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and a comparison to Greater Buenos Aires

Héctor R. Gertel
Roberto Giuliodori
Paula F. Auerbach
Alejandro F. Rodríguez

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Facultad de Ciencias Económicas
Universidad Nacional de Córdoba

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ABSTRACT

The paper seeks to apply the Dagum generating model of income distribution functions to study the location and shape parameters of a sample distribution function of individual income receivers belonging to Greater Córdoba, its evolution in 1992-2000, and the impact that the persistent increase in unemployment exerted on inequality. A comparative analysis of income distribution between the Capital region of Argentina, represented by the Greater Buenos Aires, and the rest of the country, reflected in the analysis of Greater Córdoba, is included.

1. Introduction

The paper seeks to apply the Dagum generating model[1] of income distribution functions to study the location and shape parameters of a sample distribution function of individual income receivers belonging to Greater Córdoba, its evolution in 1992-2000, and the impact that the persistent increase in unemployment exerted on inequality. In recent years there has been a growing interest in the exploration of parametric models of the distribution of income. Dagum (1977) proposed a parametric specification of a generalized logistic function that facilitates comparative analysis of different subgroups in the population, by regions or over time, and proved to render a better goodness-of-fit relative to alternative

[*] corresponding author: hgertel@eco.unc.edu.ar

[**] Héctor Gertel is with Instituto de Economía y Finanzas - Universidad Nacional de Córdoba; Roberto Giuliodori is with Departamento de Estadística y Matemática – Facultad de Ciencias Económicas – Universidad Nacional de Córdoba; Paula Auerbach is graduate student, Georgetown University; Alejandro Rodríguez is graduate student Departamento de Economía – Universidad Nacional de Córdoba. This article was partially funded by the Agencia Córdoba Ciencia (Project ACSE-034002304).

[1] A generating model is one that helps to understand the stochastic properties and explanatory powers of income distribution functions. There are three families of generating systems: the D´Addario system, the Pearson system and the generalized logistic, or “Dagum” system. They provide competing strategies to describe empirical income distribution data by using parametric estimations. Dagum type of models have proved to provide a superior goodness-of-fit for the full range of the distribution while previous and most popular models, such as those of Pareto, provide a good description of only the upper tail of the distribution. (Dagum, 1990a)

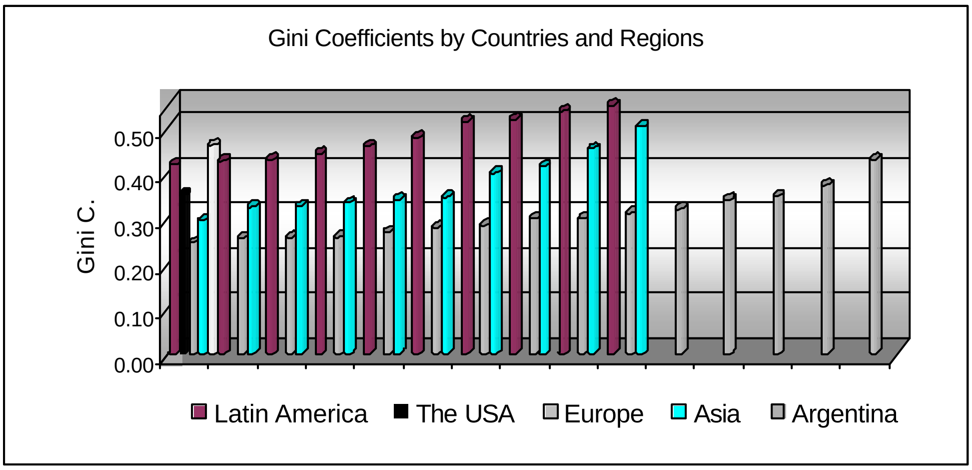
existing models. An application of these models has been done by Petrecolla and Botargues using data for Greater Buenos Aires (1997, 1999).

The main objective of the present paper is to study the most relevant characteristics of income distribution in Greater Córdoba and to perform a comparative analysis of income distribution in the Capital region of Argentina, represented by the Greater Buenos Aires, and the rest of the country reflected in the analysis of Greater Córdoba.

1.1. Inequality in Argentina: An Overview

Studies of personal income distribution in Argentina can be traced back to the 1950 decade when Modern Economic Growth as coined by Kuznets (1955) started in the country. The acceleration of growth in the country is associated with the process of inwards industrialization that ignited in mid-twenty century a process of moderate long-term improvement in GDP per capita. Inequality of personal income had remained during those years relatively stable. The adjusted Gini Coefficient of the country total household income was estimated from National Accounts sources to be at 0.40 and 0.41 for 1953 and 1961 respectively (Altimir and Beccaria, 2000). In the late seventies, together with the acceleration of inflation, the economy of the country suffered from increasing macroeconomic instability and rising income inequality. The Gini ratio of per-capita household income in Greater Buenos Aires was 0.34, 0.38 and 0.46 in 1974, 1980 and 1986 respectively (Altimir and Beccaria, 2000). The basic variables of the economy stabilized and had remained strong during the nineties when labor productivity significantly increased. Yet those were years of strong de-industrialization with growing labor unemployment: open unemployment rates jumped to an unprecedented level (18 percent of the total labor force was reported unemployed in 1998) from previous historical records of 6 to 8 percent. Inequality in the distribution of personal total income estimated from empirical distributions followed a rising trend in the nineties, the Gini ratio of per-capita total household income in Greater Buenos Aires was calculated at 0.45, and 0.47 in 1990 and 1997 (Altimir and Beccaria, 2000). Nevertheless, no clear trend emerged when estimates of the Gini ratio were obtained from parametric distribution models, as those reported in section four. Lognormal, Gamma, Sigh-Maddala and Dagum functional forms were applied to the study of personal (total) income and labor earnings distribution in Greater Buenos Aires 1992-1997. Better goodness of fit was found using a Cramer test for the Dagum model (Bortagues and Petrecola, 1999). The advantage of using parametric distributions is associated in the literature with the amount of information the parameters summarize in a very effective way. It can be concluded that today income inequality in Argentina is higher compared with previous levels found in the early fifties and sixties (Gasparini, Marchionni and Sosa Escudero, 2001, Deininger and Squire, 1996). At the same time Argentina exhibits income inequality figures located at the median position among Latin American countries. Yet, when a sample of countries that includes the newly industrializing countries of Asia, and others from Europe and Latin America is taken as a comparison group (See Figure 1), Argentina exhibits one of the worst income distribution figures.

Figure 1



Source: Table 1 in Annex

1.2. The Greater Córdoba

Córdoba is the second largest city in Argentina. Today, it has 1.6 millions inhabitants and can be considered, in terms of income distribution, as a representative case of the middle-size cities situation of the "rest of the country", that we are willing to explore and compare with the capital region, which concentrates about 13 millions inhabitants. While the later has recently been the focus of attention by several papers there is not such information available for the rest of the country.

In the study, two groups of individuals from Greater Córdoba are analyzed and then compared in each year; the first, representing the employed individuals with positive income, and the other including all the employed and the unemployed individuals in the economic active population, having positive or null income. The analysis is carried out by comparing the resulting differences in the annual mean value and confidence intervals for the parameters of both distributions from 1992 to 2000. Then, the income distribution of individuals in the economic active population is compared with that of the employed income receivers to ascertain the impact that the changes in unemployment over time have on inequality measures associated with the model. Finally, regional differences are analyzed by comparing the values of the parameters and the evolution of the parametric Gini ratio in Greater Córdoba and Buenos Aires.

The next section presents the model, followed by a discussion of data sources and methodology, the results are shown in the fourth section, the fifth section develops the comparative analysis and the last section summarizes the most important findings and conclusions.

