Introduction to Time Series Analysis

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Outline

Chapter 1. Univariate ARIMA models.

Chapter 2. Model fitting and checking.

Chapter 3. Prediction and model selection.

Chapter 4. Outliers and influential observations.

Chapter 5. Vector Autoregressive and Vector Error Correction Models.

Chapter 6. Nonlinear Time Series Modelling.

TEXTBOOKS

- ✓ Peña, D., Tiao, G.C. and Tsay, R.S. (2000). A Course in Time Series Analysis. Wiley & sons.
- ✓ Hamilton, J.D. (1994). *Time Series Analysis.* Princeton University Press.
- ✓ Box, G.E.P., Jenkins, G.M. and Reinsel, G. (1994). *Time Series Analysis: Forecasting and Control,* 3rd. Ed. Prentice-Hall, Englewood Cliffs, NJ.
- ✓ Enders, W. (2004). *Applied Econometric Time Series .* Wiley & sons.
- ✓ Lütkepohl, H. and Krätzig, M. (2004). *Applied Time Series Econometrics.* Cambridge University Press.
- ✓ Franses, P.H. and van Dick, D. (2000). *Non-linear time series models in empirical finance.* Cambridge University Press.

Grading

- Final exam (70%).
- Class participation and empirical project (30%).

WEB RESOURCES

- Global Insight.
- Time series data library.

http://datamarket.com/data/list/?q=provider:tsdl

- Macroeconomic time series

http://www.fgn.unisg.ch/eurmacro/macrodata/index.html

- Instituto Nacional de Estadística

http://www.ine.es

- Eurostats

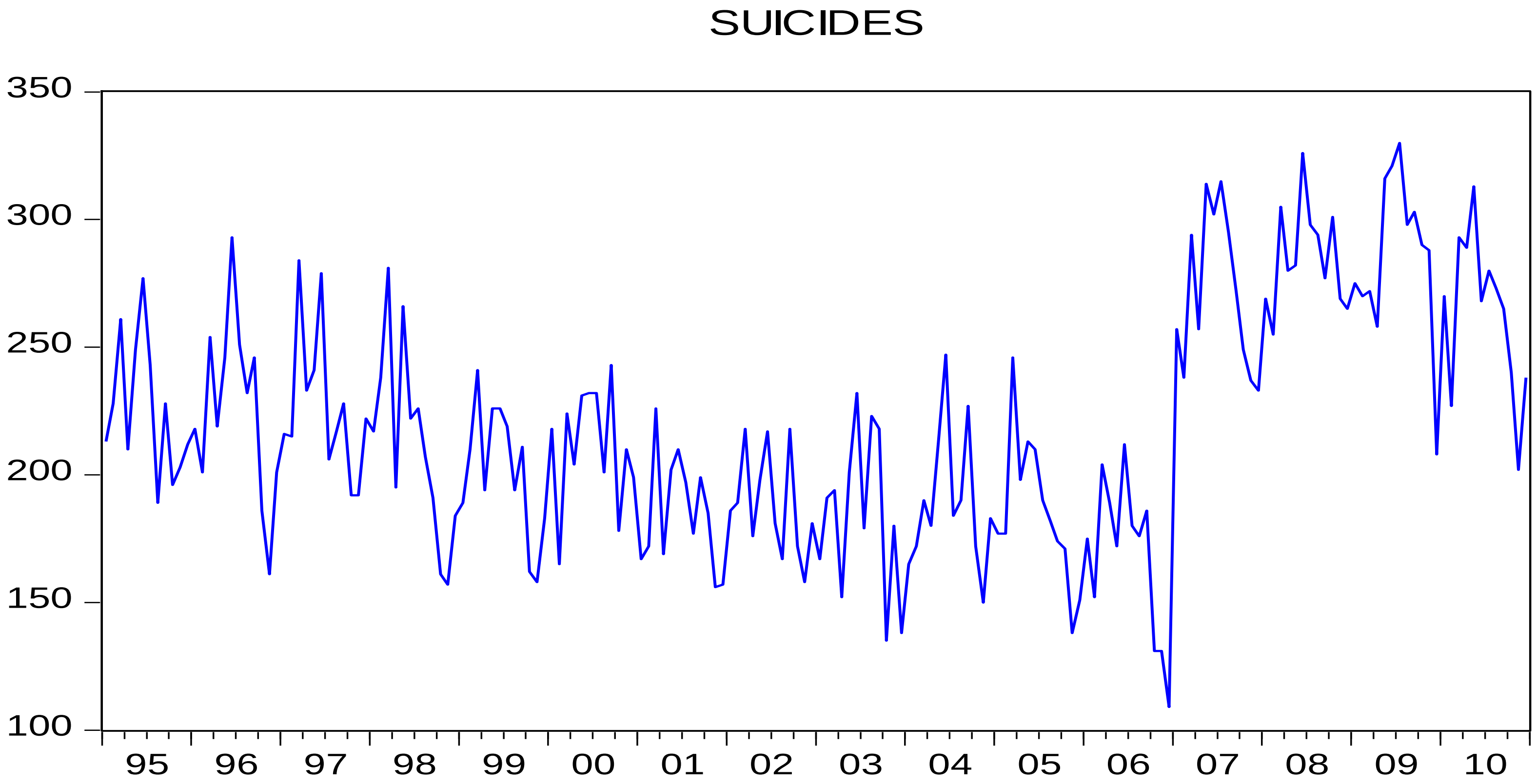
http://epp.eurostat.ec.europa.eu/portal/page/portal/eurostat/home/

Motivation

- Time series: sequence of observations taken at regular intervals of time
- Data in bussines, engineering, enviroment, medicine, etc.

MOTIVATION.

