Evaluating Equality using Parametric Income Distribution Models

An exploration of alternative effects using a Dagum Parametric Income Distribution Model

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Abstract

JEL Classification: C1, D3, I3

The paper seeks to contribute to the study of income policy effects on the distribution of personal income. A parametric model of the Dagum type will be applied to this purpose. The Dagum model of income distribution provides a good fitting to observed income distributions of developed as well as developing countries. The parameters of this model represent conditions of equality for different subgroups in the population, capture the effect of variations in the rate of unemployment and help to calculate a parametric Gini ratio. The power of the Dagum model in providing accurate descriptions of the whole range of income is supported by empirical applications based on income data from several countries: Canada, the United States, Italy and Argentina. Therefore, the model seems interesting to go beyond descriptions. In fact, through the simulation of variations in the value of the parameters assessments of subgroups who may benefit or lose when economic conditions change can be performed. Also, the sensitivity of the Gini ratio to variations in the parameters can easily be explored. An empirical study of income distribution in Greater Cordoba is used to compare the change in the situation between 1992 and 2000.

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1. Introduction

The last decade witnessed the deterioration of equality in the distribution of income and increased unemployment in most of the developed as well as developing countries. For policy-makers, these parallel developments were interpreted as bringing enough evidence in support of the idea that the rise in unemployment was behind the deteriorating conditions of equality in society. Then, to abate unemployment became a challenge in most countries, and the fight against unemployment was soon became the first goal of income policy. The scope of income policy then narrowed and became increasingly related with the practice of implementing policies and programs that, based on subsidies and income transfers, were specifically oriented to abate unemployment. So far, results around the world, perhaps with the exception of the U.S. economy, are not very encouraging. After several years of practicing this approach to income policy, unemployment has remained at the high level it reached during the previous decade. Therefore, it is time to assess whether this practice was an effective way for governments to apply income policies.

The paper seeks to contribute to the study of income policy effects on the distribution of personal income. A parametric model of the Dagum type will be applied to this purpose. The model is particularly interesting because it shows the association between the probability density function (PDF) of a population of individuals in the labor force, with certain parameters representing the level of unemployment and other economic conditions. In fact, unemployment may operate over the distribution of income through channels that are more complex and indirect than previously thought. The analysis will be based on a simulation of partial and combined effects of the parameters in the PDF of individuals in the labor force.

The distribution of income may indeed deteriorate or improve as a result of a number of factors. For example, a deterioration is said to exist when the relative distance between the first and the last decile of individuals in the labor force widens, irrespective of movements in the unemployment rate. Similarly, when the distribution moves as a whole to the left, the mean income of each decile decreases (as does the total distribution mean) and this type of movement

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is also interpreted as a deterioration, that might not be necessarily associated with changes in the level of unemployment.

The proposed model exhibits several interesting features that would help in exploring alternative explanations. First, the model is quite strong in reproducing the probability density function of economic units among the population. Empirical applications based on income data from several countries: Canada, the United States, Italy and Argentina (Dagum 1983, 1990, Botargues and Petrecolla 1999) provided strong evidence of the power of the Dagum model in producing superior descriptions of the whole range of income. Second, it does so with two parameters "beta" and "delta" that represent conditions of equality for different subgroups in the population while a third parameter "alpha" captures the effect of variations in the rate of unemployment. Third, the distinct effects associated with each of the parameters have specific economic meaning and intervene in the calculation of the parametric Gini ratio. Fourth, the Dagum function has also a forth parameter "lambda", that accounts for the monetary scale, henceforth it is a tool to adjust for inflation and to facilitate cross-country comparisons of income distributions that are expressed in different monetary units. However, the equity parameters are scale free in the Dagum model, thus, the scale parameter does not affect the measurement of the parametric Gini ratio.