2. The Model

The probability of finding an individual with income X>x is represented by $F(x) = P(X > x)$, the individual cumulative income distribution function. In a world of perfect equality, any one percent increase in the cumulative population will render a one percent increase in cumulative income. Theoretical analyses and empirical data on income distributions have shown that the rate of growth of the cumulative income proceeds at a faster pace than the rate of growth of the population of income receivers, which means that the cumulative income elasticity is decreasing (Dagum, 1990). The paper will seek to verify this result in our case.

Theoretical representations for this behavior have traditionally relied on two-parameter functions because these are relatively easy to estimate. However, the two-parameter models cannot deal with the existence of null and negative incomes or an unknown positive but small minimum value of income without the introduction of assumptions that limit their goodness-of-fit (Dagum, 1980: pp. 18). Instead, Dagum (1977, 1980) proposed a superior theoretical description for the Income-Elasticity of the Cumulative Distribution Function (CDF) with respect to the origin α as a differential equation with three or four parameters where:

$$(1)\quad \frac{d\ln[F(x)-a]}{d\ln x} = bd\left[1-\left(\frac{F(x)-a}{1-a}\right)^{\frac{1}{b}}\right], x>0, (b,d)>0,\ bd>1$$

with solution, for the CDF

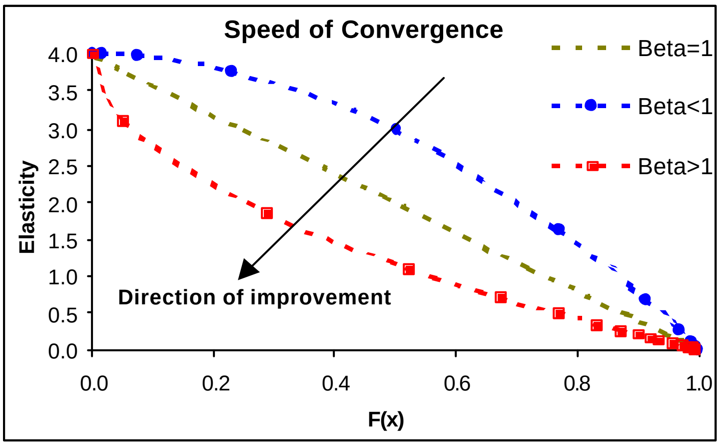
$$(2)\quad F(x) = a + \frac{(1-a)}{\left(1+lx^{-d}\right)^{b}}, (b,d,l)>0$$

as a CDF equation (2) is well defined for all δ greater than zero. However, for income distributions, given the existence of finite income mean, hence, the existence of the corresponding Lorenz curves and Gini ratios, the condition δ greater than one has to be imposed. (Dagum, 1980a: pp. 350)

β and δ are the equality (shape) parameters for the lower and upper tail of the distribution and, λ = exp. of the constant of integration, a scale parameter (Dagum, 1977: pp. 424).

This theoretical model will adjust empirical distributions with null, negative or positive starting income, represented by the value taken by α in the formula; which are, in general, unimodal and positively skewed; with income range in the (X_0, ∞), where X_0 > 0 for sample distribution of employed individuals with starting positive income. The income elasticity of the CDF with respect to the origin α of F(x) is represented by a monotonic decreasing function of F(x). The income elasticity of the CDF converges to βδ, a finite and positive value, when income X tends to zero; and it converges to zero when income X tends to infinity (Dagum, 1977: pp. 421). The speed of convergence, examined in Dagum and Lemmi (1989: pp. 131), is constant for β = 1, decreasing for β > 1, and increasing for 0<β<1. Figure 2, illustrates this relationship with hypothetical values.

Figure 2



Note: own estimates with hypothetical values.

There are three versions of the Dagum model, each accounting for specific assumptions about the population of income receivers.

(a) Dagum Type I of a CDF corresponds to origin $\alpha = 0$, then it is said to best represent the behavior of (employed) wage earners.

$$(3)\quad F(x) = \frac{1}{\left(1 + l\, x^{-d}\right)^{b}}$$

Since $\alpha = 0$, Type I contains three parameters and describes distributions starting with no income recipients having zero income, an assumption commonly adopted to study the income distribution of employed wage earners, with the exception of Pareto (Dagum, 1983a). Parameters β and δ are scale free and help to explain equality.

(b) Dagum, Type II and Type III, include the fourth parameter α having specific economic meaning, then

$$(4)\quad F(x) = a + \frac{(1-a)}{\left(1 + l x^{-d}\right)^{b}}$$

is the corresponding four parameter CDF.

Type II is defined for $0 < \alpha < 1$, and α measures income units with zero or negative income; in Dagum (1977) it has been interpreted as a proxy for the rate of unemployment (because it indicates the percentage of population in the labor force having no positive earnings). If unemployment is high, α becomes critical to understand inequality in the labor force (Dagum, 1983a). Thus, α is an inequality parameter, while λ is a scale parameter that would allow time or space comparison between distributions when income data is expressed in different monetary units.

Type III is an appropriate representation of sample income distribution of total income receivers (wage earners plus those with income from property) because it starts accumulating income of a population with initial positive earnings (Dagum, 1983a).

The Gini ratio[2] associated with the Dagum model, is calculated through formula (5)

$$(5)\quad G = (2a - 1) + (1 - a)\frac{\Gamma(b)\Gamma\left(2b + \frac{1}{d}\right)}{\Gamma(2b)\Gamma\left(b + \frac{1}{d}\right)}$$

where $\Gamma(.)$ is the complete Gamma function specified in Dagum (1977: pp. 424).

The Gini ratio is an increasing function of α and, as proved in Dancelli (1986), tends to zero, perfect equality, with increasing values of β and/or δ; and tends to 1, perfect inequality, with decreasing values of β and/or δ. This means that there is an improvement of the income distribution when the values of β and/or δ are increased. More specifically, a movement of the central part of the distribution to the right, towards the upper tail, has the implication that a mass of "middle class" individuals receives a higher income, reflected in increased values of δ [∂ Gini / $\partial \delta < 0$]. In a similar way increasing values of β would signal the welfare improvement of low-income people [∂ Gini / $\partial \beta < 0$].

3. Model Estimation and Data

The Dagum model was fitted to the observed sample income data of employed and total active population reported in Greater Córdoba by the Permanent Household Survey ("EPH"). This study covers the period 1992-2000, during these years, the national currency,

[2] The Gini concentration ratio, or Gini coefficient is an aggregate inequality measure and can be obtained from information included in the construction of a Lorenz curve by calculating the ratio of the area between the diagonal and the Lorenz curve divided by the total area of the triangle in which the curve lies. This definition implies that Gini coefficients can vary anywhere from zero, perfect equality, to one, perfect inequality. This traditional approach produces an underestimation for the value of Gini coefficients because income distribution data are presented by class intervals and to calculate the area under the Lorenz curve only the middle points of the intervals are considered (See Dagum, 1987: pp. 531). Todaro observed that the Gini coefficient calculated from empirical income distributions typically lies between 0.50 and 0.70 for highly unequal income distributions, and it is on the order of 0.35 to 0.20 for countries with relatively equitable distributions (Todaro, 1997: pp. 146).

the peso, was linked to the dollar by a one to one relationship because of the Convertibility Law of 1991.The stability of the peso and low inflation, running below 1 percent per year on average, would facilitate intertemporal comparability of results.

The data were obtained from the Permanent Household Survey that is collected twice a year, in May and October. The results reported in this paper are based on the May survey. However, no significant differences have been found when the October survey was processed instead.

The parameters of the model were estimated using STATA 7 and the Econometric Package for Income Distribution (EPID) software developed by Dagum and Chiu's (1991) with similar output. In both cases, estimates for the parameters of the model were obtained using the non-linear least square method which applies an algorithm that searches for the minimization of the sum of the squared deviations of the actual from the fitted cumulative density function. Only results obtained through the use of STATA are reported, including the confidence intervals of the parameters, the adjusted R^2 and the "t" and "F" tests. The corresponding Gini ratio was estimated from the parameters of the model applying the Mathcad 8 software to solve equation (5).

