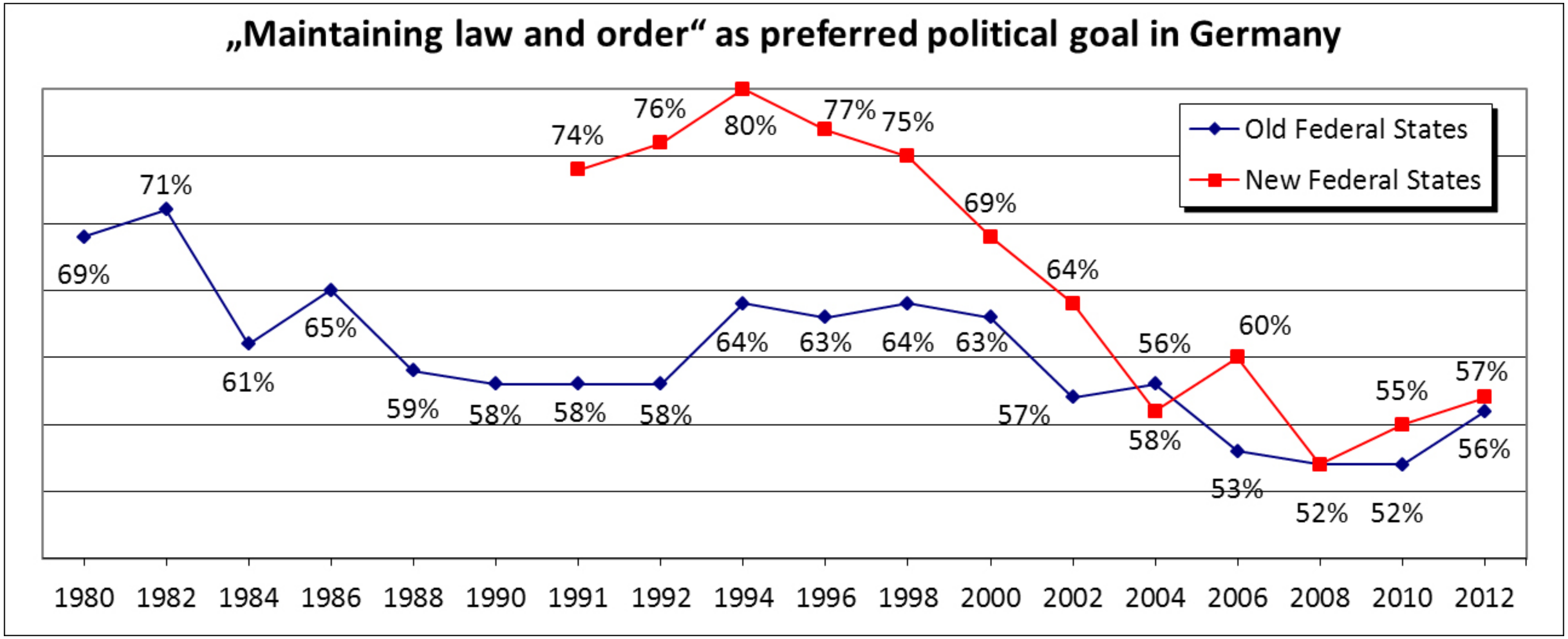
Class 8: Time Series and Index Numbers



Motivation

Thus far, we have studied the characteristics of a sample of data. However, in many situations, these characteristics can change over time:

Unemployment, inflation, the price of beer, consumption of ice-creams

We want to study the changes in the value of a variable over time.

Recommended reading:

- For Spanish readers, the chapter on Laspeyres and Paasche in Dados y Datos is quite nice.