The Dagum model is applied to data representing individuals in the labor force in Greater Cordoba. First, the cumulative and the probability density functions are calculated from the May Household Survey (EPH- Encuesta Permanente de Hogares of 1992 and 2000) applying equation (2). Then, the 1992 estimates of the position parameters in the distribution (the modal, median and mean income) are taken as a benchmark, a modification in the original parameters of the equation is simulated and the relative and joint effects on the mode, the median and the mean income and on the Gini ratio of altering the original alpha, beta and delta values of the PDF are evaluated. Third, two scenarios will be constructed: (i) the data for 2000 would help to set up the less desirable economic scenario while (ii) the 1992 data will be used to simulate the more desirable scenario. Three questions will be explored here: (i) how sensitive is the value of the Gini ratio to changes in the average rate of unemployment?, (ii) what sort of changes can be predicted for the welfare of economic units at the lower tail of the distribution when the measures of location in the distribution (the mean, the median and the modal income) moves up or down by a certain amount? and, (iii) what would be the impact of these movements on middle and upper-middle individuals in the labor force? The next section reviews the model. The following develops a parametric simulation exercise. This will help to examine how movements in the parameters of the model (alpha, beta and delta) will affect the modal, median and mean income and the Gini ratio. The paper concludes with an application of the simulation

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exercise to assess the relative contribution of alpha, beta and delta in explaining the observed change in the distribution of income in Greater Cordoba between 1992 and 2000.

2. The model

Theoretical representations of income distribution have traditionally relied on two-parameter cumulative density functions because these are relatively easy to estimate. However, the two-parameter models cannot deal with the existence of null and negative incomes or an unknown positive but small minimum value of income without the introduction of assumptions that limit their goodness-of-fit (Dagum, 1980). Instead, Dagum (1977, 1980) proposed a superior theoretical description based on the observed characteristic of regularity of income-elasticity in observed income distributions. In general, Income-Elasticity of the Cumulative Distribution Function (CDF) with respect to the origin α of F(x), is represented as a monotonically decreasing function of F(x). The mathematical representation of this argument is a differential equation, with three or four parameters where:

$$(1) \quad \frac{d\ln[F(x)-\alpha]}{d\ln x} = \beta_1\left[1-\left(\frac{F(x)-\alpha}{1-\alpha}\right)^{\beta_2}\right], x>0, (\beta_1,\beta_2)>0$$

with solution, for the CDF

$$(2) \quad F(x) = \alpha + \frac{(1-\alpha)}{\left(1+\lambda x^{-\delta}\right)^{\beta}}, (\beta,\delta,\lambda)>0, \beta=\frac{1}{\beta_2}, \delta=\beta_1\beta_2$$

with derivative F´(x) = f(x), the PDF.

The CDF equation (2) is well defined for all δ greater than zero. However, for income distributions, where the existence of finite income mean is assumed, hence, the existence of the corresponding Lorenz curves and Gini ratios, the condition δ greater than one has to be imposed. (Dagum, 1980)

β and δ are the equality (shape) parameters for the lower and upper tail of the distribution and, λ = exp. of the constant of integration, a scale parameter (Dagum, 1977).

The Gini ratio[1] associated with the Dagum model, tends to zero, perfect equality, with increasing values of β and/or δ; and tends to 1, perfect inequality, with decreasing values of β

[1] The Gini concentration ratio, or Gini coefficient is an aggregate inequality measure and can be obtained from information included in the construction of a Lorenz curve by calculating the ratio of the area between the diagonal and the Lorenz curve divided by the total area of the triangle in which the curve lies. This definition implies that Gini coefficients can vary anywhere from zero, perfect equality, to one, perfect inequality. This traditional approach produces an underestimation for the value of Gini coefficients because income distribution data are presented by class intervals and to calculate the area under the Lorenz curve only the middle points of the intervals are considered (See Dagum, 1987: pp 531). Todaro observed that the Gini coefficient calculated from empirical income distributions typically lies between 0.50 and 0.70 for highly unequal income distributions, and it is on the order of 0.35 to 0.20 for countries with relatively equitable distributions (Todaro; 1997: pp. 146).

and/or δ. This means that there is an improvement of the income distribution when the values of β and/or δ are increased.

This theoretical model will adjust empirical distributions with null, negative or positive starting income, represented by the value taken by α in the formula; which are, in general, unimodal and positively skewed; with income range in the (X_0, ∞), where $X_0 > 0$ for sample distribution of employed individual with starting positive income.

