3.3: Time Series and Index Numbers

1. Time series:
   - Plots
   - Components

2. Index numbers:
   - Simple indices
   - Simple aggregate indices
   - Weighted aggregate indices: Laspeyres, Paasche, Edgeworth, Fisher
   - The RPI

Recommended reading:

- Los índices de Laspeyres y Paasche en comic.

Motivation

Thus far, we have studied the characteristics of a sample of data. However, in many situations, these characteristics can change over time:

Unemployment, inflation, the price of beer, consumption of ice-creams

We want to study the changes in the value of a variable over time.

Time Series

¿What is a time series?

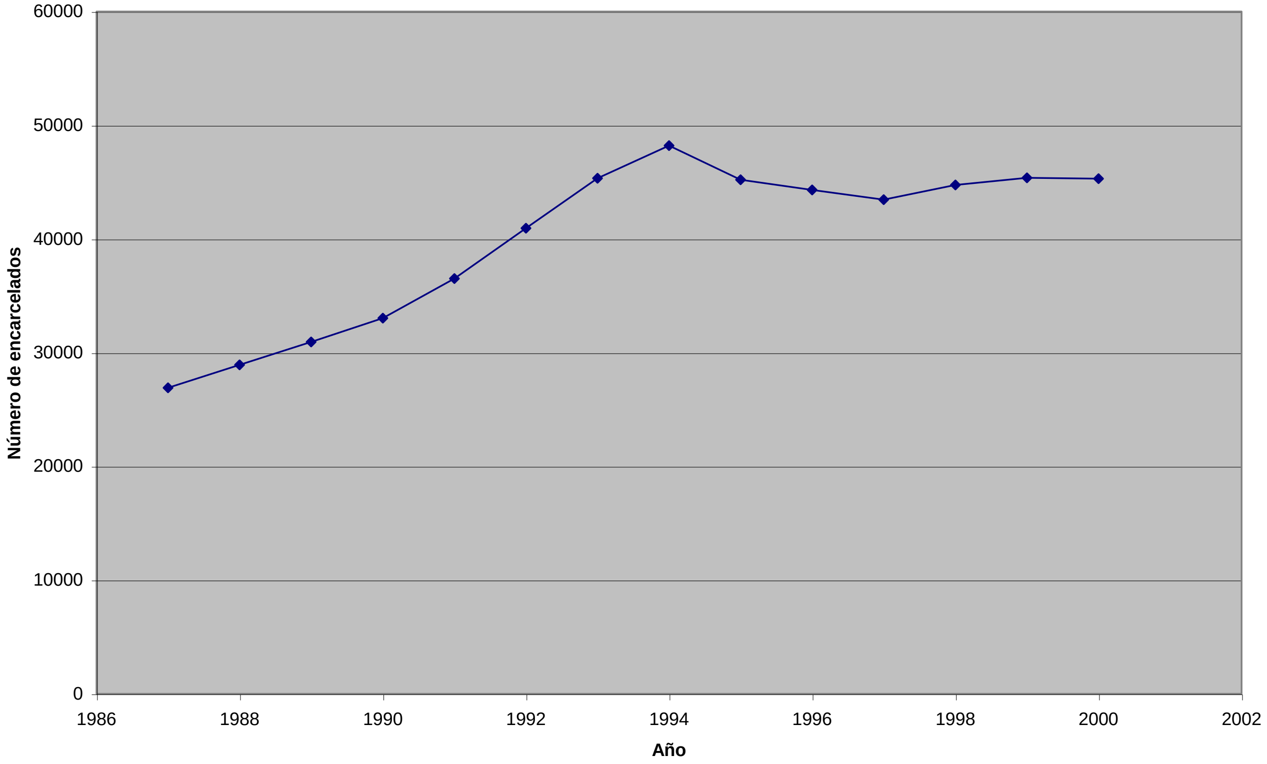
It is a set of measures, ordered according to a time index, of a variable of interest.

Año	Número de encarcelados
1987	26905
1988	28917
1989	30947
1990	33035
1991	36512
1992	40950
1993	45341
1994	48201
1995	45198
1996	44312
1997	43453
1998	44747
1999	45384
2000	45309

¿How does the population of prisoners change over time?