Index

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2. Index numbers:
   - Simple indices
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   - The Retail Price Index

Time Series

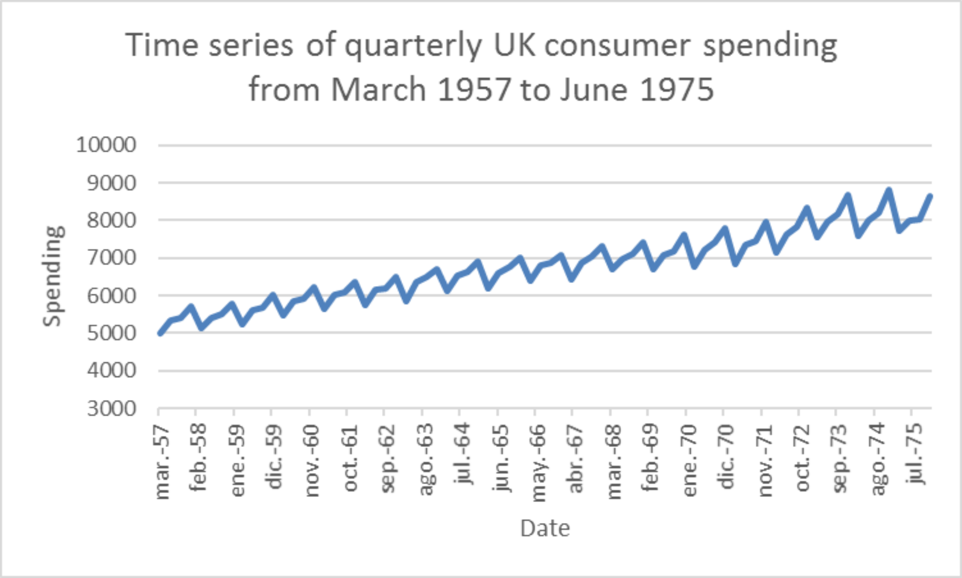
Period	Yt
mar-57	4996
jun-57	5342
sep-57	5401
dic-57	5698
mar-58	5120
jun-58	5409
sep-58	5511
dic-58	5789
mar-59	5224
jun-59	5628
sep-59	5690
dic-59	6029
mar-60	5463
jun-60	5863
sep-60	5916
dic-60	6221
mar-61	5661
jun-61	6016

¿What is a time series?

It is a set of measures, ordered according to a time index, of a variable of interest.

Here we show a snippet of a time series of consumer expenditure in the UK.

The time series graph



What are the characteristics of this series?

Characteristics of a time series

Trend effect: the tendency of a time series to increase or decrease over time.

- We consume more beer now than 20 years ago.
- Since the introduction of the points system, the number of fatal accidents has gone down.

Seasonal effects: fluctuations that occur regularly (each week, month or season for example).

- Beer drinking goes up in summer or at weekends.
- There are more traffic accidents on Friday afternoons or on “puentes, operación salida, …”

Time series decomposition

We typically want to estimate the seasonal and trend components individually.

Two models are commonly used:

Additive model: $y_t = T_t + S_t + I_t$

Multiplicative model: $y_t = T_t \, S_t \, I_t$

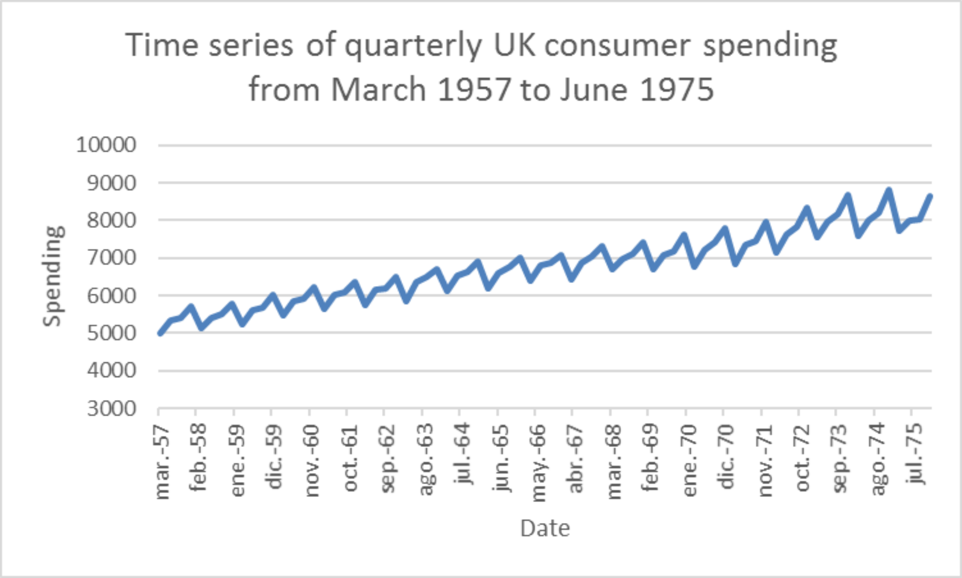
- T_t is the trend component
- S_t is the seasonal component
- I_t is the irregular or error component

How do we decide if a series is additive or multiplicative?