There are three versions of the Dagum model, each accounting for specific assumptions about the population of individuals in the labor force, in this study Dagum Type II Model has been estimated. It corresponds to $0 < \alpha < 1$, then

$$(3)\ F(x) = \alpha + \frac{(1-\alpha)}{\left(1+\lambda x^{-\delta}\right)^{\beta}}$$

is a four parameter CDF[2].

Here, $0 < \alpha < 1$ measures income units with zero or negative income; in Dagum (1977) it has been interpreted as a proxy for the unemployment rate (because it indicates the percentage of population in the labor force having no positive earnings). If unemployment is high, α becomes critical to understand inequality in the labor force (Dagum, 1983). Thus, α is an inequality parameter**,** while λ is an scale parameter that would allow time or space comparison between distributions when income data is expressed in different monetary units. Dagum (1977) proved that the parametric Gini Ratio can be obtained from the corresponding Lorenz curve of the CDF and it is an increasing function of α and a decreasing function of β and δ. The corresponding formula is:

$$(4)\ G = (2\alpha - 1) + (1-\alpha)\frac{\Gamma(\beta)\Gamma\left(2\beta + \frac{1}{\delta}\right)}{\Gamma(2\beta)\Gamma\left(\beta + \frac{1}{\delta}\right)}$$

where Γ(.) is the complete Gamma function specified in Dagum (1977).

3. Data and Methodology

Table 1 shows the estimated values for alpha, beta and delta that resulted from application of equation (3) to a sample of individuals in the labor force belonging to Greater Cordoba, from 1992 to 2000[3] and the corresponding parametric Gini ratio, calculated from (4).

[2] For $\alpha = 0$, a three parameter CDF is obtained. This version is useful to analyze population of income receivers excluded the unemployed. (i.e it.includes only the individuals having positive income)

[3] H.Gertel, R.Giuliodori, P. Auerbach and A. Rodríguez (2001) An estimation of personal income distribution using the Dagum Model with an application to Cordoba 1992-2000. IEF. Universidad Nacional de Córdoba

Table 1: Parameters of Dagum Type II Model and Gini ratio for Greater Cordoba 1992-2000

Period	Gini	Alpha	Beta	Delta
1992	0.46066	0.03881	2.78953	2.02892
2000	0.48057	0.05992	0.61438	2.49399
Variations	**4%**	**54%**	**-78%**	**23%**

Source: own estimates, based on May EPH-INDEC.

The parametric Gini ratio increased from 0.46 to 0.48 in 1992-2000. The value of alpha represents a total of 3.9 per cent unemployed, on the total sampled population of may 1992 but 6.0 per cent in may 2000[4] . Beta shows a decreasing trend over the period assuming a value of 2.8 in 1992, and 0.6 in 2000. Delta was 2.0 in 1992 and 2.5 in 2000. The estimated R^2 for equation (2) is 0.99464 and 0.99543 in 1992 and 2000, respectively.

4. The alpha, beta and delta effects on income distribution equality

From equation (4) it is clearly seen that any increase in alpha, a proxy measure for unemployment, will cause the Gini ratio to rise, holding beta and delta constant. This finding might help to explain the actual support most policy-makers are now giving to the intuitive idea that increased inequality in income distribution resulted from increased unemployment (Gasparini et.al., 2000). Beta and delta summarize information about the density, or frequency distribution of individuals in the labor force at different income levels. It can easily be seen from (2) that beta and delta have a joint effect on the shape of the distribution. Because the joint effect expresses itself through the expanded Gamma function, the independent, partial contribution to equality of beta or delta may not be easily assessed through the standard practice of setting a linear function. Instead, by introducing a numerical simulation, partial effects can be examined with certain detail. In fact, using (2) and moving beta and delta, holding the other constant, it was found: (i) any increase (decrease) in beta with delta constant would represent an improvement (deterioration) in equality. Similarly, any increase (decrease) in delta would indicate improvements (deterioration) in equality; (ii) turning back to formula (2), beta and delta have opposite signs; therefore, the joint beta-delta effect summarizes the partial improvement (deterioration) of equality associated with a rise (decrease) in beta together with the partial deterioration (improvement) due to a decrease (rise) in delta; (iii) the delta effect outweigh the beta effect in our simulations.

