Home Work 4: Continuous time Markov Chains

Stochastic Processes

Deadline April 4, 2013

1. Obtain the transition function of the Poisson process, namely

$$p_t(i,j) = e{-}\lambda t \frac{(\lambda t)^{j-i}}{(j-i)!}\mathbf{1}_{\{j\geq i\}}$$

   from its generator via Kolmogorov equations

2. Consider a population in which each individual splits into two at rate λ.

   - Justify why the process $X(t) =$ number of individuals at time t has jump rates

$$q(i,j) = \lambda n \mathbf{1}_{\{j=i+1\}}$$

   - Starting with only one individual, prove $X(t) \sim Geometric(e^{-\lambda t})$

3. Complete the proof of $\pi p_t = \pi$ is equivalent to $\pi G = \mathbf{0}$, discussed in the classroom.

4. Compute the stationary distribution of a $M(\lambda)/M(\mu)/s$, with $\lambda < \mu$.

5. A salesman flies around the cities A, B and C as follows. He stays in each city for a exponential amount of time. The time mean that he stays is 1/4, 1/5 and 1/6, accordingly he is in A, B or C.

   - Find the limiting fraction of time he spends in each city
   - Simulate several sample paths of the process