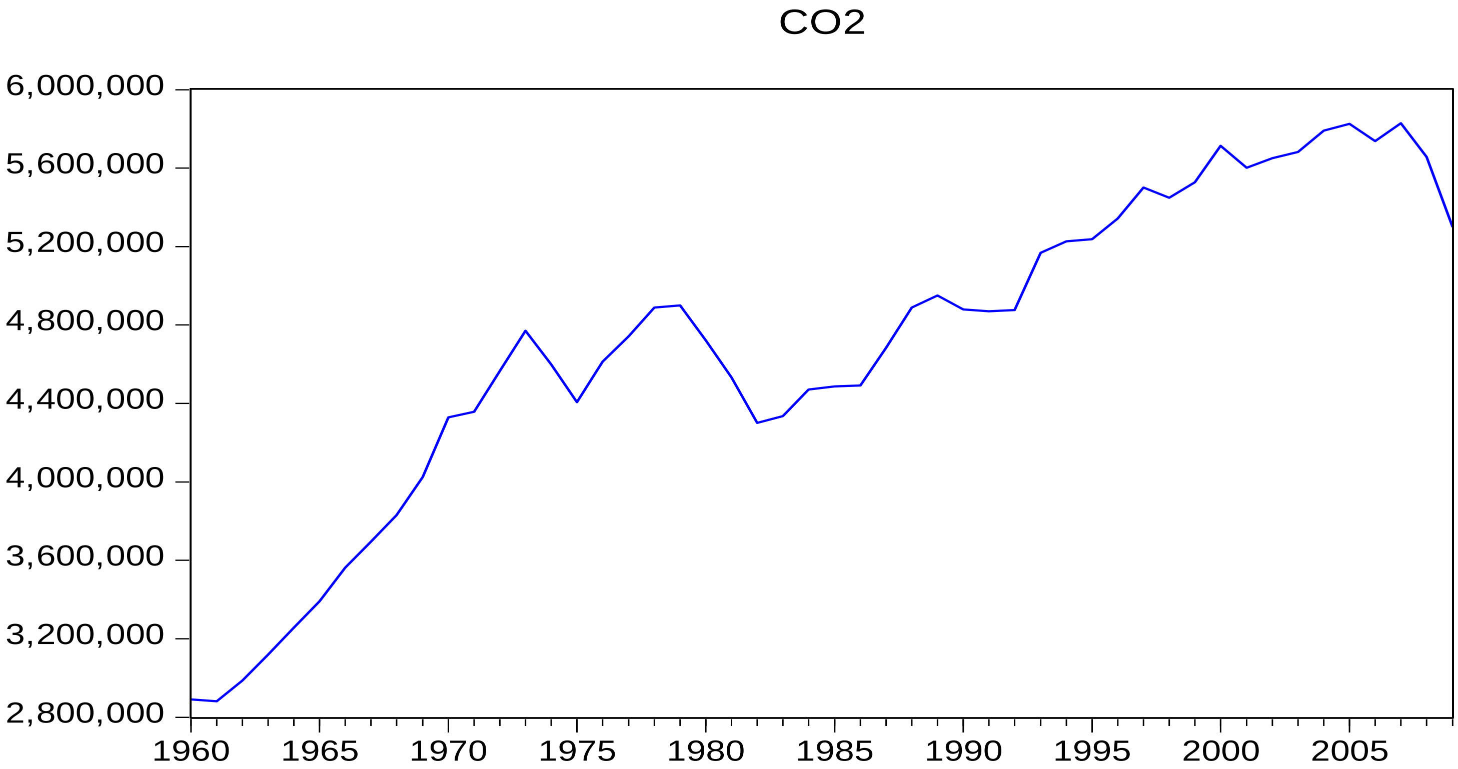
SUICIDES IN SPAIN, TOTAL NUMBER



Source: INE

MOTIVATION.