3.1.Employed Income Receivers. Greater Córdoba, 1992-2000

Table 1 summarizes the main characteristics in the actual income distribution for the sample of Employed Income Receivers (EIR) of the Greater Córdoba, between 1992 and 2000.

Table 1: Basic Indicators of the Sample Data for Employed Income Receivers - Greater Córdoba

	1992	1993	1994	1995	1996	1997	1998	1999	2000
Sample Population	1,583	1,506	1,495	1,227	1,107	1,124	965	1,036	1,096
Mean Income ($)	482.1	567.4	662.6	655.2	610.6	577.5	549.9	591.4	549.9
Std. Error Mean	10.97	15.21	22.12	16.16	21.34	15.86	17.03	19.63	14.93
Median Income ($)	350	400	450	500	420	400	400	400	400
Variance	190,650	348,215	731,758	320,422	503,951	282,886	279,902	399,161	244,170
Skewness	3.08	7.53	11.26	3.48	7.33	4.12	4.69	4.64	2.60
Kurtosis	13.05	118.83	209.79	19.71	84.41	28.93	35.76	31.25	9.49
Minimum ($)	10	20	23	20	12	10	20	18	10
Maximum ($)	4,000	12,500	20,000	6,000	11,000	7,000	7,000	7,000	4,000

Note: Greater Córdoba includes Córdoba Capital, Villa Allende and Saldán. The Mean Income is calculated from the monthly individual total income that includes wage, earnings, profits, rents, unemployment insurance, scholarships and other incomes as stated in question 47T of EPH. The exchange rate was $ 1= u$s 1 during the whole period.
Source: own estimates, based on May EPH-INDEC.

The monthly mean income of sample EIR was $482 in 1992, it reached a peak of $663 in 1994 and decreased to $550 in 2000. The reported minimum income is close to zero in every year while the maximum was $4,000, in 1992 and 2000, and it reached a peak of $20,000 in 1994.

3.2.Economic Active Population. Greater Córdoba - 1992-2000

Table 2 summarizes the main characteristics in the actual income distribution for the sample of individuals in the Economic Active Population (EAP), those employed income receivers with positive income and the unemployed with positive or null income, at Greater Córdoba, between 1992 and 2000.

Table 2: Basic Indicators of the Sample Data for Economic Active Population - Greater Córdoba

	1992	1993	1994	1995	1996	1997	1998	1999	2000
Sample Population	1,668	1,626	1,727	1,476	1,382	1,405	1,141	1,222	1,276
Mean Income ($)	462.0	531.0	581.9	557.6	507.3	476.3	476.1	519.5	484.6
Std. Error Mean	10.7	14.5	19.8	14.7	18.1	13.9	15.4	17.7	13.7
Median Income ($)	320	400	400	410	400	390	360	400	373
Variance	189,952	340,911	678,831	319,527	453,292	272,157	269,371	381,142	240,397
Skewness	3.0	7.3	11.1	3.2	7.2	3.9	4.5	4.5	2.5
Kurtosis	12.9	116.6	215.2	18.3	87.8	27.6	34.9	31.1	9.5
Minimum ($)	0	0	0	0	0	0	0	0	0
Maximum ($)	4,000	12,500	20,000	6,000	11,000	7,000	7,000	7,000	4,000

Note: Greater Córdoba includes Córdoba Capital, Villa Allende and Saldán. The Mean Income is calculated from the monthly individual total income that includes wage, earnings, profits, rents, unemployment insurance, scholarships and other incomes as stated in question 47T of EPH. The exchange rate was $ 1= u$s 1 during the whole period.
Source: own estimates, based on May EPH-INDEC.

The monthly mean income in the sample of EAP was $462 in 1992, it reached a peak of $582 in 1994 and it decreased to $484 in year 2000. The reported maximum income fluctuated between $4,000, in 1992 and in 2000, and a peak of $20,000 was reached in 1994. The difference between the mean income of the EIR and the EAP had an important increased from 1992 to 2000 reflecting the effect of increased unemployment rate, from 4.8 percent in 1992 to 13.4 in year 2000, as it can be seen from Table 3. The rate of activity was 37.9 percent of total population in 1992, and it increased to 41.0 percent in year 2000. The employment rate followed a different path, it decreased from 36.1 percent in 1992 to 31.3 in 1996, and then it slightly recovered to 35.5 percent in 2000. Meanwhile the underemployment rate increased from 10.3 percent in 1992 to 14.4 percent in year 2000.

Table 3: Main Indicators of Greater Córdoba Labor Market.

Rates in %	1992	1993	1994	1995	1996	1997	1998	1999	2000
Activity (EAP/TP)	37.9	38.4	38.5	38.0	37.8	38.8	39.2	40.1	41.0
Employment (E/TP)	36.1	35.8	35.5	32.2	31.3	31.5	34.4	34.4	35.5
Unemployment (U/EAP)	4.8	6.8	7.8	15.2	17.2	18.6	12.5	14.2	13.4
Underemployment (UE/EAP)	10.3	8.6	10.2	10.8	13.7	13.3	13.8	11.2	14.4

Note: EAP stands for Economic Active Population, TP stands for Total Population, E stands for Employed Population, U stands for Unemployed Population, UE stands for Underemployed Population (people who work less than 35 hours a week). The exchange rate was $ 1= u$s 1 during the whole period.
Source: May EPH-INDEC.

4. Results

The results have been grouped in two parts: first, those corresponding to the estimation of the Dagum Type I model with data from the sample population of EIR reporting positive income; and second, those corresponding to the estimation of the Dagum Type II model with data from the EAP, which includes the employed and the unemployed (with positive or null income), that is the total labor force. Each of these parts includes the reported results for the cumulative income elasticity from 1992 to 2000, the Alpha, Beta, Delta and Lambda parameters and the associated Gini ratios. Finally, the section concludes presenting two comparisons: the first covers the resulting distribution of income of the two populations of income receivers in Greater Córdoba; and the second concentrates on a regional comparison between Córdoba and Buenos Aires.

4.1.Employed Income Receivers

The main purpose of this section is to report the results of an estimation of the CDF belonging to the population of the Employed Income Receivers (EIR) with positive income that will require the use of the three parameter Dagum Type I model specified in section 3.

The Cumulative Income Elasticity: The cumulative income elasticity [$\varepsilon_{X:F}$] of EIR, reflects the percentage change in cumulative population divided by the percentage change in income. The fitted values in Table 4 are those of the three parameters of the Dagum Type I model and have been estimated from the differential equation (1).

Table 4: Cumulative Income Elasticity by Income Level of EIR Greater Córdoba (fitted values)

Income (in $1000)	$e_{X:F}$	
	Year 1992	**Year 2000**
0.000	5.730	2.255
0.120	4.059	2.120
0.240	2.141	1.729
0.360	1.191	1.281
0.480	0.732	0.918
0.600	0.489	0.661
0.720	0.347	0.486
0.840	0.258	0.366
0.960	0.199	0.283
1.080	0.158	0.223
1.200	0.128	0.180
1.500	0.082	0.112
2.000	0.046	0.060
2.500	0.029	0.037
3.000	0.020	0.024
4.000	0.011	0.013

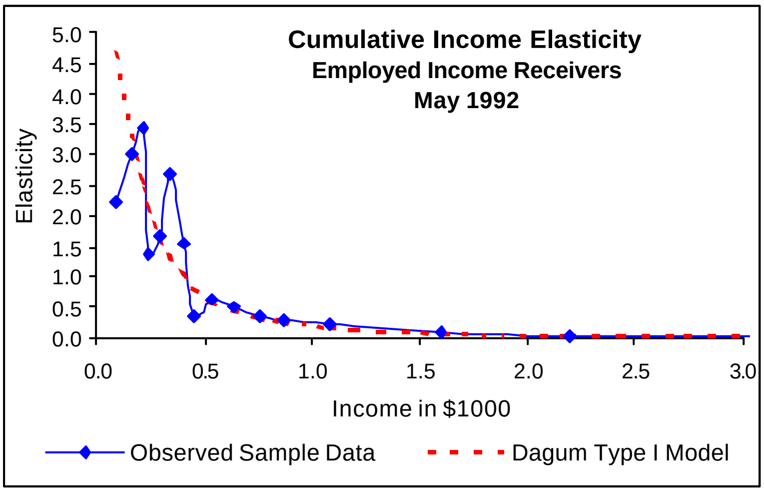
Source: own estimates, based on May EPH-INDEC.