Números de encarcelados por año en España tomados de Eurostat

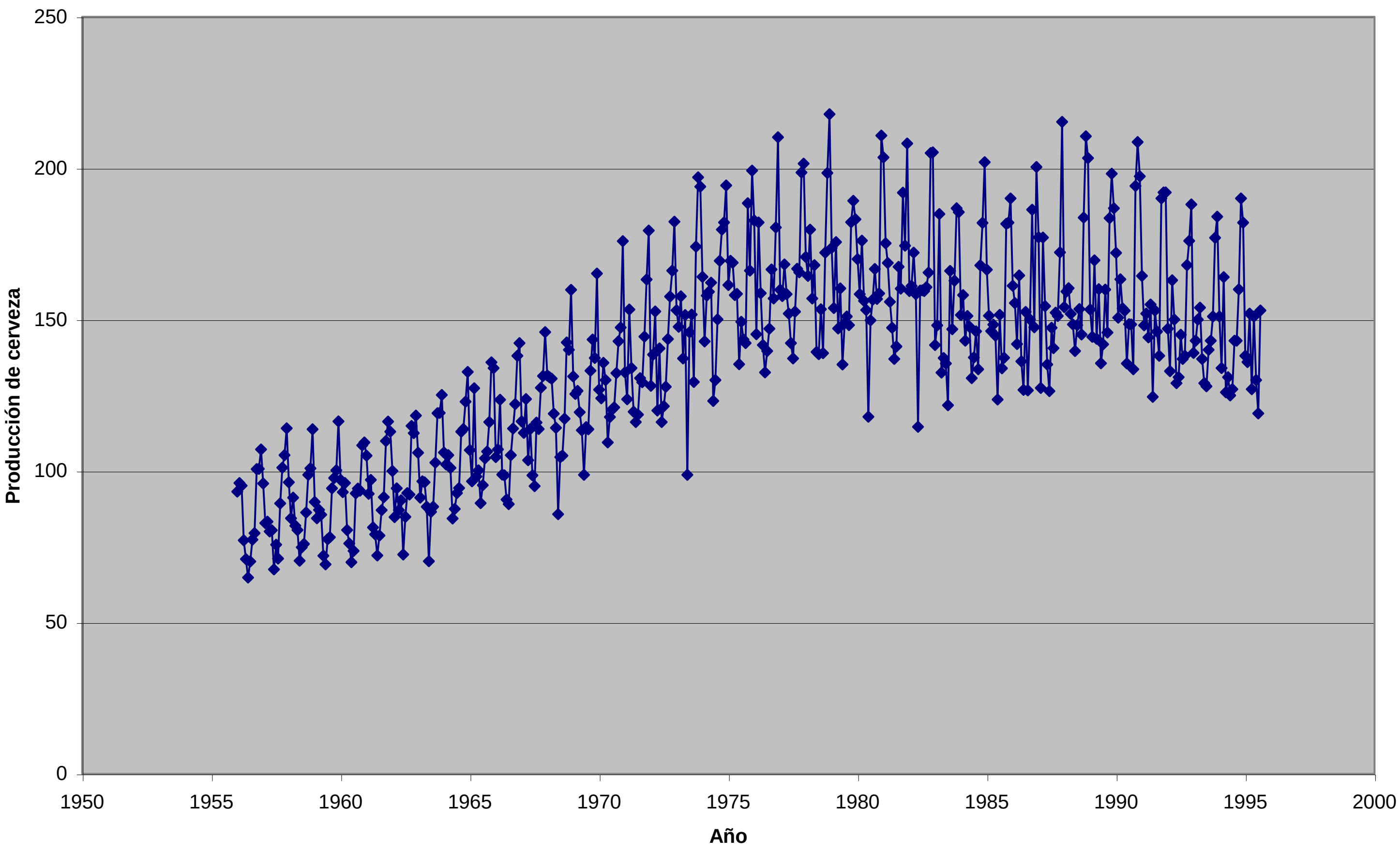
The time series graph



¿What are the characteristics of this series?