Choosing the time series model

In a multiplicative model, for a time series with an increasing trend, the seasonal variation increases over time.

In an additive model, we would expect seasonal variation to be more constant.



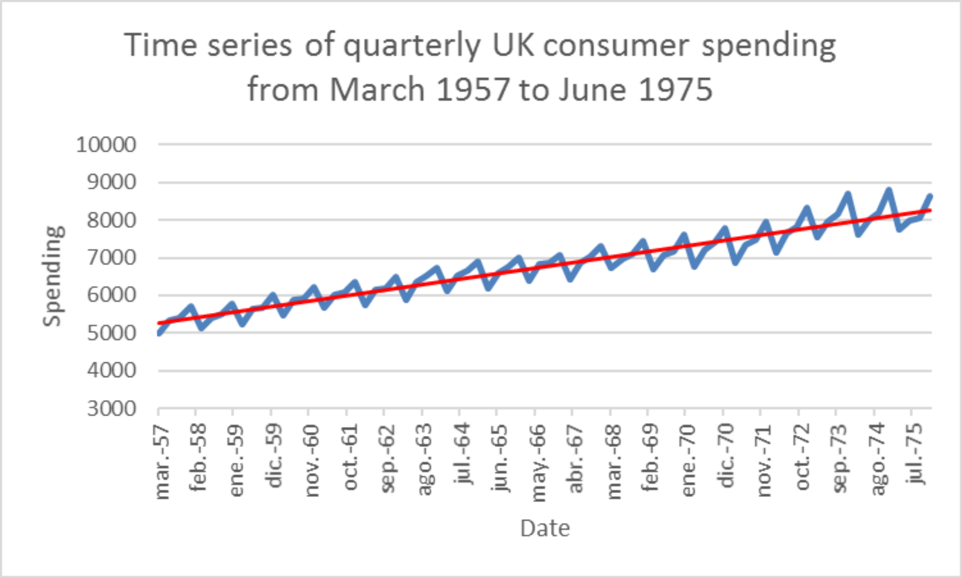
Which model do you think would best represent this data?

Estimating the trend: regression

It looks like a line would go well through this series:

We could assume a regression model:

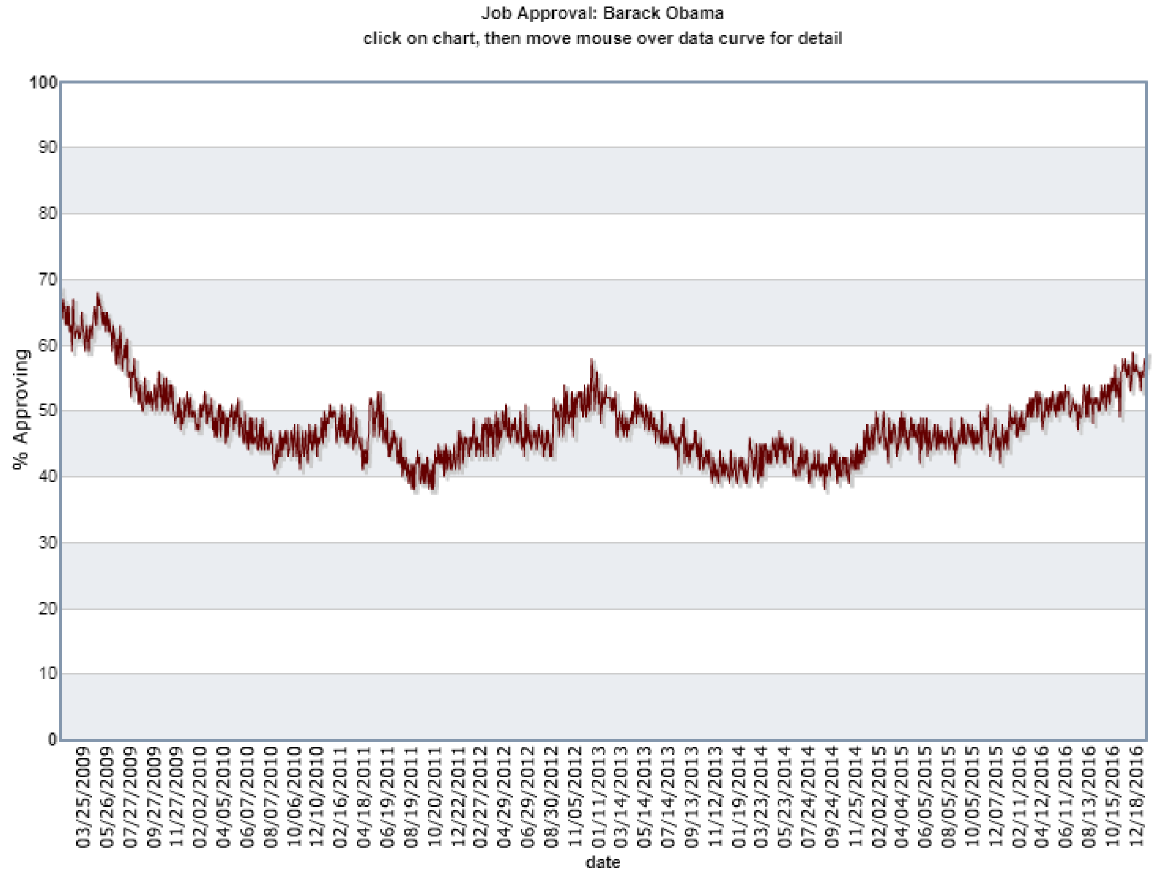
$$T_t = \alpha + \beta t$$



Estimating the trend

Most time series don't increase so regularly:

To remove the seasonal effects and leave the trend, we could use a moving average.



Weekly series of President Obama's approval rating.

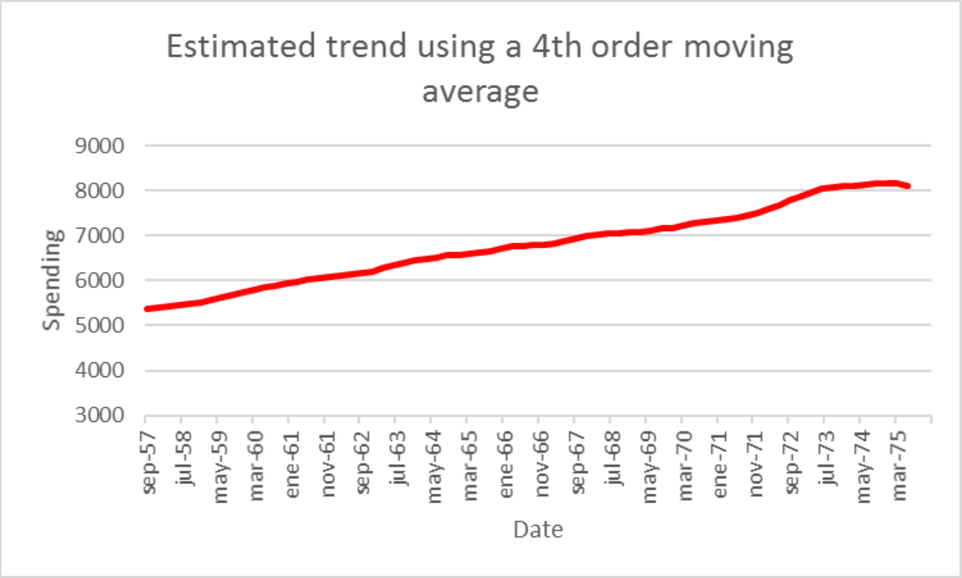
Estimating the trend: moving averages

Our data are quarterly.