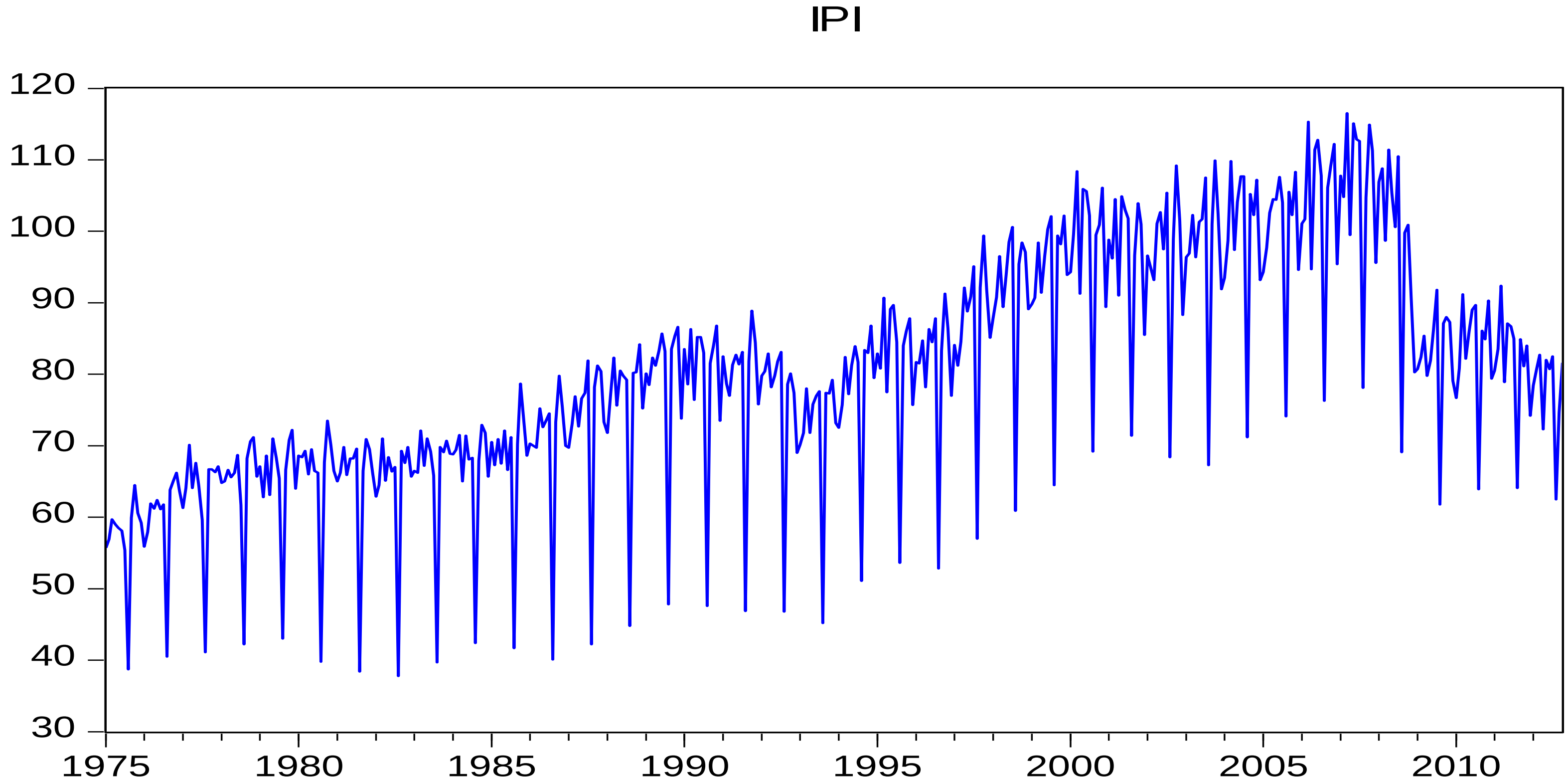
CO2 EMISSIONS IN THE USA



Source: World Databank

Motivation.

INDUSTRIAL PRODUCTION INDEX (SPAIN)



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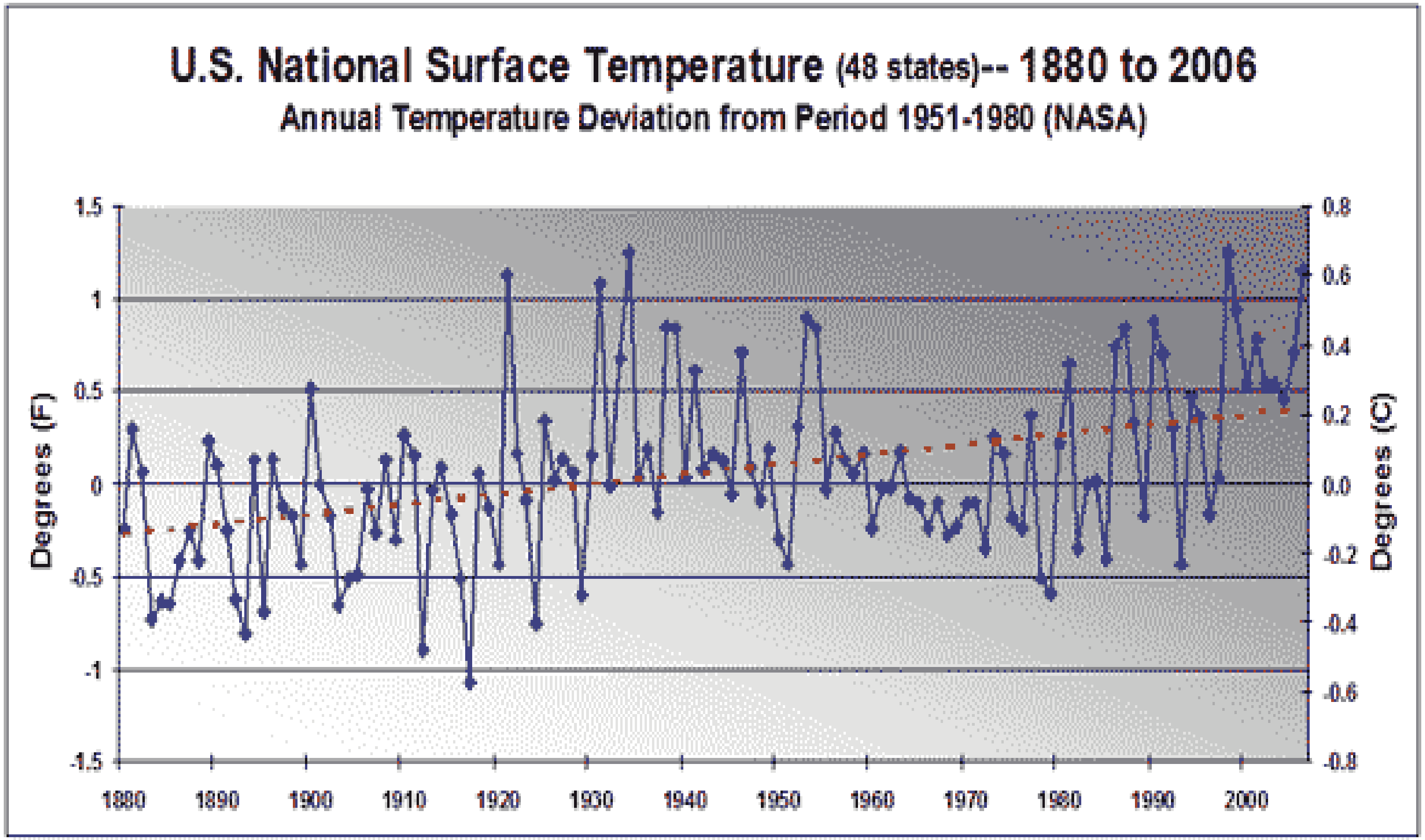
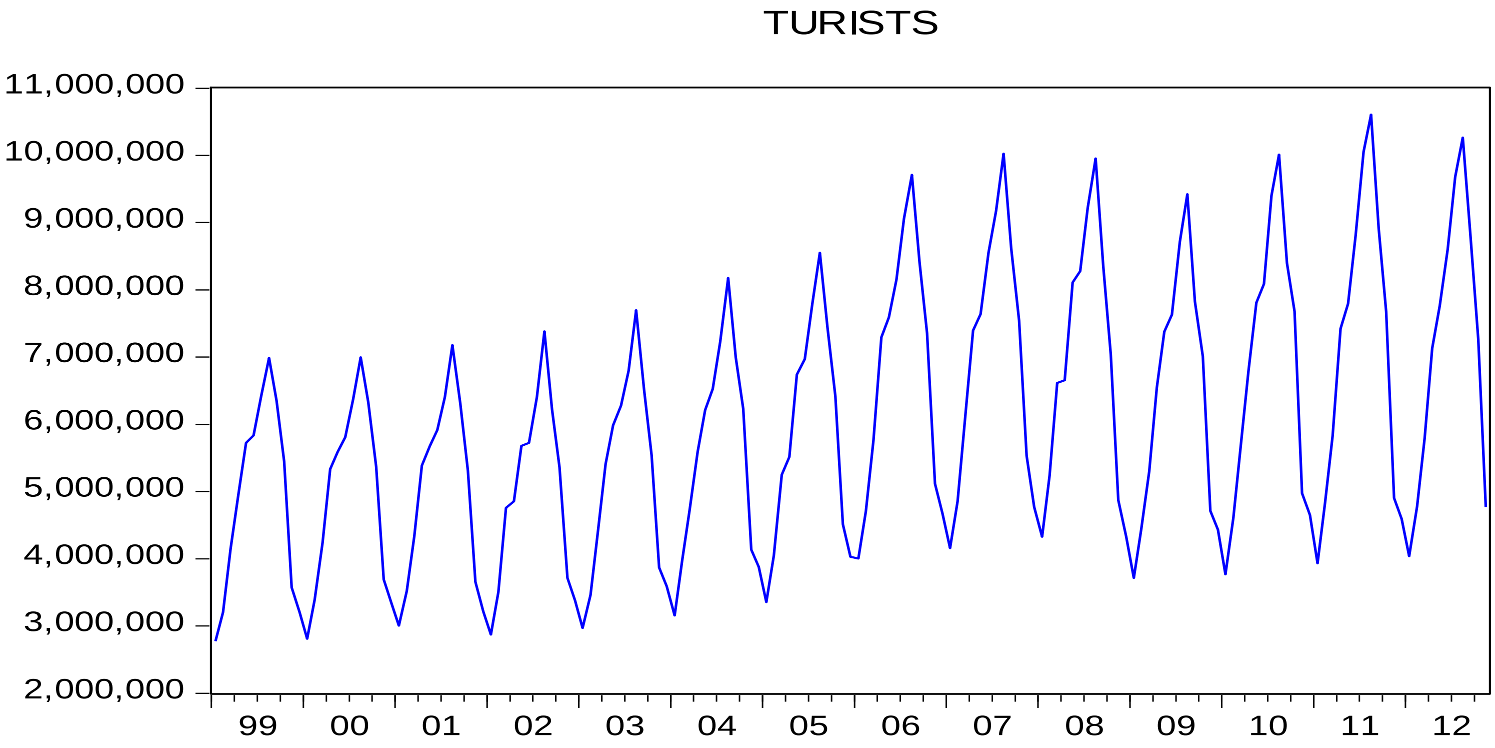
MOTIVATION.



