Home Work 1: Random Walk

Stochastic Processes

2013

Below, $\{X_n, n \geq 1\}$ is a Random Walk with $X_0 = 0$.

1. Show the moment generating function of X_n is

$$M_{X_n}(t) = \mathbb{E}\left[e^{tX_n}\right] = \left(pe^t + qe^{-t}\right)^n$$

2. Verify

$$\mathbb{P}\left(X_n = x | X_0 = 0\right) = \binom{n}{\frac{1}{2}(n+x)} p^{\frac{1}{2}(n+x)} q^{\frac{1}{2}(n-x)}, \quad \text{for} \quad -n \leq x \leq n \quad \text{and} \quad n+x \quad \text{even},$$

   is a symmetric funtion of x if and only if $p = q$. Interpret this result.

3. What is the probability that the walk returns to the origin in the sixth step?

4. Show $\mathbb{P}(X_{n+1} = x) = p\mathbb{P}(X_n = x - 1) + q\mathbb{P}(X_n = x + 1)$

5. Let τ_n be the time of the first visit to n. Show

$$\mathbb{P}(\tau_n < \infty) = \begin{cases} 1 & \text{si} \quad p \geq 1/2 \\ \left(\dfrac{p}{q}\right)^n & \text{si} \quad p \leq 1/2 \end{cases}$$

   Interpret this result.

6. Consider the Gambler's ruin problem, with initial amout k and maximun value N. Discuss the probability of ruin as function of p, N y k, fixing two parameters and varying one.