Figures 3 and 4 illustrate the actual and fitted relationship of the cumulative income elasticity to income level in 1992 and 2000 respectively. The cumulative income-elasticity corresponding to the fitted income distribution function is shown by the dotted lines. In Figure 3 the fitted income distribution starts accumulating with [$\varepsilon_{X:F}$=5.73] at the origin; then the elasticity decreases smoothly to [$\varepsilon_{X:F}$=1] at about the $475 level and falls to almost zero for income levels above $2,500.

In Figure 4 the fitted income distribution shows a pattern of stable regularity in its entire domain. Its maximum value, found at the origin, is [$\varepsilon_{X:F}$=2.25]. Then it decreases smoothly with increased income levels as predicted in the model, it reaches [$\varepsilon_{X:F}$=1] at about the $475 level and rapidly converges to zero at income levels higher than $2,000.

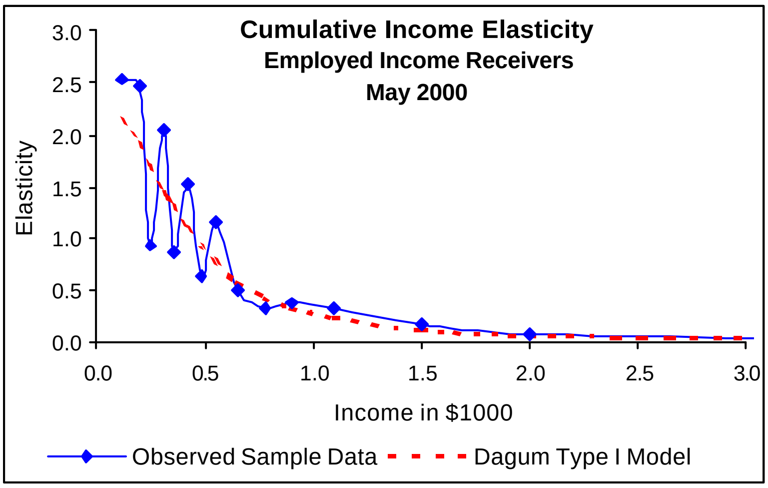
The continuous line shows the pattern exhibited by the cumulative income elasticity when it is calculated from actual income distribution data. In 1992 as well as 2000, the elasticity calculated from actual and fitted values follow a similar pattern when the whole income range is considered. Yet, for actual income distribution data, the relationship appears to be less stable at the bottom of the income scale (i.e. below $500); although this may be the result of the high number of responses found in the survey with declared income of $200, $300 or $400.

Figure 3



Note: own estimates, tables are available from the authors.

Figure 4



Note: own estimates, tables are available from the authors.

The Fitted Cumulative Density Function: The CDF corresponding to the Dagum Type I model is the solution of the differential equation (1), which is the cumulative income elasticity that was discussed above. Table 5 summarizes the results corresponding to the Dagum Type I model.

Table 5: Sample Estimates of the Parameters of the Dagum Type I Model EIR - Greater Córdoba - 1992-2000

Year	Beta	Delta	Lambda*	df	Adj-R^2	*F*
1992	*2.8275* (16.44)	*2.0261* (117.20)	*0.0331* (18.57)	*1580*	*0.9987*	*392994*
1993	*1.9106* (23.39)	*2.1599* (117.15)	*0.0651* (26.30)	*1503*	*0.9988*	*425895*
1994	*2.5438* (18.97)	*2.0421* (121.31)	*0.0648* (20.01)9	*1493*	*0.9988*	*429293*
1995	*2.1862* (19.38)	*2.2480* (107.27)	*0.0757* (20.60)	*1224*	*0.9988*	*339350*
1996	*1.7719* (20.63)	*2.2918* (96.02)	*0.0727* (23.23)	*1104*	*0.9987*	*286677*
1997	*1.2881* (26.98)	*2.4377* (95.89)	*0.0953* (31.69)	*1121*	*0.9987*	*298078*
1998	*1.0394* (32.36)	*2.5392* (97.33)	*0.1050* (40.39)	*962*	*0.999*	*333723*
1999	*1.0609* (28.50)	*2.4648* (86.31)	*0.1151* (34.52)	*1033*	*0.9987*	*268439*
2000	*0.9974* (33.04)	*2.2604* (95.31)	*0.1307* (39.30)	*1093*	*0.9989*	*319274*

Note: * Lambda stands for an income of $1,000. t values reported between brackets.
Source: own estimates, based on May EPH-INDEC.

The sixth column from the left indicates Adjusted R^2 as a measure of the goodness-of-fit of the parametric estimate of the CDF to actual data. The high Adjusted R^2 confirms the goodness-of-fit of the Dagum Type I model applied to Greater Córdoba sample population of income receivers in all the years. Lambda, the scale parameter, helps in transforming the currency units of the income data in comparative country analysis (Dagum and Lemmi, 1989: pp.144). The last column shows the F statistics that is evaluated with the corresponding degree of freedom (df) reported in the fifth column. The parameters Beta and Delta are scaleless and help to interpret equality. A rise in Beta values would reflect a welfare improvement in the groups of population at the lower tail of the distribution, those with the lower income levels. The value of Delta will increase with improvement in the welfare situation of population groups at the middle and upper tail of the income distribution. The decreasing values for Beta shown in Table 5, column 2, would suggest that the welfare situation of low-income earners of Greater Córdoba had deteriorated between 1992 and 2000. A lower Beta causes a widening of the lower tail of the distribution implying, for example, that the quantity of income receivers in the first two deciles (or those with monthly income below $100, or the ones in the informal labor market) increased from 1992 to 2000. At the same time, a decline in Beta would tend to lower the mean value of the distribution and will increase the variance. According to EPH data, workers in the informal job market, as a percentage of total employment, grew from 33.4 percent in May 1992 to 41.6 percent in May 2000.

To reinforce this interpretation, suppose for a moment that the mass of income receivers is concentrated as close to zero as possible, this would imply a very small value for Beta (close to zero). This mean that the F(x) rapidly tends to 1. Consequently any increase in the value of Beta would represent an improvement in income distribution as the mass of income receivers is moving to the right of the distribution. In terms of Table 5 the different values of Beta obtained for 1992 – 2000, clearly show that the economic situation of income receivers worsened as Beta values followed a decreasing trend. In terms of Figure 2, the elasticity curve is moving up and to the right, which confirms the deterioration of the income distribution in Córdoba between 1992 and 2000.

The value of Delta increased from 1992 to 1998 and decreased thereafter with an overall gain for the 1992-2000 period, implying that the relevant middle and high income groups might have partially contributed to counterbalance other causes of inequality during the last decade in Greater Córdoba. In fact, according to the model, the trend in the Delta coefficient would imply that the mass of income of middle and upper income receivers shifted to the right during the decade, with the exception of the last two years when higher income receivers might have accused a welfare loss.

Table 6 provides supplementary information to assess the significance of the change in the estimated parameter values over time. For example, using the confidence intervals of Beta, it is possible to asses whether the parameter is increasing, stable or decreasing. As the upper bound of the confidence interval for every year in the period 1996 - 2000 lies below the lower bound reported yearly in 1992 - 1995, the trend of Beta is definitely decreasing. Similarly, but in the opposite direction, it can be seen that the trend of Delta is increasing, except for the last two years.