Characteristics of time series

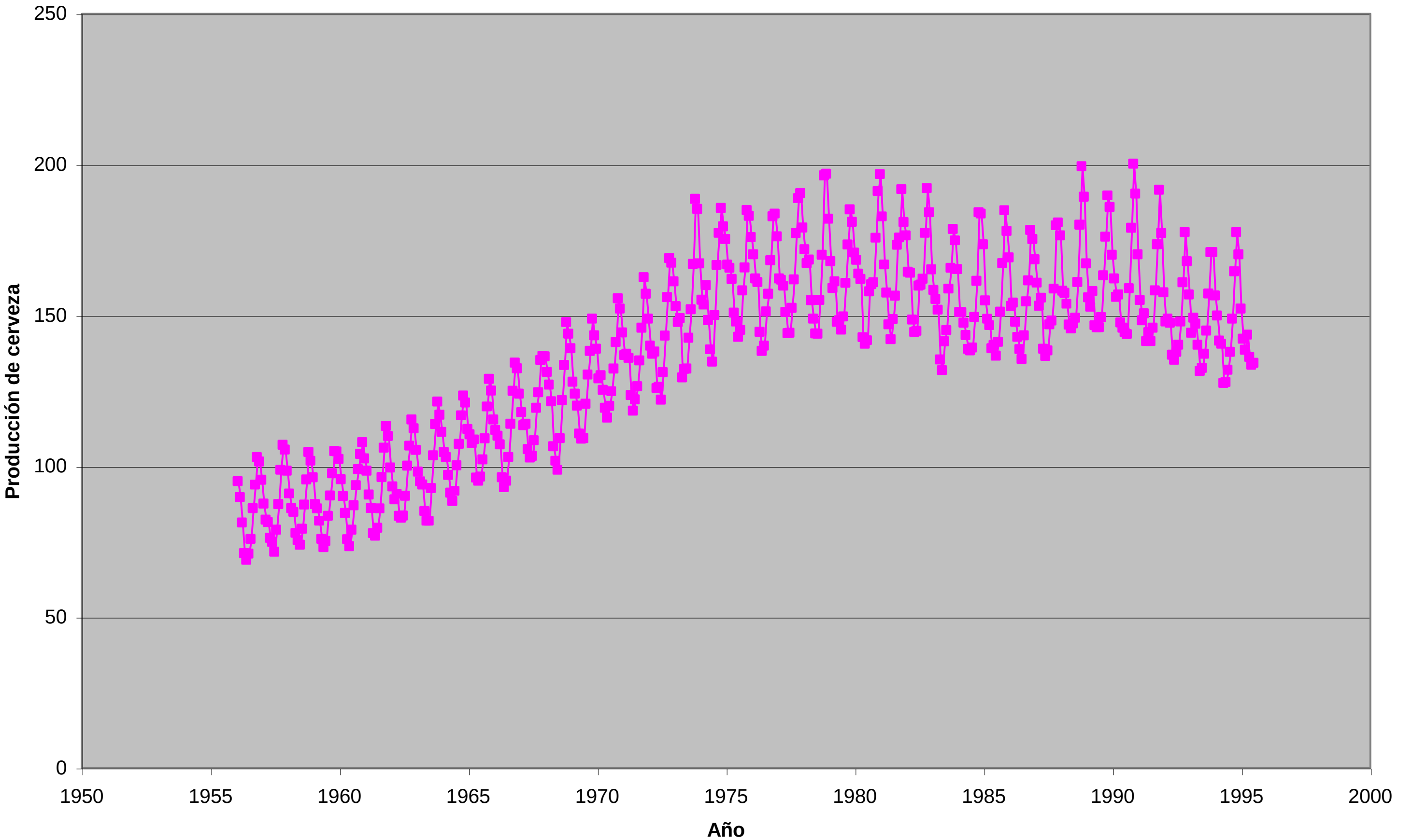
A monthly series of beer production in Australia. There is a strong seasonal effect and a positive trend in the first part of the series.