Beta summarizes information that represents more strongly the effect of events on the density of individuals in the labor force at the left of or located near the modal income; thus, it has the visual effect of either sharpening or flattening the peak of the distribution. It is in this

rather limited sense that we will attach a localized effect to beta movements; and because the distribution of income is skewed to the right, the beta effect is predicted to be stronger in accounting for local conditions at the lower tail of the distribution (for example, having stronger impact on individuals located below and close to the modal income level than individuals in any other frequency range in the distribution). The parameter delta captures effects of a more generalized nature. An increase in delta will sweep the income distribution to the right. However, although the mean, the median and the modal income will all move to the right, the distance between the mean and the mode will became smaller, suggesting that equality in income distribution improves with a rise in delta. Thus, the parametric Gini depends on the behavior of parameters having an important economic impact on equality. While the parameters beta and delta have a joint effect on the shape of the distribution, alpha has an immediate association with the unemployment rate and its independent contribution to equality can be numerically explored through (4). Be $\alpha = 0.06$, a value representing the ratio of the number of unemployed over the total population at time t and having a Gini ratio of 0,48, as it was in year 2000, using (4) the resulting value for the combined beta and delta effect is 1.447 ($[0.48 – (2\alpha – 1)]/(1-\alpha) = 1.447$). Next, by holding the beta-delta effect constant at 1.447 and moving alpha, a table can be constructed to estimate the sensitivity of the Gini ratio to changes in alpha. Table 2 shows the resulting Gini ratio for hypothetical values of alpha, holding the beta-delta effect constant at 1.447

Table 2: Parametric Gini ratio estimates

Alpha*	Gini ratio
0.06	0.48
0.054	0.476
0.0486	0.474
….	…..
0.0300	0.463

Note: *given a beta-delta combined effect of 1.447
Source: own estimates, based EPH-INDEC.

The table 2 illustrates that cutting unemployment by half will have a small impact on the Gini ratio since it improved only a 3.5 per cent, from 0.48 to 0.463. Similarly, it would be possible to evaluate the response of the Gini ratio to changes in beta and delta.

A more specific exercise will help assessing the partial effect of a change in alpha, beta and delta respectively. Simulations will be carried out using 1992 and 2000 data on individuals in

[4] Since there is a systematic relationship between the active and the total population, the value of alpha, the unemployed over the total population, is related to the usual measure for unemployment, the unemployed over the active population, through the formula alpha= a + bU +e;

the labor force in Greater Cordoba. The Dagum PDF will be used to estimate the parameters and the modal, median and mean income that will be the benchmark in our comparison.

For this exercise alpha will be cut in half and beta and delta will be doubled holding the others parameters constant. Figure 1(a) shows the cutting alpha by half increases the mean income by 2 per cent, the median by 1.7 per cent and has no effect on the mode. Overall it contributes to decrease the Gini ratio (slightly improving equality) by 2.4 per cent.

Figure 1(b) indicates the effect of a hundred per cent increase in beta. The mean income is increased by 41 per cent, the median by 45 per cent and the mode by 48. Its impact on the Gini ratio is a slight improvement of 3.3 per cent.

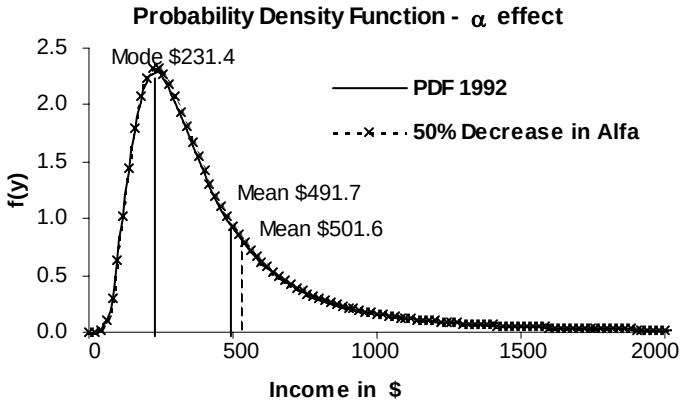
The partial effect of delta in Figure 1(c) is shown to rise the mean income by 30 per cent, the median by 72 per cent and the mode by 123 per cent. The Gini ratio is improved by 48 per cent.