U.S. National Surface Temperature (48 states)-- 1880 to 2006
Annual Temperature Deviation from Period 1951-1980 (NASA)
Degrees (F)
Degrees (C)
1.5
1
0.5
0
-0.5
-1
-1.5
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
1880
1890
1900
1910
1920
1930
1940
1950
1960
1970
1980
1990
2000

MOTIVATION.

OVERNIGHT STAYS IN SPANISH HOTELS



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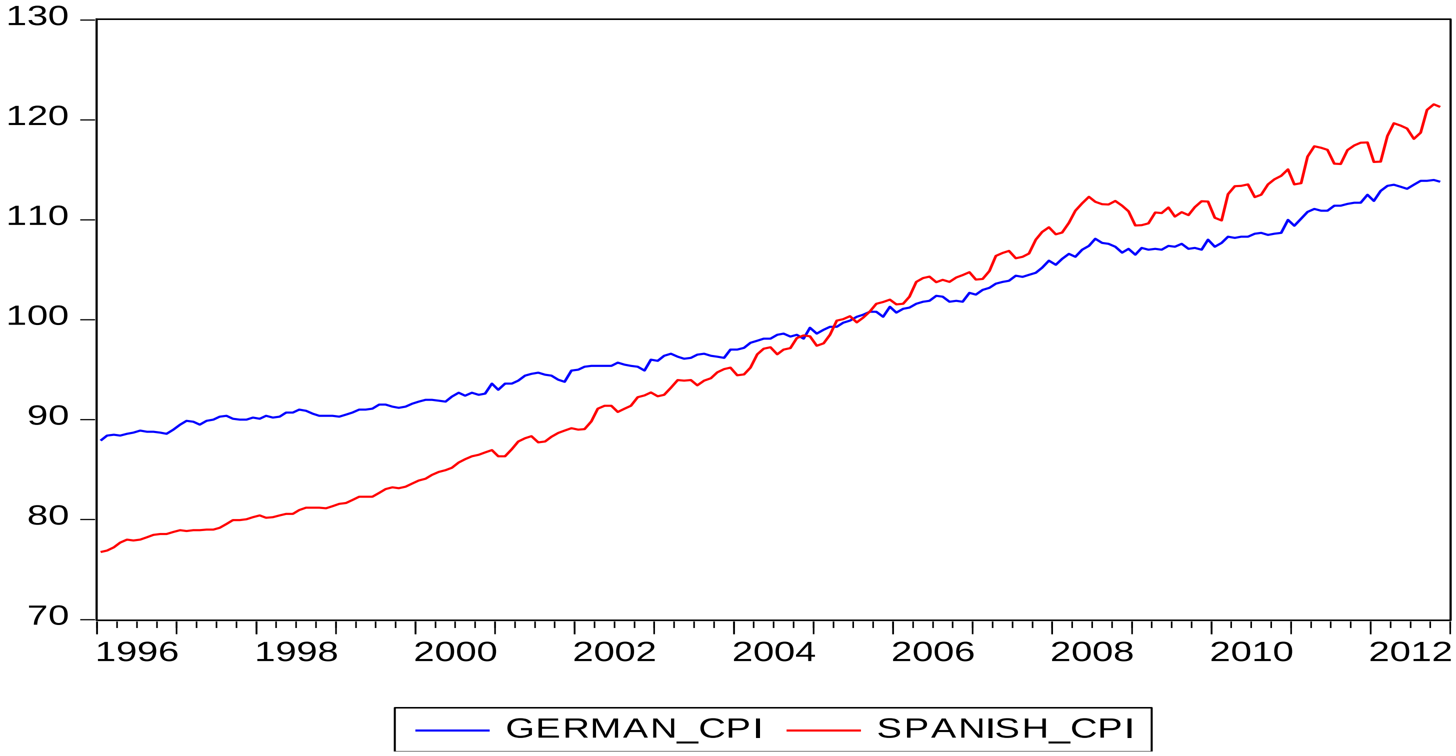
MOTIVATION.