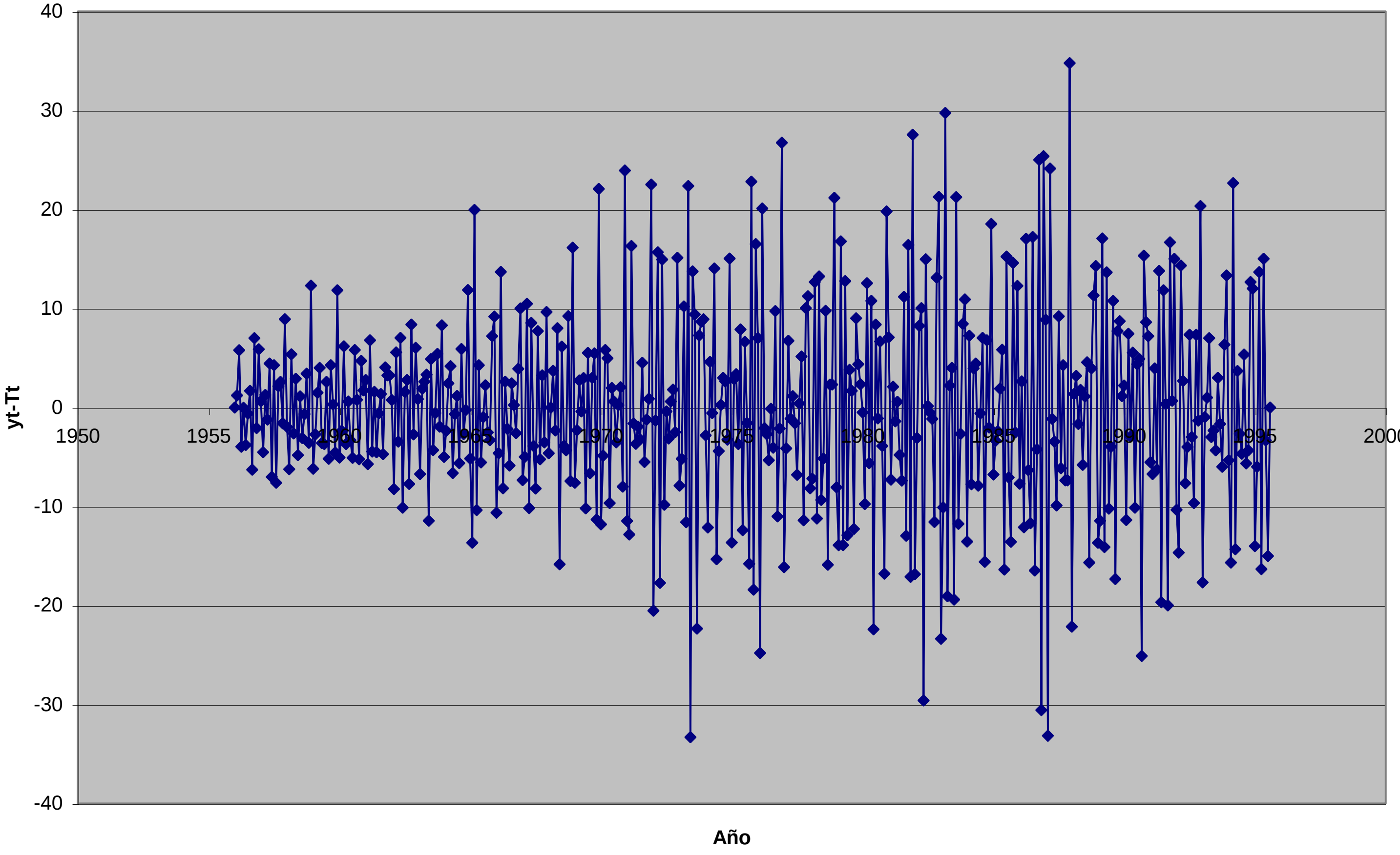
Decomposition of time series

How do we estimate the trend and seasonal effects?
Suppose an additive model $y_t = T_t + E_t + I_t$