A preliminary interpretation of these results would be that the delta effect is stronger than the beta effect for a wider range of individuals in the labor force, because of its property of sweeping the distribution to the right towards normality, meaning that a greater number of individuals will concentrate around the mean income. The effect of beta is stronger on the left side of the distribution positively affecting low individuals in the labor force, which explains the rise in the mean income of the distribution. Figure 2 shows similar finding for year 2000.

Figure 1

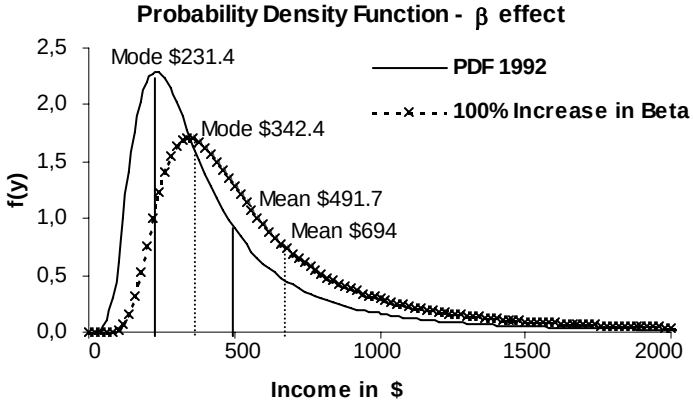
The effects of α, β and δ movements on the probability density function – Year 1992

	1992	Δα= -50%
Mean	491.74	501.66
Median	338.13	343.73
Mode	231.42	231.42
Gini ratio	0.4606	0.4497



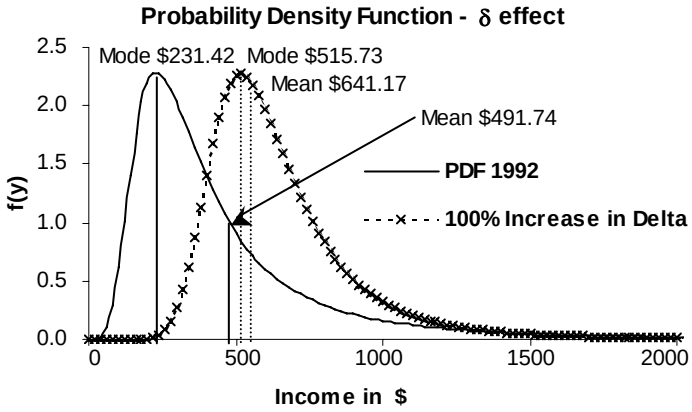
(a)

	1992	Δβ= 100%
Mean	491.74	694.07
Median	338.13	492.05
Mode	231.42	342.45
Gini ratio	0.4606	0.4455



(b)

	1992	Δδ = 100%
Mean	491.74	641.17
Median	338.13	581.49
Mode	231.42	515.73
Gini ratio	0.4606	0.2377



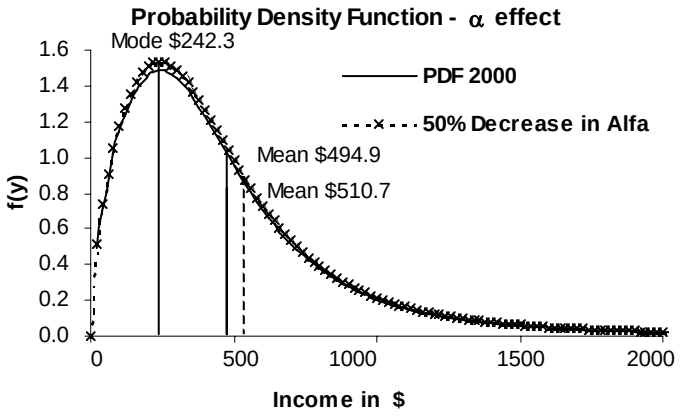
(c)

Note: figures were obtained from estimation of PDF.