We might expect the same quarters to behave the same way (e.g. higher in summer, lower in winter).

At time t, take an average of 4 consecutive time periods:

$$T_t = \frac{1}{4}\,(y_{t-2} + y_{t-1} + y_t + y_{t+1})$$



(This doesn’t quite work as it is should be centred at t-½. The practical shows how to get round this.)

We can see that the seasonal variation has been removed.

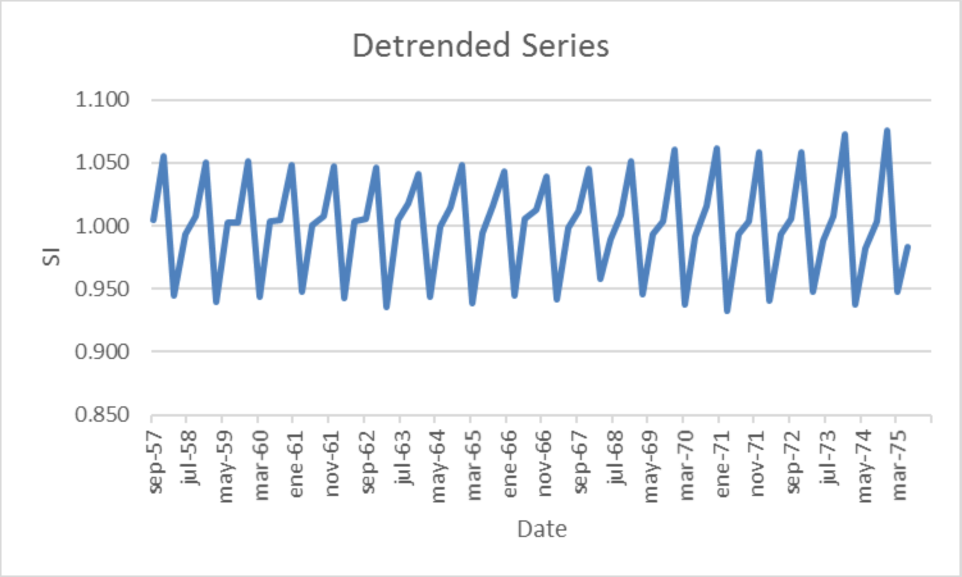
Detrending the series

Under a multiplicative model:

$$y_t = T_t\, S_t\, I_t$$

so

$$S_t\, I_t = y_t\, /T_t$$

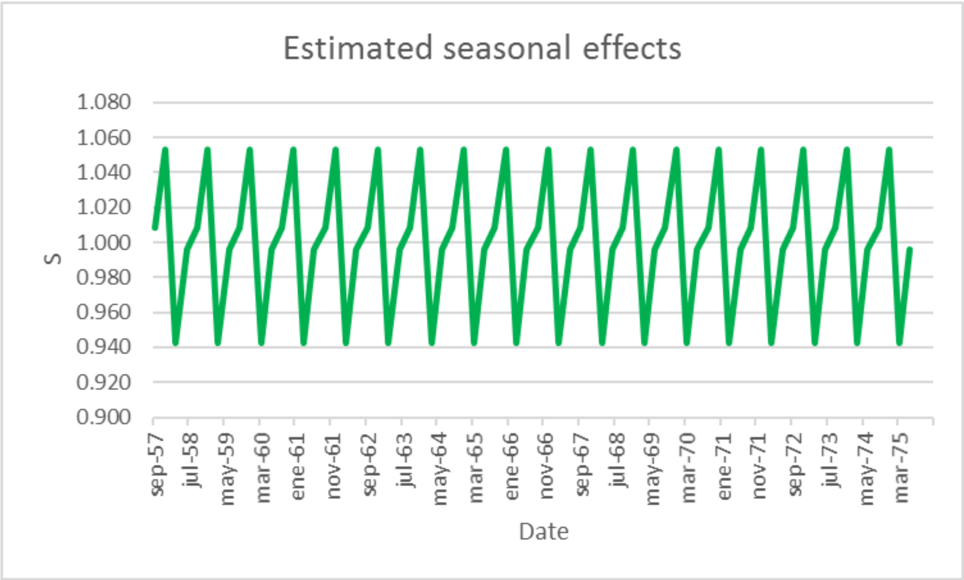


We are left with the seasonal variation and the error.