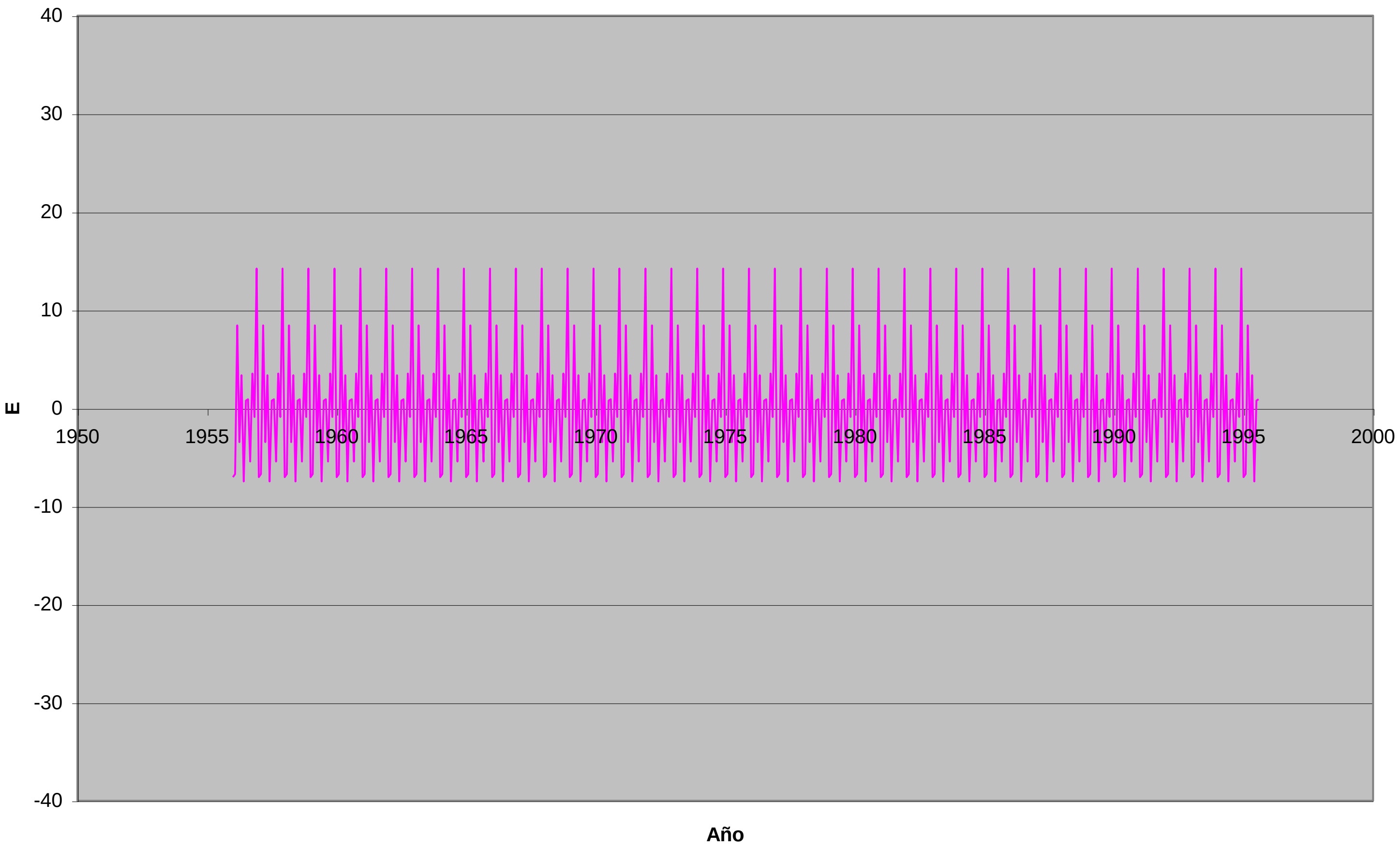
The trend can be estimated using regression or moving averages.