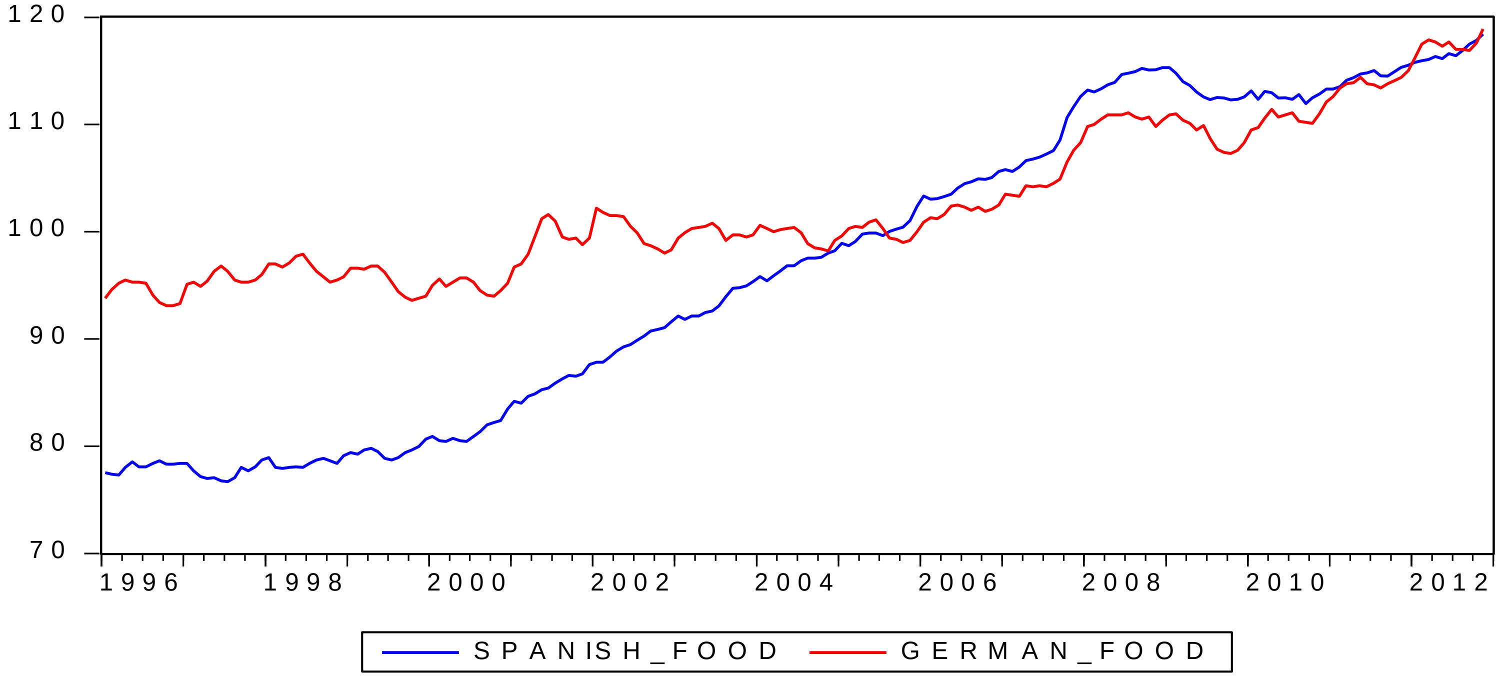
CONSUMER PRICE INDEX IN GERMANY AND SPAIN



Source: Eurostats

MOTIVATION.

CONSUMER PRICE INDEX FOR FOOD AND NON-ALCOHOLIC BEVERAGES IN GERMANY AND SPAIN



Source: Eurostats

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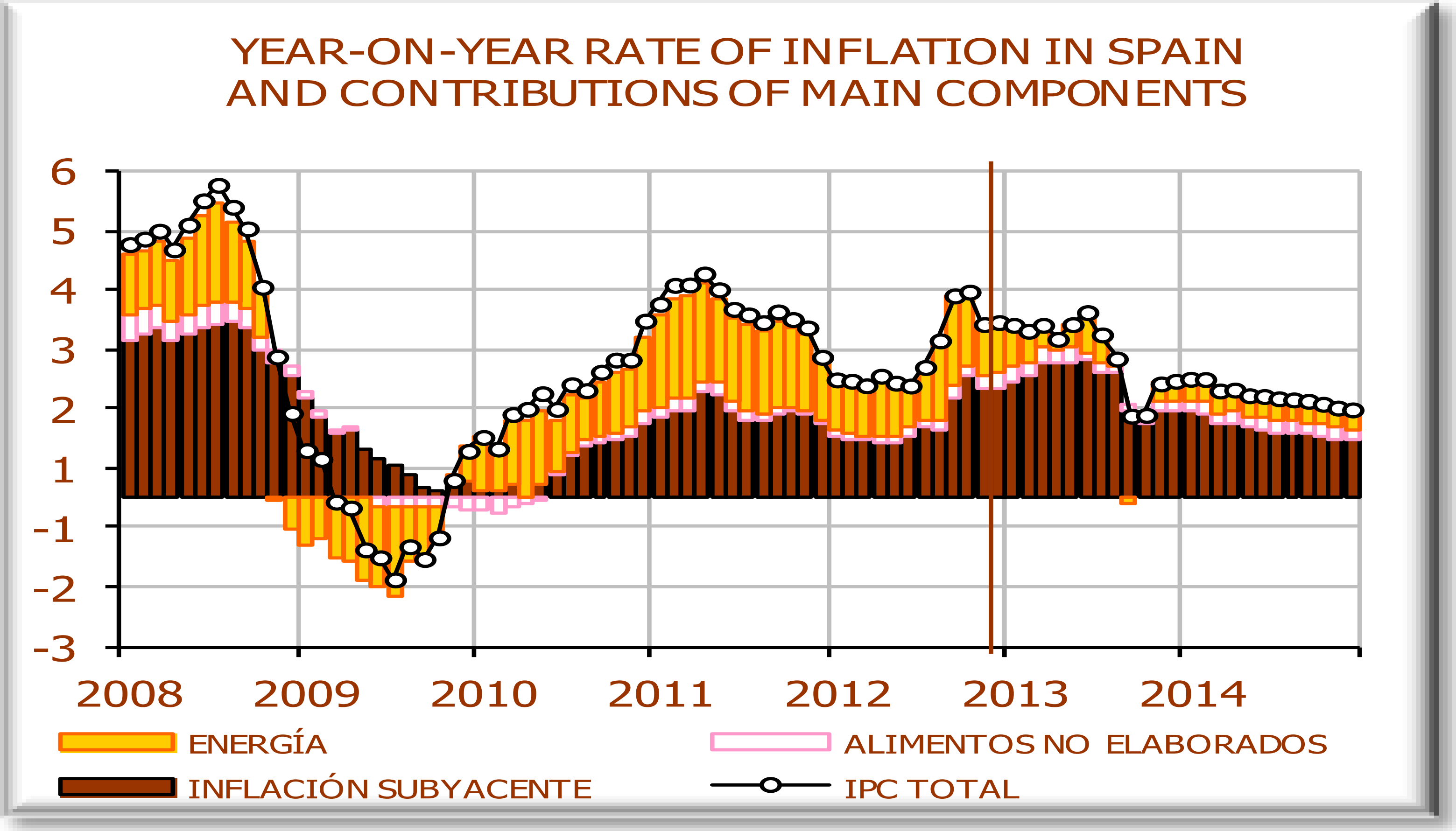