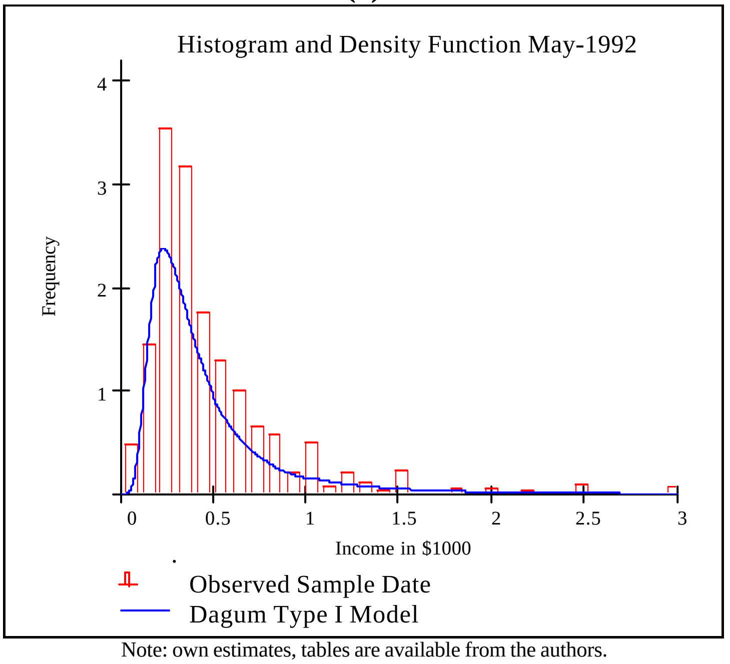
Table 6: Sample Estimates of the 95 percent Confidence Intervals for the Parameters of Dagum Type I Model. EIR - Greater Córdoba - 1992-2000

Year	Beta		Delta		Lambda	
	Lower	Upper	Lower	Upper	Lower	Upper
1992	2.4902	3.1648	1.9922	2.0600	0.0296	0.0366
1993	1.7504	2.0708	2.1237	2.1961	0.0602	0.0699
1994	2.2808	2.8068	2.0091	2.0751	0.0584	0.0711
1995	1.9649	2.4075	2.2068	2.2891	0.0685	0.0830
1996	1.6033	1.9404	2.2450	2.3386	0.0666	0.0789
1997	1.1944	1.3818	2.3878	2.4876	0.0894	0.1012
1998	0.9763	1.1024	2.4880	2.5904	0.0999	0.1101
1999	0.9878	1.1339	2.4087	2.5208	0.1085	0.1216
2000	0.9382	1.0567	2.2138	2.3069	0.1242	0.1372

Note: STATA was used to estimate the 95 percent confidence intervals.
Source: own estimates, based on May EPH-INDEC.

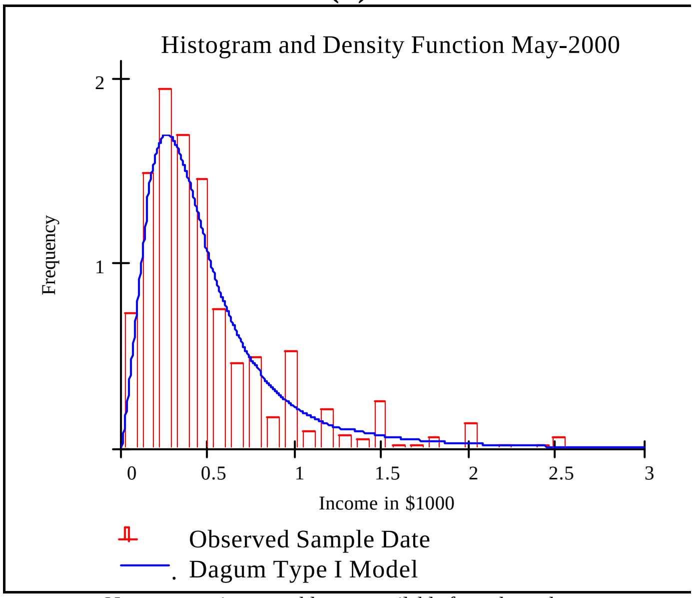
Figures 5 (a) and (b) provide a visual representation of the estimated Dagum Type I model and the observed values for the density function corresponding to the years 1992 and 2000. They confirm the goodness-of-fit obtained from the estimation, as it can be seen in Table 6, through the values of F and t-tests.

Figure 5
(a)



Note: own estimates, tables are available from the authors.

(b)



Note: own estimates, tables are available from the authors.

The Gini Coefficient: The Gini ratio reported here was calculated from the Dagum Type I model using formula (5). The 95 percent confidence intervals were estimated through the intervals of the parameters. A non-parametric estimate for the Gini ratio (Slottje, 1999) is also reported. The area below the Lorenz Curve was estimated through numerical integration using about 46 points jointly determined by the cumulative income percentiles and the cumulative percentiles of income receivers. Then, a comparative analysis of both Gini ratios was performed (Table 7).

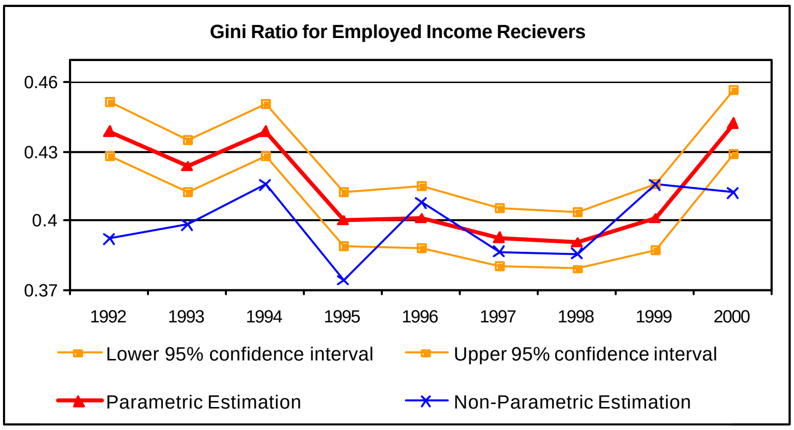
Table 7: Gini Ratio Estimations using Parametric and Non-parametric Methods. EIR - Greater Córdoba - 1992-2000

Year	Dagum Type I Model			Non-parametric Gini ratio
	Gini ratio	95% confidence intervals		
		Lower	Upper	
1992	0.43913	0.42778	0.45171	0.39186
1993	0.42355	0.41256	0.43545	0.39822
1994	0.43875	0.42781	0.45072	0.41530
1995	0.40007	0.38871	0.4126	0.37430
1996	0.40075	0.3881	0.41469	0.40776
1997	0.39244	0.38008	0.40585	0.38646
1998	0.39088	0.37884	0.40382	0.38614
1999	0.40121	0.38746	0.41613	0.41578
2000	0.44261	0.42922	0.45692	0.41206

Note: The Gini ratio estimated using the MATHCAD for the Dagum Type I model is a parametric one and the Gini ratio observed from sample data was calculated through a non-parametric method.
Source: own estimates, based on May EPH-INDEC.

The Gini ratios estimated using the Dagum model are U shaped for the period 1992-2000 implying that income distribution of the EIR sample population first improved, from 1992 to 1998, and then deteriorated. The non-parametric estimations of the Gini ratio show a less definite trend, and they are generally lower than the Gini ratio obtained from the parametric model. The resulting different trends in theoretical and empirical Gini calculations are shown in Figure 6.

Figure 6



Note: own estimates, see Table 7.

4.2.Economic Active Population

This section reports the results of an estimation of the CDF belonging to the Economic Active Population (EAP) that will require the use of the four parameter Dagum Type II model specified in section 2.