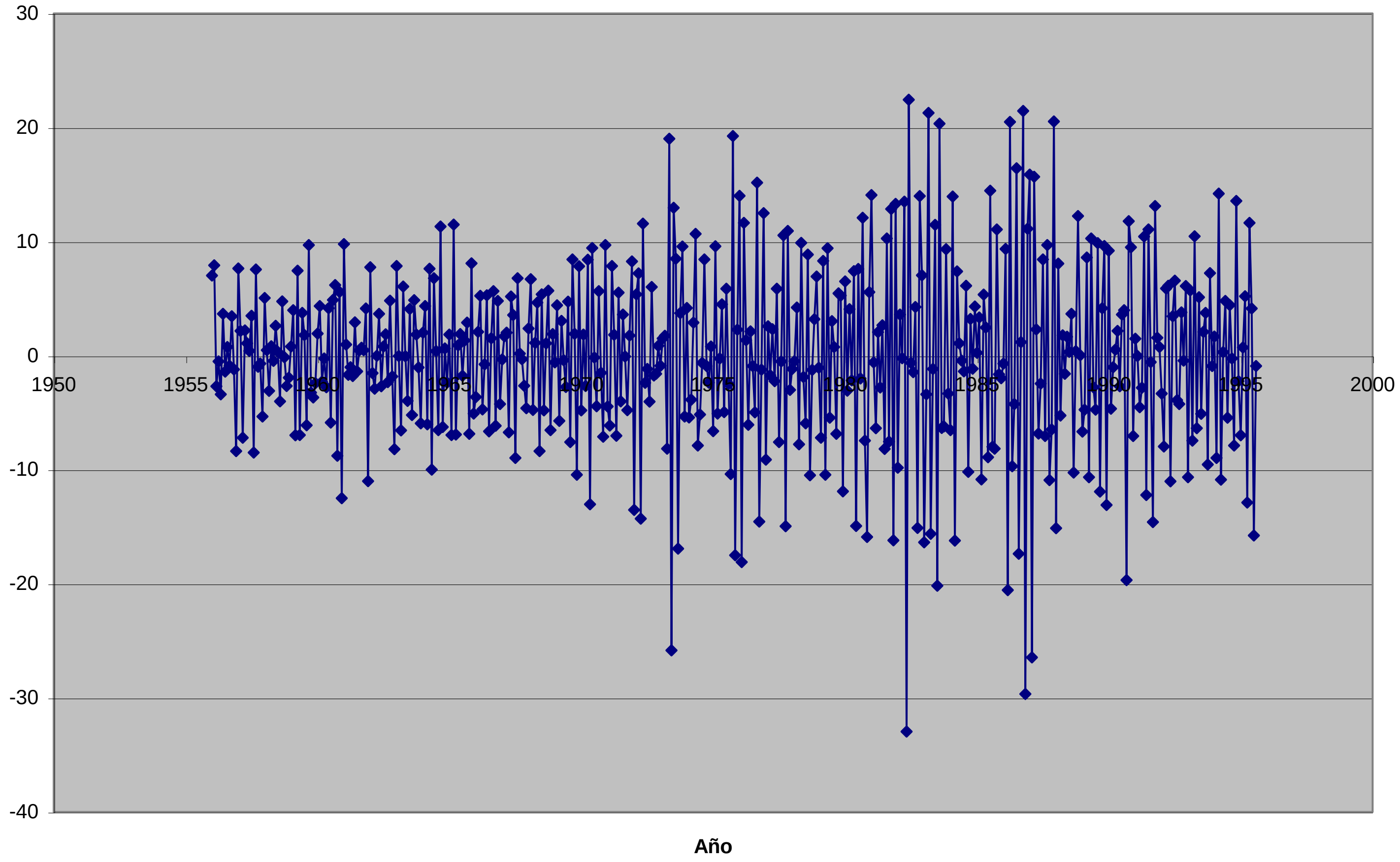
Now we take the trend out of the series…



and calculate the seasonal effect…



Getting rid of the seasonal effect we are left with a series of irregular variations.



Index Numbers

An index number is an indicator designed to describe the changes in a variable over time, that is its evolution over a given time period.

• the evolution in the quantity of a determined product or service or of a group of products or services (e.g. quantities produced or consumed).
• the changes in the **price** of a product or service or a group of such.
• the changes in the value of a product or service or a basket of such.

Simple indices

We want to look at the changes in the prisoner population relative to the year 1987.

Año	Número de encarcelados	Índice
1987	26905	100
1988	28917	107,478164
1989	30947	115,02323
1990	33035	122,783869
1991	36512	135,707118
1992	40950	152,202193
1993	45341	168,522579
1994	48201	179,152574
1995	45198	167,99108
1996	44312	164,698012
1997	43453	161,505296
1998	44747	166,314811
1999	45384	168,682401
2000	45309	168,403642

← = (45309/26905)*100%

The number of prisoners in 2000 has increased by 68% with respect to the prisoner population in1987.

Aggregate indices

In many occasions, we are not interested in comparing the prices (quantities or values) of individual goods, but in comparing these for groups of products.

Article	Prices		Simple indices	
Year	2007	2009	2007	2009
Milk	10	12	100	120
Cheese	15	20	100	133,3
Butter	80	80	100	100

Simple aggregate indices

The most basic index is simply the arithmetic mean of all the indices

$$I_{2009} = (120+133{,}3+100)/3 = 117{,}76$$

Alternatives are geometric or harmonic means or aggregate indices.