Source: Instituto Flores de Lemus

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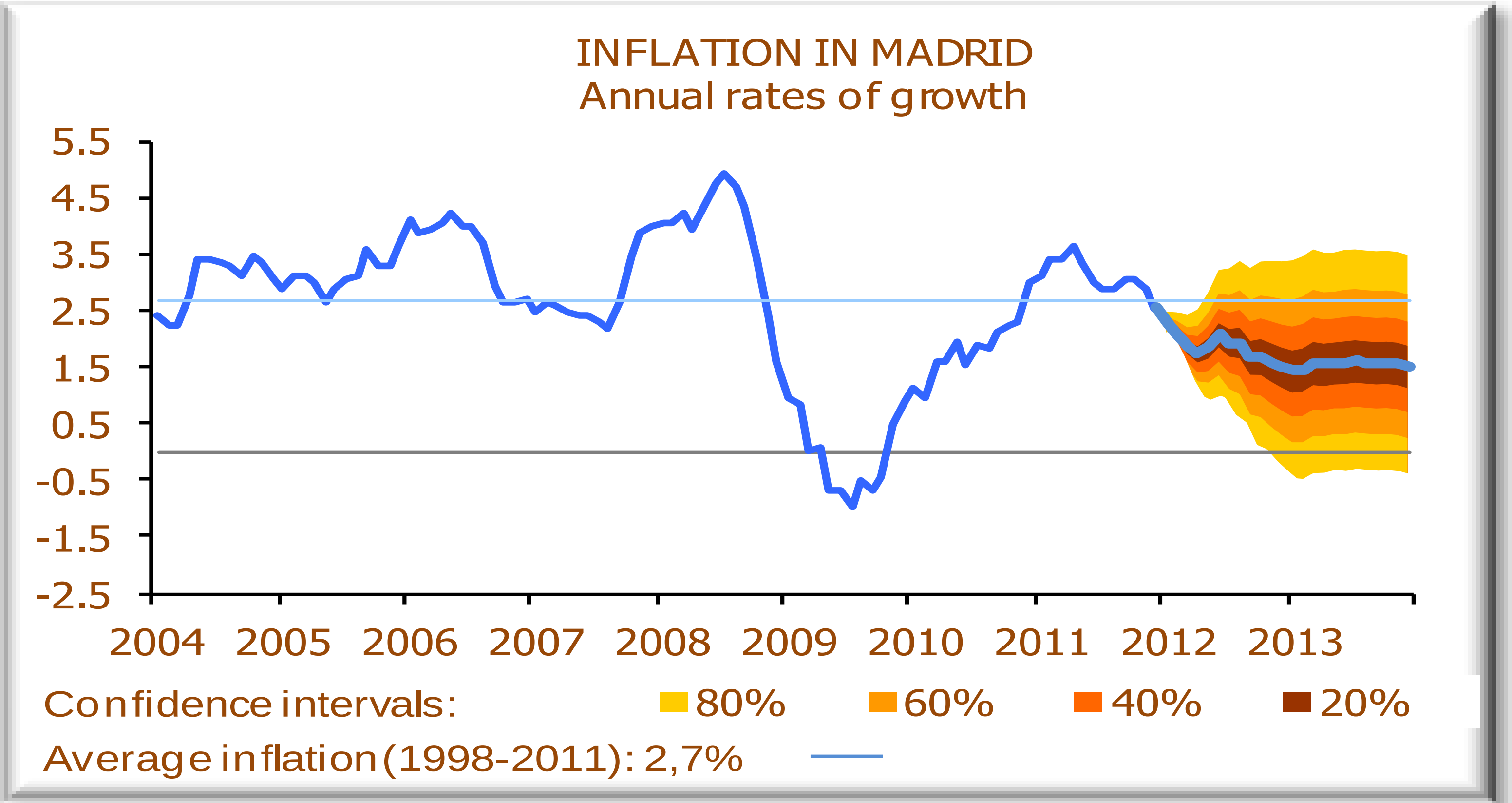


Source: Instituto Flores de Lemus

MOTIVATION.

IBEX 35
^IBEX
2 ene, 2013
9.000
8.500
8.000
7.500
7.000
6.500
6.000
5.500
© Yahoo!
mar-12
may-12
jul-12
sep-12
nov-12
ene-13
Volumen
800,0
600,0
400,0
200,0
0,0
Millones

MOTIVATION.