Figure 2

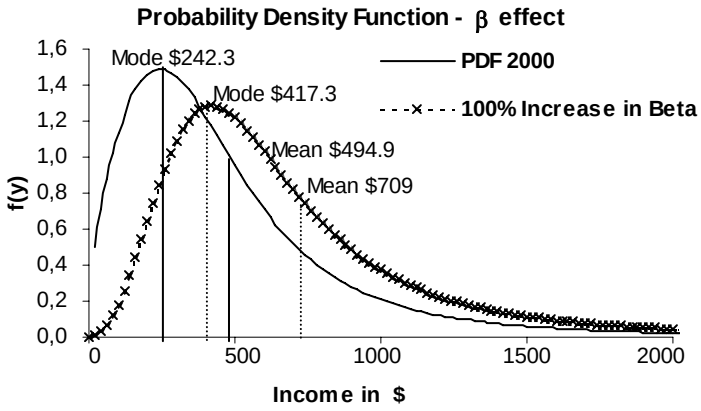
The effects of α, β and δ movements on the probability density function – Year 2000

	2000	Δα = -50%
Mean	494.92	510.70
Median	360.38	372.14
Mode	242.33	242.33
Gini ratio	0.4806	0.4640



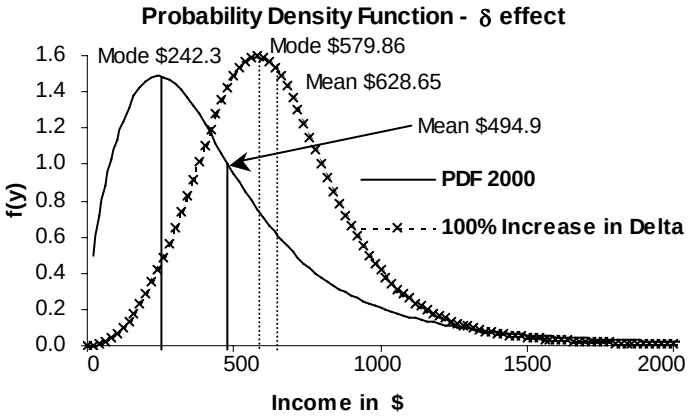
(a)

	2000	Δβ =100%
Mean	494.92	709.09
Median	360.38	548.81
Mode	242.33	417.31
Gini ratio	0.4806	0.4230



(b)

	2000	Δδ = 100%
Mean	494.92	628.64
Median	360.38	600.31
Mode	242.33	579.85
Gini ratio	0.4806	0.2804



(c)

Note: figures were obtained from estimation of PDF.

The fourth section has produced results that should help to illuminate some of the most plausible implications for income policy design. For example, if unemployment is cut in half (i.e. from a rate of 18 to 9 per cent) how would the Gini ratio register this significant improvement in employment conditions? The answer is that no significant improvement in the Gini ratio can be expected from acting solely on unemployment. Similarly, beta and delta summarize the effect of movements in the mean, the median and the mode, the measures of location of the distribution, henceforth, they help to simulate impacts on the Gini ratio of stylized movements in the relative position of individuals in the labor force at the lower, the middle and upper-middle percentiles in the distribution. Through equation (2) it is seen that beta and delta move simultaneously in opposite directions. However, the net effect would follow the direction of the delta effect, that resulted from a wider, rather than a narrow interpretation of what an income policy should eventually be. Thus, the exercise has an immediate economic meaning.

5. A parametric estimation of income inequality in Greater Cordoba, from 1992 to 2000

Table 3 shows the mean income in 1992 and 2000 of individuals in Greater Cordoba and the relative impact of changes in alpha, beta and delta on the Gini ratio and on the mean income of individual by income level.

