Stochastic Process: Exercise # 3

TA Session

March 7th, 2013

Problem 1 *Solve the following exercise.*

Let $\{N(t)\}$ be Poisson process with a rate λ and its arrival times given by $\{S_n, n = 0, 1, \ldots\}$. Evaluate the expected sum of squares of the arrival times occurring before t,

$$E(t) = \mathbb{E}\left[\sum_{n=1}^{N(t)} S_n^2\right]$$

where we define $\sum_{n=1}^{0} S_n^2 = 0$.

Problem 2 *Solve the following exercise.*

Insurance claims are made at times distributed according to a Poisson process with rate λ; the successive claim amounts are independent random variables having distribution G with mean μ, and are independent of the claim arrival times. let S_i and C_i denote, respectively, the time and the amount of the ith claim, The total discounted cost of all claims made up to time t, call it $D(t)$, is defined by

$$D(t) = \sum_{i=1}^{N(t)} e^{-\alpha S_i} C_i$$

where α is the discount rate and $N(t)$ is the number of claims made by time t. What is the expected value of $D(t)$?

Problem 3 *Arrivals at the Infamous orthopedist's Office*

Limping customers arrive at the infamous orthopedist's office according to a homogeneous Poisson process with rate $\alpha = 1/10$ minutes. Careful observation by Harry's niece reveals that t the doctor does not bother admitting patients until at least three patients are in the waiting room. If $\Gamma_n = E_1 + \ldots + E_n$, where E_j are iid unit exponential random variables, then $\{E_j/\alpha\}$ are iid exponentially distributed random variables with parameter α. Thus, if a Poisson process has rate $\alpha = 1/10$, its points can be represented as $\Gamma_n = 10\Gamma_n$. What is the expected waiting time until the first patient is admitted to see the doctor? What is the probability that nobody is admitted in the first hour to see the doctor?

Problem 4 *Location of Competition*

Harry wonders who the nearest competitor to his restaurant would be if restaurants were geographically distributed relative to this restaurant as a spatial Poisson process with rate $\alpha = 3$ per square mile. Let R be the distance of the nearest competitor and let $d(r)$ be a disc of radius r centered at Harry's restaurant. What is the expected distance to the nearest competitor?

Problem 5 *Rush Hour at Harry's restaurant*

Harry's restaurant is well known for serving great food, but it is filthy and hence not for those with weak stomachs. during rush hour,customers arrive at the restaurant according to a Poisson process of rate α. Customers peek in the door and with probability q they decide the filth is not for them and depart; with probability p they enter and eat. What is the distribution of the waiting times between entrances of customers into the restaurant? What is the mean and variance of this waiting time?

Problem 6 *Short Noise Processes*

This class of processes is a superposition of iid random impulses. Assume electrons arrive according to a homogeneous Poisson process with rate α on $[0, \infty)$. An arriving electron produces an electrical current whose intensity t time units after arrival is $w(t)$. Typical choices for w are exponential functions:

$$w(t) = \exp\{-\theta t\}, \quad \theta > 0$$

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If arrivals occur at Γ_n, then the total current at time t is

$$X(t) = \sum_{i=1}^{N((0,t])} w(t - \Gamma_i),$$

What is the expectation of total current at time t? (We may use the order statistic property to determine it.)