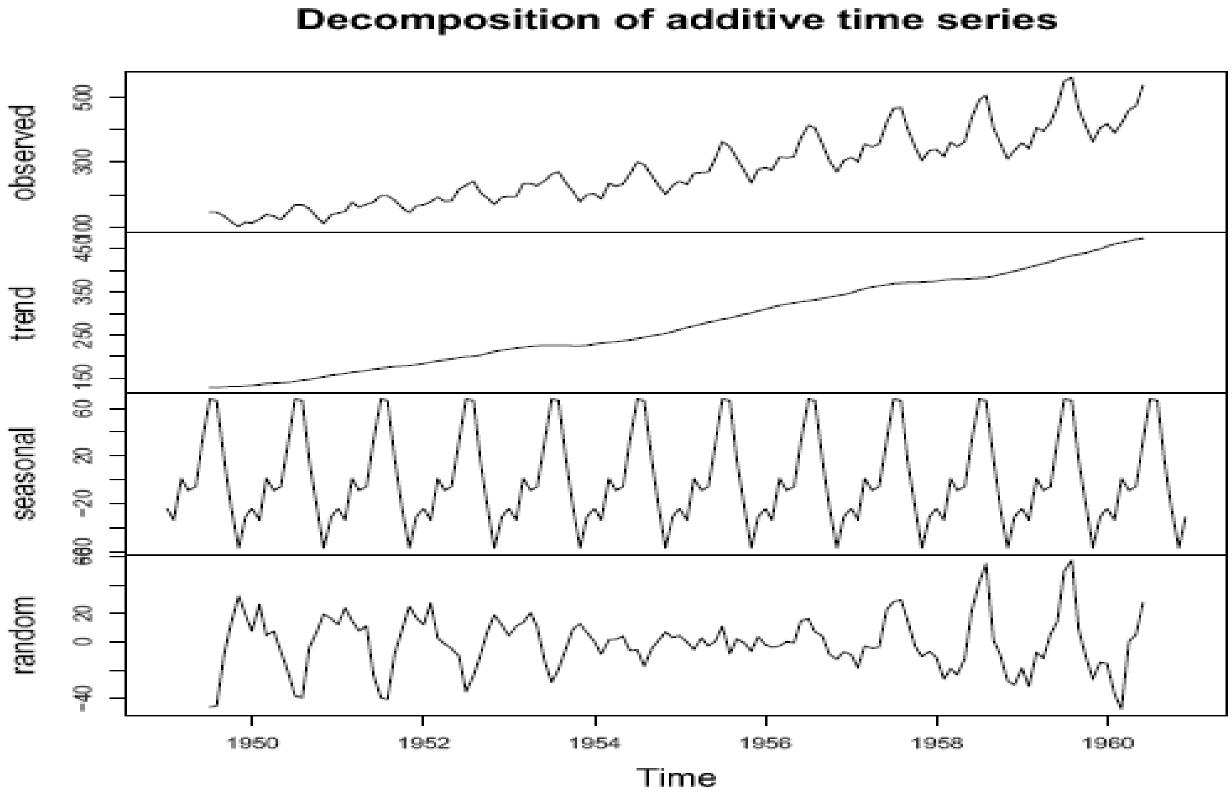


Figure: Additive seasonal decomposition of the airline data

Motivation.

- Knowledge of the dynamic structure will help to produce accurate forecasts of future observations and design optimal control schemes.

Main objectives

- Understanding the dynamic structure of the observations in a single series.
- Ascertain the leading, lagging and feedback relationships among several variables.
- Analyzing the impact of other variables that can be used for policy decisions.
- What to do when there are breaks in the evolution of the series.

Chapter 1

Univariate ARIMA Models

Recommended readings: chapters 1 to 3 of Peña *et al.* (2000)

Chapter 1. Contents.

1.1. Introduction.

Definitions, examples,

1.2. Properties of univariate ARMA.

ARMA weights.

Stationarity and covariance structure.

Autocorrelation function.

Partial autocorrelación function.

1.3. Model especification.

1.4. Examples.

Introduction

- **Stochastic Process.** Real-valued random variable Z_t that follows a distribution $f_t(Z_t)$.

- **The T-dimensional** variable $Z_{t_1}, Z_{t_2}, \dots, Z_{t_T}$ will have a joint distribution that depends on $t_1, \dots, t_T$.

INTRODUCTION

- Time series $z_{t_1}, z_{t_2}, \ldots, z_{t_T}$ will denote a particular realization of the stochastic process.

- To simplify notation $z_1, z_2, \ldots, z_T$.