Table 3: Mean Income and Gini ratio for changes in the value of the parameters α, β and δ by income level

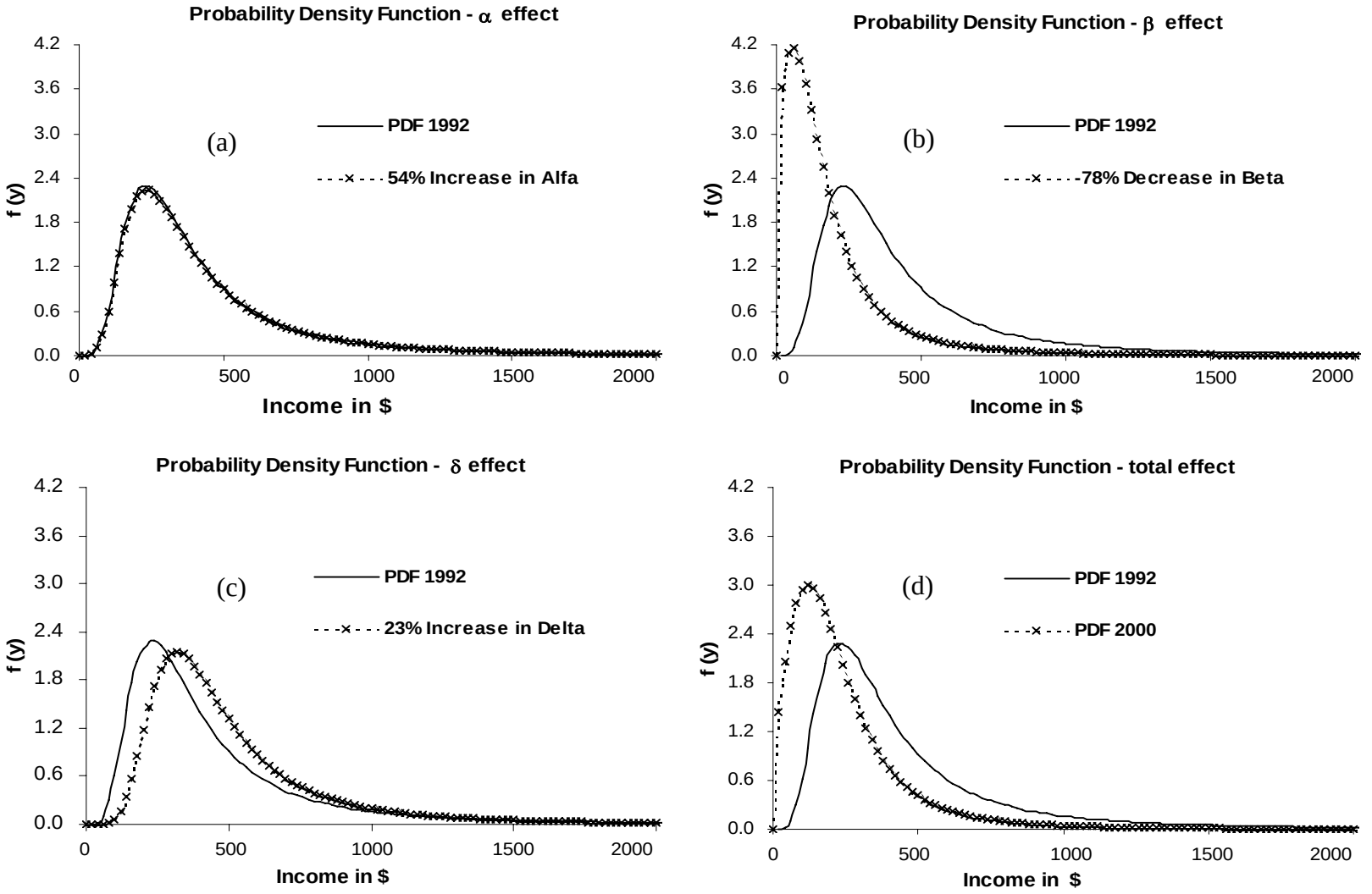
Income Levels	Mean Income (1) 1992	Mean Income (1) 2000	Total Effect	Decomposition of the effects: Partial effects (2) α (+54%)	Partial effects (2) β (-78%)	Partial effects (2) δ (+23%)	Cross effect αβ, αδ, βδ
Lower (1 - 2 deciles)	151.16	90.85	-60.31 -39.9%	-5.30 -3.5%	-126.00 -83.4%	62.99 41.7%	8.00 5.3%
Middle (3 to 6 deciles)	291.11	291.56	0.45 0.2%	-6.64 -2.3%	-193.42 -66.4%	74.25 25.5%	126.26 43.4%
Upper-middle (7 - 8 deciles)	495.42	552.48	57.06 11.5%	-6.98 -1.4%	-287.31 -58.0%	68.65 13.9%	282.70 57.1%
Gini Ratio (3)	0.46066	0.48057	0.02 4.3%	0.0118 2.6%	0.0966 21.0%	-0.0861 -18.6%	-0.0026 -0.6%