The Fitted Cumulative Density Function: The fitted CDF corresponding to the EAP is obtained by solving the differential equation (1) that now corresponds to the Dagum Type II model with four parameters. The parameter Alpha is now included and will assume positive values, measuring the extent of unemployment (when income X tends to zero, the CDF, the probability to find individuals with income less or equal to X, tends to Alpha). Therefore, an increase in Alpha would negatively affect equality. The parameters Beta and Delta are the equality parameters in the model and Lambda is a scale parameter. Table 8 summarizes the values of Alpha, Beta, Lambda and Delta estimated from the Dagum Type II model for individuals in the EAP having positive and null income, in Greater Córdoba, for the period 1992-2000.

Table 8: Sample Estimates of the Parameters of the Dagum Type II Model. EAP - Greater Córdoba - 1992-2000

Year	Alpha	Beta	Delta	Lambda*	df	Adj-R^2	*F*
1992	*0.0388* (20.56)	*2.7895* (14.10)	*2.0289* (112.74)	*0.0334* (15.83)	*1664*	*0.9988*	*339616*
1993	*0.0439* (25.91)	*1.5024* (23.23)	*2.2261* (107.51)	*0.0793* (26.78)	*1622*	*0.9989*	*364230*
1994	*0.0638* (42.92)	*1.1584* (25.58)	*2.2283* (92.54)	*0.1313* (28.40)	*1723*	*0.9986*	*309669*
1995	*0.0732* (44.80)	*0.8774* (25.43)	*2.5067* (73.67)	*0.1714* (28.76)	*1472*	*0.9983*	*222395*
1996	*0.0808* (46.08)	*0.6926* (24.73)	*2.6686* (60.46)	*0.1557* (32.01)	*1378*	*0.9980*	*176239*
1997	*0.0817* (47.58)	*0.5385* (27.95)	*2.8440* (57.95)	*0.1935* (37.32)	*1401*	*0.9981*	*180672*
1998	*0.0654* (38.75)	*0.6266* (30.70)	*2.7819* (70.66)	*0.1511* (42.25)	*1137*	*0.9988*	*231417*
1999	*0.0640* (35.33)	*0.6611* (28.47)	*2.6760* (67.61)	*0.1663* (37.16)	*1218*	*0.9986*	*212130*
2000	*0.0599* (34.65)	*0.6144* (32.49)	*2.4940* (73.92)	*0.1914* (41.25)	*1272*	*0.9987*	*247208*

Note: * Lambda stands for an income of $1,000. t values reported between brackets
Source: own estimates, based on May EPH-INDEC.

We can see in Table 8, that the reported F and t-tests are strong. The column of Adjusted R^2 provides an additional measure of the goodness-of-fit of the parametric estimate of the CDF to actual data. As it was previously explained, the parameters Alpha, Beta and Delta are scaleless and help in interpreting equality. Also a growing Alpha reflects the increased number of individuals having no income in the EAP. Observing column 2 in table 8, the values of Alpha are consistent with the real unemployment rate for each year. For example, the observed unemployment rate for 2000 of about 13 percent of the EAP is equivalent to approximately 6 percent of the Total Population, the value of Alpha for that year. More generally, Alpha increased from 1992 to 1997, had a sharp decrease in 1998 and again in 2000. In spite of the decrease, the value for 2000 exceeded that of 1992 by almost 60 percent. A rise in Beta values reflects the welfare improvements in the population groups at the lower tail of the distribution, those with the lower income levels, while Delta increment implies improvement in the welfare situation of population groups at the middle and upper tail of the income distribution. Table 8, column 3, shows a strong decrease in Beta values, from 2.8 to 0.6 in 1992 and 2000 respectively. This would suggest a deteriorating trend in the welfare situation of EAP members located at the left tail of the income distribution. The increase in unemployment and the multiplication of low paid jobs in the informal job market would help to explain this. The value of Delta increased from 1992 to 1997 but decreased from 1998 to 2000.

Table 9 provides additional information to assess the goodness-of-fit of the model. For example, one intuitive way to check whether the parameters have followed a definite trend over time would be to compare if the lower bound of the confidence interval in year t lies above the upper bound of the confidence interval in year t+n (a decreasing trend between year t and t+n would be confirmed). Let´s observe the Alpha parameter. The upper limit in 1992 is 0.04 and the lower limit in 2000 is 0.05; therefore, there is strong evidence that the estimated parameters belong to different populations: the one in 1992 having less unemployment than the respective population in 2000. Similar interpretations can be drawn for Beta and Delta with respect to structural changes in equality.

Table 9: Sample Estimates of the 95 percent Confidence Intervals for the Parameters of the Dagum Type II Model. EAP - Greater Córdoba - 1992-2000

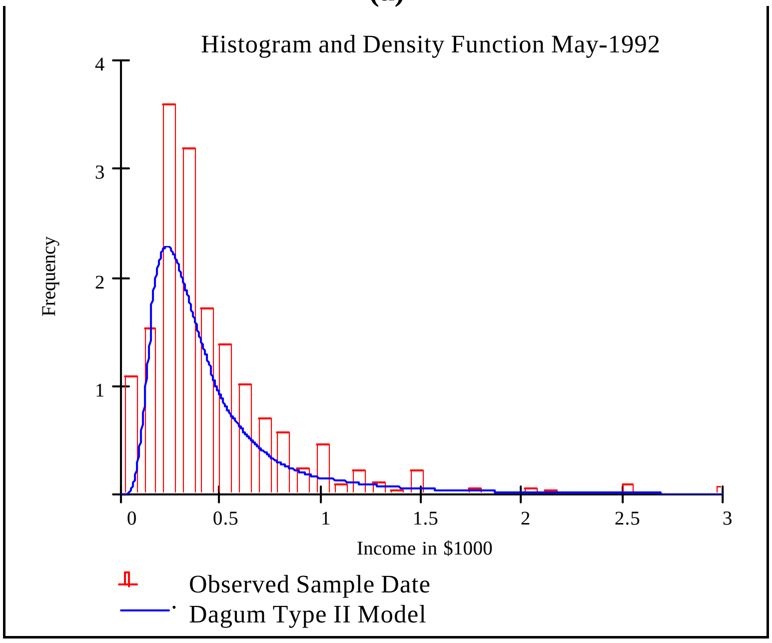
Years	Alpha		Beta		Delta		Lambda	
	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
1992	0.0351	0.0425	2.4014	3.1777	1.9936	2.0642	0.0292	0.0375
1993	0.0406	0.0473	1.3755	1.6293	2.1855	2.2668	0.0735	0.0851
1994	0.0609	0.0667	1.0696	1.2472	2.1811	2.2756	0.1223	0.1404
1995	0.0700	0.0764	0.8098	0.9451	2.4400	2.5735	0.1597	0.1831
1996	0.0774	0.0842	0.6376	0.7475	2.5820	2.7552	0.1462	0.1653
1997	0.0784	0.0851	0.5007	0.5763	2.7478	2.9403	0.1833	0.2036
1998	0.0621	0.0687	0.5866	0.6667	2.7046	2.8591	0.1441	0.1581
1999	0.0604	0.0675	0.6155	0.7067	2.5984	2.7537	0.1575	0.1751
2000	0.0565	0.0633	0.5773	0.6515	2.4278	2.5602	0.1823	0.2005

Note: STATA was used to estimate the 95 percent confidence intervals.
Source: own estimates, based on May EPH-INDEC.

Figures 7 (a) and (b) illustrate the fitted density function corresponding to the EAP in 1992 and 2000 respectively. Notice that the first interval from the left in the histogram that includes the population group with null income had an important increase between 1992 and 2000.