What is the problem with this type of index?

They don’t take the consumption of each product into account.

Article	Prices		Units consumed	
Year	2007	2009	2007	2009
Milk	10	12	50	40
Cheese	15	20	20	10
Butter	80	80	1	1

Weighted aggregate indices I: Laspeyres index

We suppose that the consumption in year t is the same as that in the base year.

$$I_t^L = \frac{\sum_{j=1}^{k} q_{j0} \times p_{jt}}{\sum_{j=1}^{k} q_{j0} \times p_{j0}} \times 100\%$$

$$\frac{\text{old quantities * new prices}}{\text{old quantities * old prices}}$$

Weighted aggregate indices II: Paasche's index

We suppose that consumption in the base year is the same as in year t.

$$I_t^P = \frac{\sum_{j=1}^{k} q_{jt} \times p_{jt}}{\sum_{j=1}^{k} q_{jt} \times p_{j0}} \times 100\%$$

$$\frac{\text{new quantities * new prices}}{\text{new quantities * old prices}}$$

Weighted indices III: Fisher and Edgeworth

Fisher's index is the geometric mean of Laspeyres and Paasche

$$I_t^F = \sqrt{I_t^L \times I_t^P}$$

The Edgeworth index uses the sum of the quantities consumed in the base year and in year t as the weight.

$$I_t^E = \frac{\sum_{j=1}^{k} (q_{j0} + q_{jt}) \times p_{jt}}{\sum_{j=1}^{k} (q_{j0} + q_{jt}) \times p_{j0}} \times 100\%$$

The Retail or Consumer Price index (RPI)

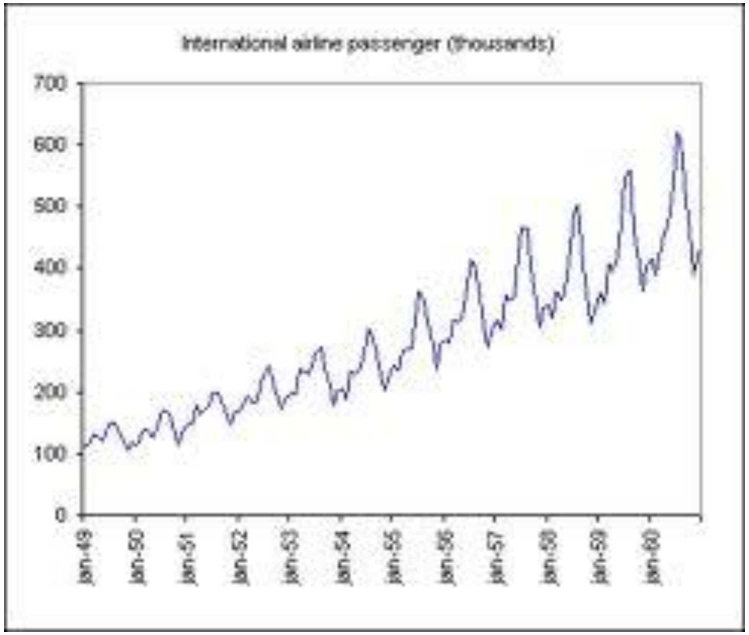
Describes the evolution of prices of consumption over time.

Every 10 years, a survey (EPF) is taken to analyze the spending habits of a large number of families. The consumption of various products which form the typical shopping basket is considered.

In the following years a Laspeyres index based on the consumption in the EPF year is calculated.

In the majority of the developed world, the RPI increases over time.

Example



The diagram shows a monthly time series of airline passengers for a particular company in the 1950's. The characteristics of this series are.

a) It is stationary.
b) It shows a seasonal effect but no trend.
c) It shows seasonal and trend effects.
d) It shows a trend but no seasonality.

Example

	Quantity of Burgers	Price of Burgers($)	Quantity of Milkshakes	Price of Milkshakes($)
2005	100	2.00	50	1.00
2006	120	3.00	75	1.50
2007	125	4.00	25	3.00

The table shows the prices and quantities of burgers and milkshake bought, on average, per day in a Madrid bar in the years 2005 to 2007. Taking the base year as 2005:

a) The Laspeyres index for 2005 is 150%.
b) The Laspeyres index for 2006 is 150%.
c) The Laspeyres index for 2007 is 150%.
d) None of the above.