Note: (1) Mean Income in pesos. (2) Partial effects shown as mean income differences in pesos and in percentage change relative to 1992.(3) Gini ratio varies from 1 (perfect inequality) to 0 (perfect equality)
Source: own estimates, based on EPH-INDEC.

The total variation of 54 per cent in alpha had a small and decreasing effect by income level, of 3.5%, 2.3% and 1.4%, respectively. The beta variation (-78%) had an important impact but rapidly decreasing by income level, of 83%, 66% and 58%, respectively. The delta variation (23%) had a moderate but rapidly decreasing impact by income level, of 42%, 26% and 14%, respectively.

Figure 3

The effects of α, β and δ movements on the probability density function – Year 1992 to 2000



Note: figures were obtained from estimation of PDF.

The individual impact of alpha, beta and delta on the Gini ratio is a deterioration of 2.6% and 21.0%, and an improvement of 18.7%, respectively. The combined effect on the Gini is a net deterioration of 4.3%.

6. Final Remarks

A numerical calculation of the parametric Gini helped in assessing the relative contribution to equality of proportionate changes in alpha, beta and delta. The paper provided a simple exercise in simulation trying to assess the sensibility of the Gini ratio to changes in the value of the parameters. First, changes in the parameters were simulated to assess the impact on the estimated values of the position parameters in the income distribution equation (the mean, the median and the modal income), the benchmark, or control values obtained from equation (2). The relative impact on the lower, middle and upper-middle tail in the distribution was also investigated. The findings tend to confirm that a (positive) delta effect shows a greater impact over a wider range in the distribution, sweeping the modal income rapidly to the right. The beta effect operates in the opposite direction. However, it is weaker than the delta effect. Hence, if delta is positive, equality is likely to improve because the mass of individuals around the mean income increases. In the last section, the Gini ratio, resulting from equation (4) was recalculated with the 1992 and 2000 mean income of individuals in the labor force in Greater Cordoba.

This numerical approach helped to improve our understanding about the partial contribution of each of the parameters to the Gini ratio. However, at this stage, the exercise should be understood only as a crude approximation to the individual contribution of alpha, beta and delta to equality in income distribution. It is clear from the structure of equation (4) that interaction effects exist and more work is required to improve our understanding about each parameter net and cross contribution to variations in the Gini index of equality. Lastly, the parametric Gini ratio has an immediate advantage over its empirical equivalent in exploring plausible outcomes associated with alternative treatments to abate poverty or improve equality in income distribution. Changes in the value of the parametric Gini ratio can immediately be related with policies whose effects are summarized through movements in alpha, beta and delta. For example, would a policy that generates jobs for 10 per cent of the unemployed have a sizable impact on the Gini ratio, the most common measure for inequality? If localized subsidies are directed at the poorest individuals in the labor force (i.e. those below the modal income), how would the Gini ratio improve? Will introducing a tax cut that benefits a wider range of individuals in the labor force, most of them localized in the middle and upper-middle range in the distribution, have a weak or a strong effect in

the Gini ratio? The parametric Gini ratio also helps in assessing the differential power of alternative policies directed to specific policy objectives, for example, to increase the mean income in the population by 10 per cent.

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