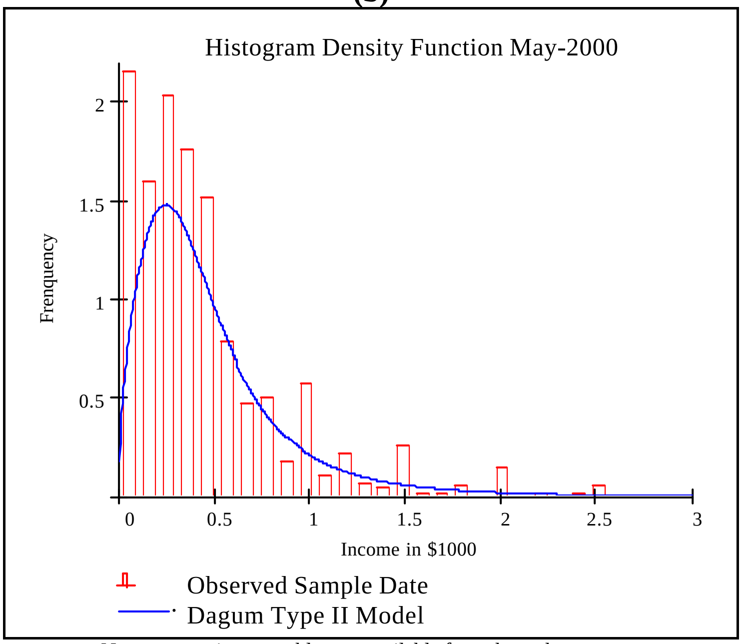
Figure 7

(a)



Note: own estimates, tables are available from the authors.

(b)



Note: own estimates, tables are available from the authors.

The Gini Coefficient: The Gini ratio reported here was calculated from the Dagum Type II model, hence it is sensitive to changes in the Beta and Delta equality parameters and the Alpha inequality parameter as well. The 95 percent confidence intervals were estimated through the intervals of the parameters. Table 10 reports a non-parametric estimate for the Gini ratio, also a comparative analysis of both estimates is performed

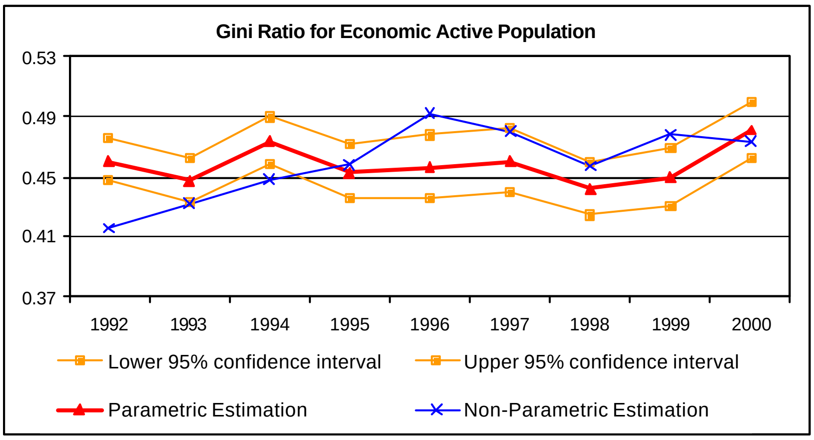
Table 10: Gini Ratio Estimations using Parametric and Non-parametric Methods. EAP - Greater Córdoba - 1992-2000

Year	Dagum Type II Model			Non parametric Gini ratio
	Gini ratio	95% confidence intervals		
		Lower	Upper	
1992	0.46065	0.44681	0.47592	0.41600
1993	0.44731	0.43359	0.46204	0.43160
1994	0.47363	0.45858	0.48974	0.44786
1995	0.45290	0.43553	0.4716	0.45784
1996	0.45550	0.43533	0.4774	0.49194
1997	0.46020	0.43966	0.48238	0.48014
1998	0.44173	0.42408	0.46058	0.45722
1999	0.44914	0.43033	0.46922	0.47744
2000	0.48056	0.46254	0.49964	0.47324

Note: The Gini ratio estimated using MATHCAD for the Dagum Type II model is a parametric one and the Gini ratio observed from sample data was calculated through a non-parametric method.
Source: own estimates, based on May EPH-INDEC.

The parametric Gini exhibits relative stability, with variations in the interval (0.44, 0.47) from 1992 to 1999, it then increased to a peak of 0.48 in year 2000. The non parametric Gini followed a rising trend from 0.41 to 0.49 in 1992-1996; then it decreased to 0.45 in 1998 and finally it increased up to 0.47 in 2000. At the beginning of the period, the parametric Gini was above the non-parametric but the second went to the top after 1995 and they reached similar values in 2000. This is clearly seen in Figure 8, below.

Figure 8



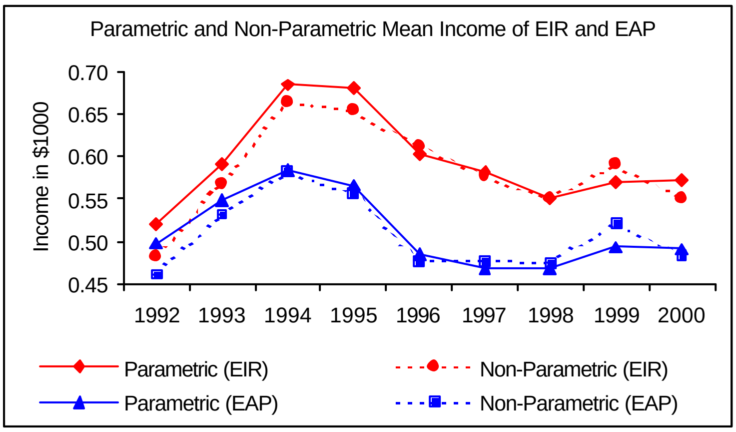
Note: own estimates, see Table 10.

5. Comparative Analysis

5.1. Employed Income Receivers versus Economic Active Population in Córdoba

Figure 9 shows the mean income of both the Employed and the Economic Active Population for the period 1992-2000. The two continuous lines reflect the results from parametric estimations, using Dagum Type I for the EIR and Dagum Type II for the EAP. The dotted lines indicate the non-parametric estimations from the sample data.

Figure 9



Note: Parametric estimations for EIR used Dagum Type I model and for EAP used Dagum Type II model.
Source: own estimates, tables are available from the authors.

The mean income in the sample of EIR is higher relative to the mean income of the sample for EAP. In 1992-1994 the difference between the two means remained stable. However, starting in 1995 the difference widens. The movements in the parameters of equality may help to understand what causes this.

The equality parameter Beta exhibits the same decreasing tendency in both population groups meaning that the economic welfare of population subgroups, mostly at the lower tail of the distribution, has worsened. However, the average rate of decrease is higher for the economic active population (17.2 against 12.2 percent), reflecting the increasing weight of individuals with zero and close to zero income that are included only in the EAP.

The parameter Delta starts in 1992 with similar values and exhibits an increasing trend in both population groups during 1992-1997. In 1998-2000 the trend is reversed but for the whole period Delta values increased. This may imply that the rise in unemployment has not affected the relative welfare of the middle and upper income subgroups in both populations.

The parameter Alpha is only included in the Dagum Type II model because it specifically measures the effect of unemployment on the income distribution of the Economic Active Population. During the period 1992-2000, it had an increasing trend that reflected the growing unemployment rate.

The comparative analysis of the parametric Gini ratio for the EIR and EAP shows higher values (more inequality) for the EAP summarizing the effects of Alpha, Beta and Delta reported in Tables 7 and 10.

5.2.Regional disparities: Córdoba versus Buenos Aires in the nineties

Comparative analysis of personal income distribution in different regions of Argentina has an interest of its own. Parametric income distribution functions are particularly helpful in assessing whether the pattern of personal income distribution in geographic regions differs. However, the main economic, social and institutional debates on economic development of Argentina that took place during the nineties, were concerned about “the Capital region” versus the “rest of the country” and centered on questions of poverty, malnutrition, intergovernmental grants and provision of public goods. But discussions about more specific personal income distribution issues have not been as visible yet. The purpose of this section is to illustrate, through a comparative analysis of parametric income distribution functions, how personal income distribution in the Capital region of Argentina, represented by the Greater Buenos Aires[3], differs from the rest of the country, reflected by Greater Córdoba[4].