Estimating the seasonal component

To estimate the seasonal component, we could simply asume this is the same in every season (all winters the same, all summers the same, …).

We can estimate the effect of being in summer as the average of all summers.

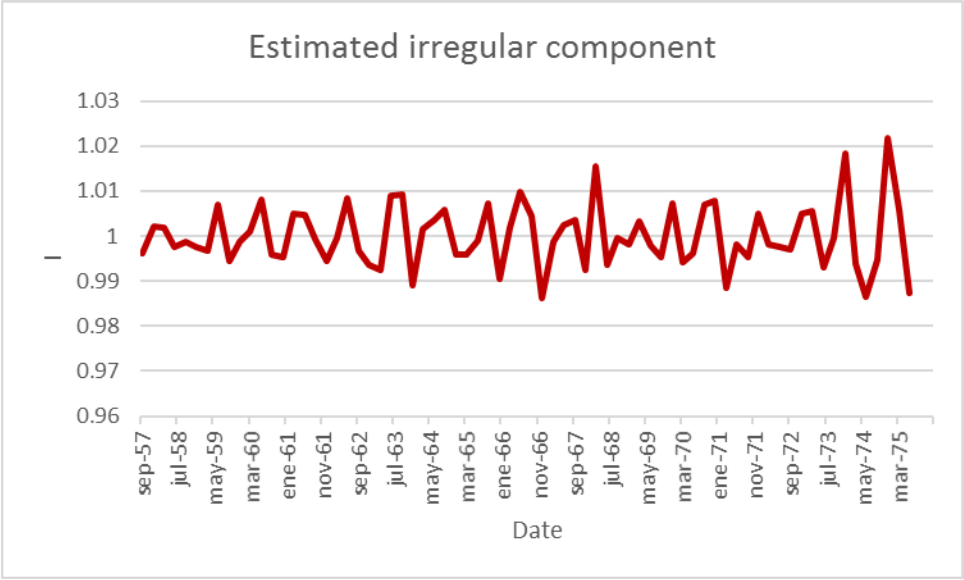


The irregular component

After removing the seasonal effect, we are left with the irregular component.

This should look "random" or stationary.

A stationary time series is a series without obvious trend or seasonal components.



Index Numbers

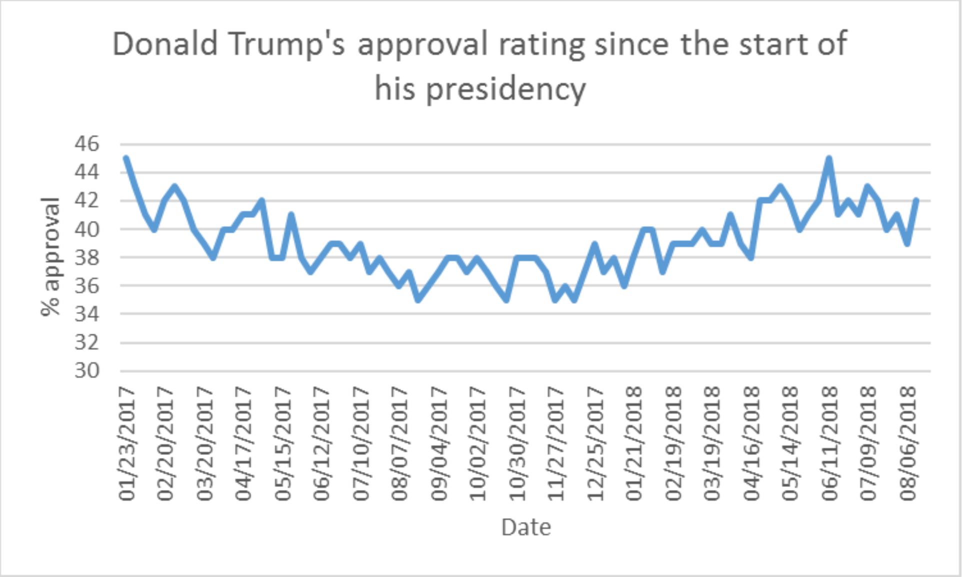
An index number is an indicator designed to describe the changes in a variable over time, that is its evolution over a given time period.

