# Stochastic Processes
Master in Mathematical Engineering and
Master in Business and Quantitive Methods
Universidad Carlos III de Madrid
12 Feb.– 16 Apr., 2013

Lecturers and instructors:
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Recommended bibliography:
R. Durrett Essential of Stochastic Processes
1st. Edition (1999) available in the library

# Stochastic Processes

The passage of time plays a vital role in the understanding of the complex world we observe. So, the mathematical models most consider random quantities changing over time. Such models are called stochastic processes and are of many types. In this course, we will consider the most basic models and wellknown. Many of them are motivated from real phenomena of the nature, finance, engineering, etc. They are

1. Markov Chains (MC)
2. Poisson Processes
3. Renewal Processes
4. Continuous time MC
5. Martingales
6. Brownian Motion and Diffusions

# Program

**February**

W1: Introduction (12) / TAS-1: Probability Review (14)

W2: MC (19, 21)

W3: TAS-2: MC (26) / Poisson Processes (28)

**March**

W4: TAS-3: PP (05) / Renewal Processes (07)

W5: TAS-4: RP (12) / Continuous Time MC (14)

W6: TAS-5: CT MC (19) / Martingales (21)

**April**

W7: TAS-6: MTG (2) / Brownian Motion (4)

W8: TAS-7: MB (16)

**Grade:** 70% homeworks and 30% final work (oral defense).

# Today

**Very short SP introduction**

First classification, sample paths, finite-dimensional distribution.

**Probability Review**

Random experiments, sample spaces, family of events.

Probability, including "$\sigma$-properties"

Random variables, distribution functions, joint distributions, independence, expected values, moment generating.

Conditional expectation

Limit laws

**Two classical examples of SP**

The simple Random Walk: construction, properties, gambler's ruin.

Branching Process: construction, properties, extinction probability.

# Definition

A **stochastic process** is a family of random variables $\{X_t, t \in T\}$ defined on the same probability space $(\Omega, \mathcal{F}, P)$ and with the same range, called **state space** and usually denoted by $S$. So, $X_t : \Omega \to S$ for every $t \in T$.

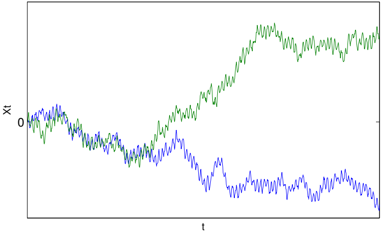
- When $T = \{0, 1, 2, \dots\}$, we say $\{X_t, t \in T\}$ is a **discrete time** stochastic process.
- When $T$ is an interval of real numbers, we say it s a **continuous time** stochastic process.

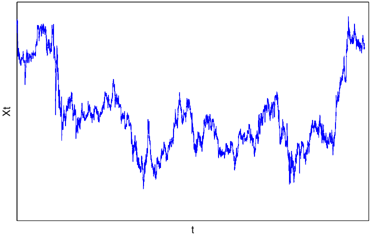
Stochastic processes with $T \subset \mathbb{R}^n$, and $n > 1$ are called *random fields* and are commonly used in modeling of *spatially structured data*.

In this course, $T$ will be (almost surely) a subset of $\mathbb{R}$ and the processes that we will discuss are used in modeling *temporally structured data.*

# Sample paths

Each $\omega \in \Omega$, defines a function $T \to S$, $t \to X_t(\omega)$, that we call **sample path** .




- The left figure shows two sample paths of the same discrete time SP obtained by adding *random shocks aleatorios* to a periodoc function.
- The right signal corresponds to the price evolution of Acerinox, at Madrid market.

# Review of probability

1. Random experiments, sample spaces, family of events.
2. Probability, “including $\sigma$-properties”
3. Random variables, distribution functions, joint distributions, independence, expected values, moment generating.
4. Conditional expectation
5. Limit laws

# Two examples

1. Simple Random Walk
    1. Sample path
    2. Transition probailities
    3. Return time
    4. Gambler's ruin
2. Branching Process
    1. Representation
    2. Some properties
    3. Extinction propabilities