Table 11 summarizes the resulting 1992 and 1997 values of parameters Beta, Lambda and Delta for a Dagum Type I Income Distribution Function and the associated Gini Coefficient estimated for Greater Córdoba and Greater Buenos Aires for EIR.

Table 11: Comparative Values of Parameters in Greater Córdoba and Greater Buenos Aires.

Year	Greater Córdoba				Greater Buenos Aires			
	Beta	Lambda*	Delta	Gini C.	Beta	Lambda*	Delta	Gini C.
1992	2.8280	0.0331	2.0260	0.4392	2.8820	0.0590	2.0470	0.4335
1997	1.2882	0.0953	2.4377	0.3924	1.0730	0.2120	2.1890	0.4514

Note:* Lambda, the scale parameter in the distribution, represents units of income in $1000
Source: Greater Córdoba: own estimations; Greater Buenos Aires: Botargues and Petrecolla (1999)

As it was previously shown by the standard Gini Coefficient analysis performed in section fourth, between years 1992 and 1997 there was an improvement of 10.6 percent in Córdoba (a decrease from 0.439 to 0.392), at the same time it occurred a deterioration of 4.1 percent in Buenos Aires (rising from 0.434 to 0.451).

The Beta and Delta parameters provide information that helps to assess the relative position of the mass of the population. An increase in Beta would reflect that the group of individuals located at the poorest segments in the population are loosing weight, or that the poverty situatio n at the left of the distribution is now less acute. Turning to Table 11, however, the value of the Beta coefficient for Córdoba and Buenos Aires in 1997 was lower than the corresponding 1992 value, hence the relative number of individuals further at the left of the distribution (i.e. persons to the left of the modal income) increased. Therefore, it can be concluded that deterioration in overall income distribution, in both regions, might be associated with the intensity of the observed decrease in Beta values in the period under study.

An increase in Delta would reflect rising concentration around the mean value of the distribution, therefore it can be interpreted as rendering information that help to asses movements in middle and upper middle income groups. Once again in Table 12, the observed value of the Delta coefficient was indeed higher in 1997 for Córdoba and Buenos Aires than the corresponding 1992 value, meaning that in both cases the relative number of

[3] Botargues and Petrecolla, (1999)

individuals concentrated around the mean of the distribution (i.e. persons to the right of the modal income) increased. Therefore, it can be concluded that the observed change in Delta values in both regions positively contributed to increase equality in overall income distribution between 1992 and 1997.

Finally, the exercise has shown that the classical Gini Coefficient analysis can be further improved by using parametric income distribution functions because the parameters give precious additional information. In Greater Córdoba, income distribution improved during the 1992-1997 period. Decreasing Gini values, might be a plausible result associated with strong improvements visualized in the relative position of middle income individuals (i.e. those having better job market opportunities) despite the deterioration in the relative situation of persons at the lower tail of the distribution. In Buenos Aires, the relative improvement of income groups around the central values of the distribution had less impact compared to Córdoba, and was clearly not enough to overcome the deterioration observed for those in the lower tail of the distribution. Henceforth, the corresponding Gini ratio for Buenos Aires increased.

6. Final remarks

This paper intended to shed some light over three questions: (i) are differences to be found in the shape of personal income distribution of the employed and of the total active population, in middle-size cities?; (ii) does unemployment play any role in explaining part of these differences?; (iii) what can comparative analysis of Greater Córdoba and Greater Buenos Aires tell us about the distinct pattern of personal income distribution in middle-size cities and the Capital region?

Parametric Dagum generating functions were applied to this purpose. They exhibit several advantages over empirical calculations. The Dagum model contains location and shape parameters that help to record movements in the mass of the distribution that cannot be derived from empirical calculation of the distribution curve, but are key in discussing what the traditional Gini coefficient analysis of inequality is trying to tell us.

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[4] See section 4.

Concerning the first two questions above, the comparative analysis covered the employed and the economic active population in Greater Córdoba from 1992 to 2000. The Dagum distribution model with three parameters, (the Beta and Delta providing equality information and Lambda, the scale parameter) and the four-parameter distribution model (adding the Alpha proxy parameter for unemployment that captures the change in the total number of unemployed individuals with null income) were used to study the personal distribution of income of employed individuals and the economic active population, respectively. Parametric Gini ratios were derived from both models. It was shown that in both cases, the equality parameters Beta and Delta followed different and independent trajectories. Beta helps to understand movements in the lower tail of the distribution and its decrease between 1992 and 2000 implied that the welfare situation of individuals in the lower percentiles of the distribution had worsened. The Delta coefficient reflects the effects of movements in the middle and upper-middle part of the distribution. During the period analyzed Delta increased indicating that the welfare of individuals in the upper tail improved. Computing the joint effect of these movements, the Delta increase may have partially offset the weakening situation in the lower tail signaled by Beta.

Considering the specific case of the Economic Active Population, the value obtained for the Alpha coefficient in the four-parameter model confirmed the negative effect of increasing unemployment on equality.

These results have direct consequences for explaining movements in the Gini coefficient. While the empirical Gini does not allow separating the sources of variations, the use of the Dagum model we have applied here suggests two main sources: the level of unemployment and the movements in the mass of the distribution.

The inclusion of the unemployed population in the income distribution analysis shifted the curve originally calculated for the employed population to the left, generating a higher Gini ratio for the Economic Active Population. Observed changes in the rate of unemployment over time affected the Gini ratio. In the case of Greater Córdoba, they caused an increase in the value of the estimated Parametric Gini ratio for the Economic Active Population in year 2000 relative to 1992.

Finally, a comparison between Córdoba and Buenos Aires was carried out for 1992 and 1997. The results confirmed the existence of important regional disparities. Analyzing the values of the estimated parameters of the Dagum model it is possible to explain some of the observed differences. It was found that between 1992 and 1997 there was a slightly income distribution improvement in Greater Córdoba as was shown by the decreasing Gini values. This result might in turn be associated with strong improvements in the relative position of middle-income individuals. On the other hand, in Buenos Aires, the relative improvement of income groups around the central values of the distribution had less impact compared to Córdoba. But this improvement was not enough to overcome the deterioration observed for individuals in the lower tail of the distribution. Thus, the corresponding parametric Gini ratio for Buenos Aires increased

The paper has shown some useful applications of parametric income distribution functions. More specifically, the model provided tools to assess sources of variation in income distribution and their impact on equality measures. Further studies should include other relevant social and economic characteristics of the population, such as education, sex and race in order to explain more accurately the movements in the mass of the personal income distribution of individuals.

Annex

Table A1: Gini Coefficients by Countries and Regions

Countries and Regions	***Gini C.***	***Countries and Regions***	***Gini C.***
Latin America		***Europe*** *cont.*	
Bolivia	0.4204	Netherland	0.2859
Jamaica	0.4290	Finland	0.2993
Ecuador	0.4300	Ireland	0.2993
Venezuela	0.4442	Germany	0.3122
Costa Rica	0.4600	Denmark	0.3209
Peru	0.4799	Norway	0.3421
Colombia	0.5151	Italy	0.3493
Chile	0.5184	Portugal	0.3744
Mexico	0.5385	France	0.4311
Brazil	0.5732	***Asia***	
Argentina	***0.4618***	Taiwan	0.2962
North America		India	0.3255
United States	0.3528	China	0.3268
Europe		Indonesia	0.3349
Hungary	0.2465	Korea, Rep. of	0.3419
Poland	0.2569	Japan	0.3482
Romania	0.2583	Singapore	0.4012
United Kingdom	0.2598	Hong Kong	0.4158
Belgium	0.2701	Thailand	0.4548
Spain	0.2790	Malaysia	0.5036

Source: Deininger, K and Squire, L (1996). Measuring income inequality: a new data set. The World Bank Economic Review 10, pp.565-91 (Table 1, Col.3)

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