• the evolution in the *quantity* of a determined product or service or of a group of products or services (e.g. quantities produced or consumed).
• the changes in the *price* of a product or service or a group of such.
• the changes in the *value* of a product or service or a basket of such.

Simple indices

Trump’s approval rating has varied considerably since he became President.

How much more or less popular is he now compared to when he started?



Simple indices

We can calculate Trump's approval relative to 23/01/2017.

Trump's approval is only 93.33% of what it was when he became President.

Date	% Approving	Index
23/01/2017	45	100.00%
30/01/2017	43	95.56%
06/02/2017	41	91.11%
13/02/2017	40	88.89%
20/02/2017	42	93.33%
27/02/2017	43	95.56%
06/03/2017	42	93.33%
13/03/2017	40	88.89%
20/03/2017	39	86.67%
27/03/2017	38	84.44%
03/04/2017	40	88.89%
10/04/2017	40	88.89%
17/04/2017	41	91.11%
24/04/2017	41	91.11%
01/05/2017	42	93.33%

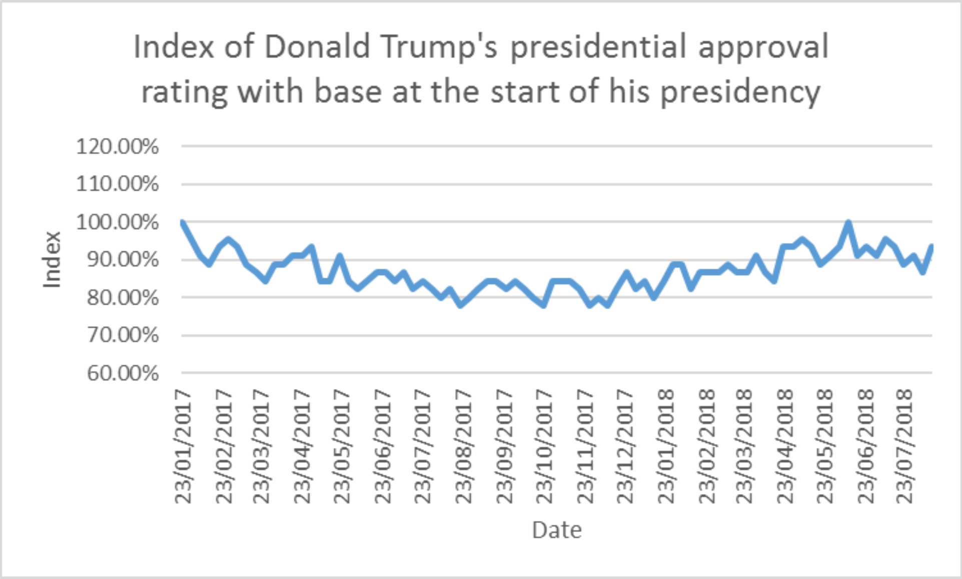
$\frac{45}{45}$ x 100%

$\frac{43}{45}$ x 100%

$\frac{42}{45}$ x 100%

Simple indices

Only once has Trump been as popular as when he became President!



Aggregate indices

In many occasions, we are not interested in comparing the prices (quantities or values) of individual goods, but in comparing these for groups of products.

Article	Prices		Simple indices	
Year	2017	2018	2017	2018
Milk	10	12	100	120
Cheese	15	20	100	133,3
Butter	80	80	100	100

Simple aggregate indices

The most basic index is simply the arithmetic mean of all the indices

$$I_{2018} = (120+133{,}3+100)/3 = 117{,}76$$

Alternatives are geometric or harmonic means or aggregate indices.

What is the problem with this type of index?

They don’t take the consumption of each product into account.

