However, at this stage, the exercise should be understood only as a crude approximation to the individual contribution of alpha, beta and delta to equality in income distribution. It is clear from the structure of equation (6) that interaction effects exist and more work is required to improve our understanding about each parameter net and cross contribution to variations in the Gini index of equality. Lastly, the parametric Gini ratio has an immediate advantage over its empirical equivalent in exploring plausible outcomes associated with alternative treatments to abate poverty or improve equality in income distribution. Changes in the value of the

parametric Gini ratio can immediately be related with policies whose effects are summarized through movements in α , β and δ . For example, would a policy that generates jobs for 10 per cent of the unemployed have a sizable impact on the Gini ratio, the most common measure for inequality? If localized subsidies are directed at the poorest individuals in the labor force (i.e. those below the modal income), how would the Gini ratio improve? Will introducing a tax cut that benefits a wider range of individuals in the labor force, most of them localized in the middle and upper-middle range in the distribution, have a weak or a strong effect in the Gini ratio? The parametric Gini ratio also helps in assessing the differential power of alternative policies directed to specific policy objectives, for example, to increase the mean income in the population by 10 per cent.

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Notes

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¹ The Gini concentration ratio, or Gini coefficient is an aggregate inequality measure and can be obtained from information included in the construction of a Lorenz curve by calculating the ratio of the area between the diagonal and the Lorenz curve divided by the total area of the triangle in which the curve lies. This definition implies that Gini coefficients can vary anywhere from zero, perfect equality, to one, perfect inequality. This traditional approach produces an underestimation for the value of Gini coefficients because income distribution data are presented by class intervals and to calculate the area under the Lorenz curve only the middle points of the intervals are considered (See Dagum, 1987: pp 531). Todaro observed that the Gini coefficient calculated from empirical income distributions typically lies between 0.50 and 0.70 for highly unequal income distributions, and it is on the order of 0.35 to 0.20 for countries with relatively equitable distributions (Todaro; 1997: pp. 146).

² The following relations between delta and the Gini ratio hold:

$$\delta \rightarrow \infty \Rightarrow G \rightarrow \alpha$$

$$\delta \rightarrow 1 \Rightarrow G = (2\alpha - 1) + (1 - \alpha) \frac{(\beta - 1)(2\beta)}{(2\beta - 1)(\beta)} = (2\alpha - 1) + (1 - \alpha)2 = 1$$

$\delta < 1 \Rightarrow G > 1$, that has no economic meaning.

³ Since there is a systematic relationship between the active and the total population, the value of alpha, the unemployed over the total population, is related to the usual measure for unemployment, the unemployed over the active population, through the formula $\alpha = a + bU + e$.