Article	Prices		Units consumed	
Year	2017	2018	2017	2018
Milk	10	12	50	40
Cheese	15	20	20	10
Butter	80	80	1	1

Weighted aggregate indices I: Laspeyres index

We suppose that the consumption in year t is the same as that in the base year.

$$I_t^L = \frac{\sum_{j=1}^{k} q_{j0} \times p_{jt}}{\sum_{j=1}^{k} q_{j0} \times p_{j0}} \times 100\%$$

$$\frac{\text{old quantities * new prices}}{\text{old quantities * old prices}}$$

Weighted aggregate indices II: Paasche's index

We suppose that consumption in the base year is the same as in year t.

$$I_t^P = \frac{\sum_{j=1}^{k} q_{jt} \times p_{jt}}{\sum_{j=1}^{k} q_{jt} \times p_{j0}} \times 100\%$$

$$\frac{\text{new quantities * new prices}}{\text{new quantities * old prices}}$$

Weighted indices III: Fisher and Edgeworth

Fisher's index is the geometric mean of Laspeyres and Paasche

$$I_t^F = \sqrt{I_t^L \times I_t^P}$$

The Edgeworth index uses the sum of the quantities consumed in the base year and in year t as the weight.

$$I_t^E = \frac{\sum_{j=1}^{k}(q_{j0} + q_{jt}) \times p_{jt}}{\sum_{j=1}^{k}(q_{j0} + q_{jt}) \times p_{j0}} \times 100\%$$

The Retail or Consumer Price index (RPI)

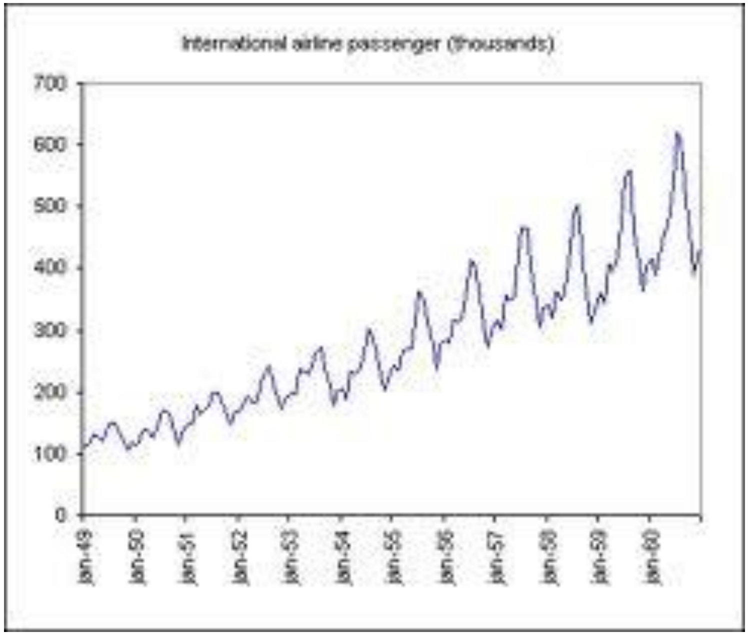
Describes the evolution of prices of consumption over time.

Every 10 years, a survey (EPF) is taken to analyze the spending habits of a large number of families. The consumption of various products which form the typical shopping basket is considered.

In the following years a Laspeyres index based on the consumption in the EPF year is calculated.

In the majority of the developed world, the RPI increases over time.

Example



The diagram shows a monthly time series of airline passengers for a particular company in the 1950's. The characteristics of this series are.

a) It is stationary.
b) It shows a seasonal effect but no trend.
c) It shows seasonal and trend effects.
d) It shows a trend but no seasonality.

Example

	Quantity of Burgers	Price of Burgers($)	Quantity of Milkshakes	Price of Milkshakes($)
2005	100	2.00	50	1.00
2006	120	3.00	75	1.50
2007	125	4.00	25	3.00

The table shows the prices and quantities of burgers and milkshake bought, on average, per day in a Madrid bar in the years 2005 to 2007. Taking the base year as 2005:

a) The Laspeyres index for 2005 is 150%.
b) The Laspeyres index for 2006 is 150%.
c) The Laspeyres index for 2007 is 150%.
d) None of the above.