INTRODUCTION

Assumptions

1. The process is stationary.

2. The joint distribution of $z_1, z_2, \ldots, z_T$ is a multivariate normal distribution.

INTRODUCTION

Implications of stationarity

- The joint distribution remains constant over time
- It is then true that

$$f(z_1, z_2, \ldots, z_T) = f(z_{1+k}, z_{2+k}, \ldots, z_{T+k})$$

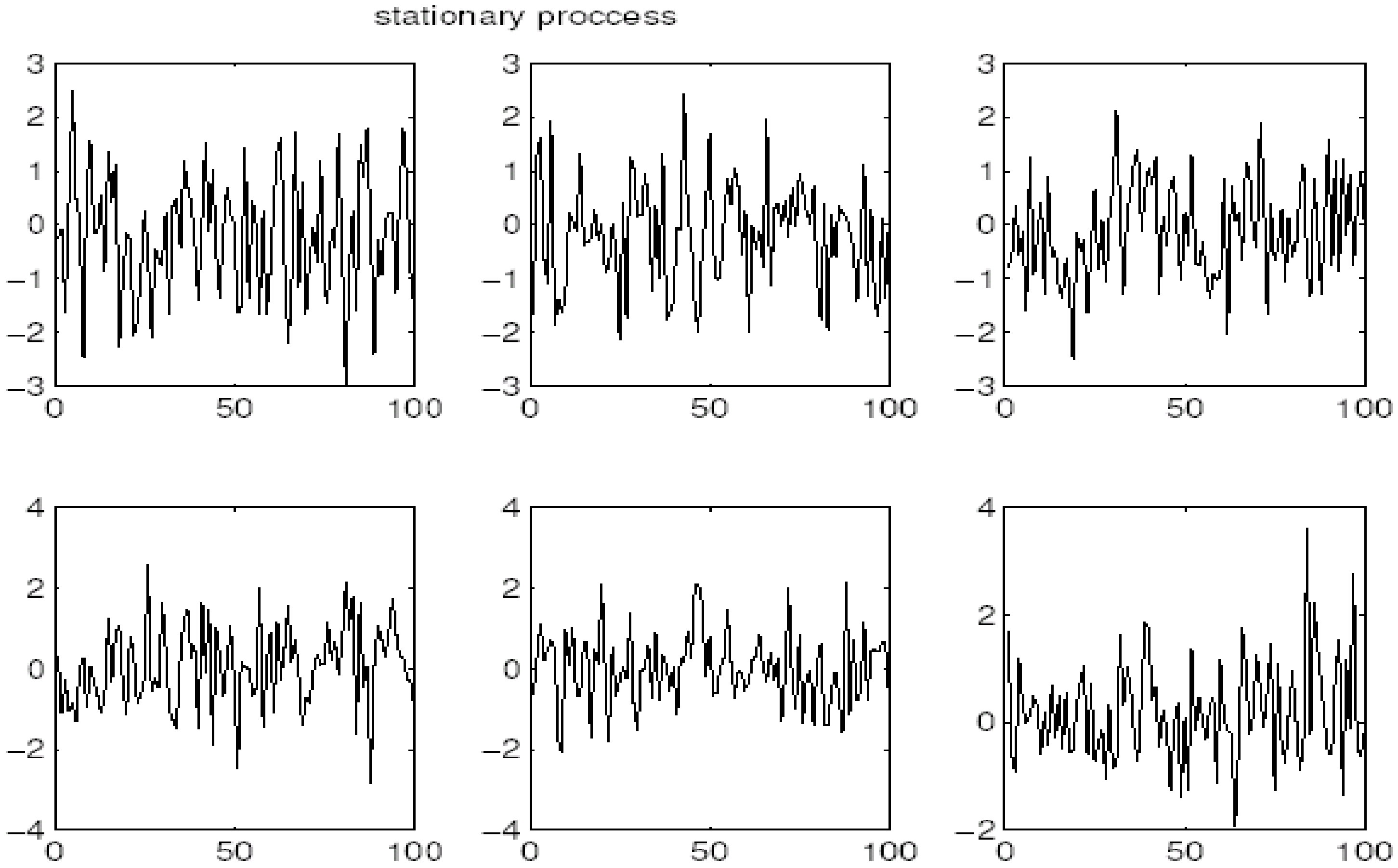
$$Ez_t = \mu_z; \qquad Vz_t = V_z$$

INTRODUCTION

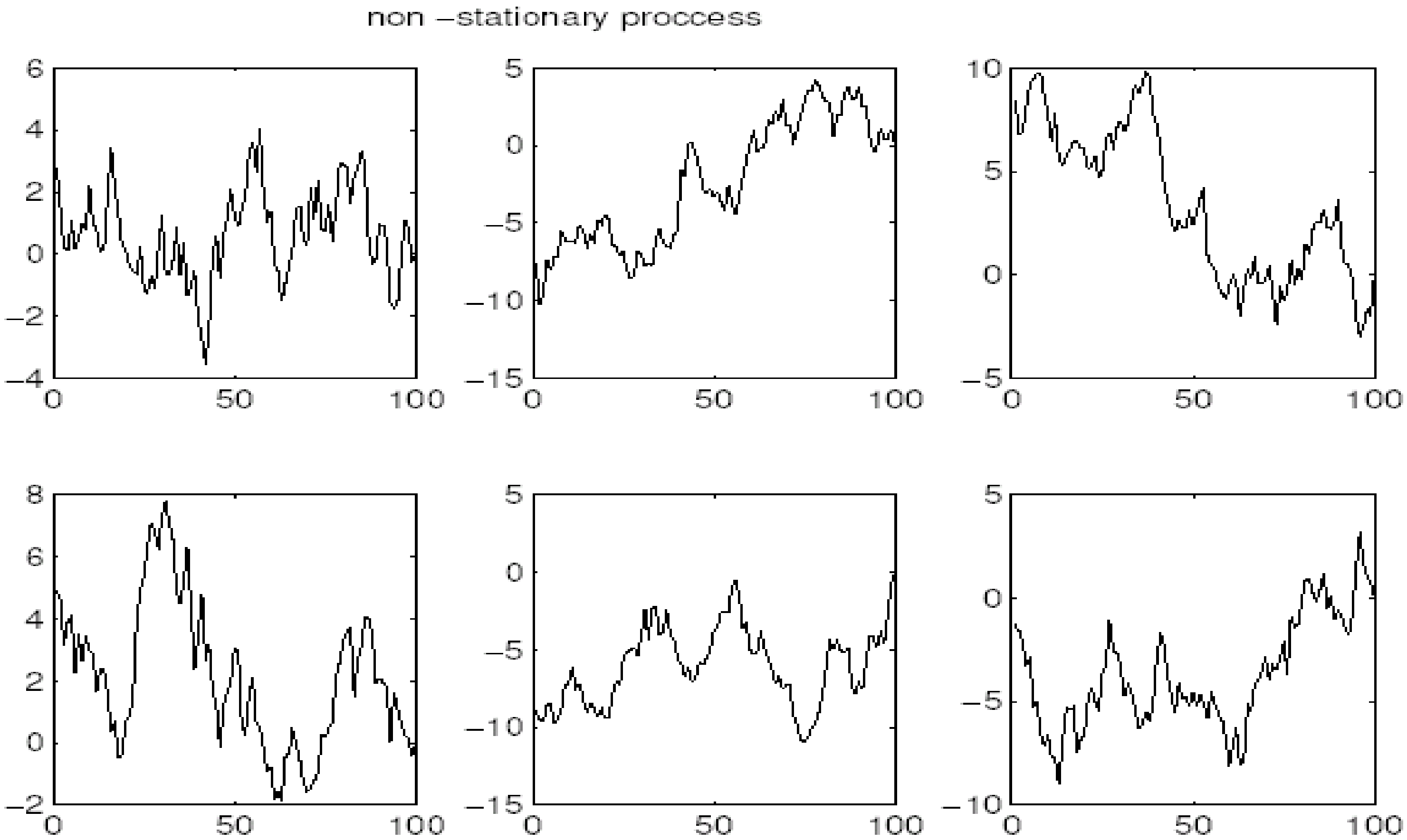
- Stationarity implies a constant mean and bounded deviations from it.

- Strong requirement, few actual economic series will satisfy it.

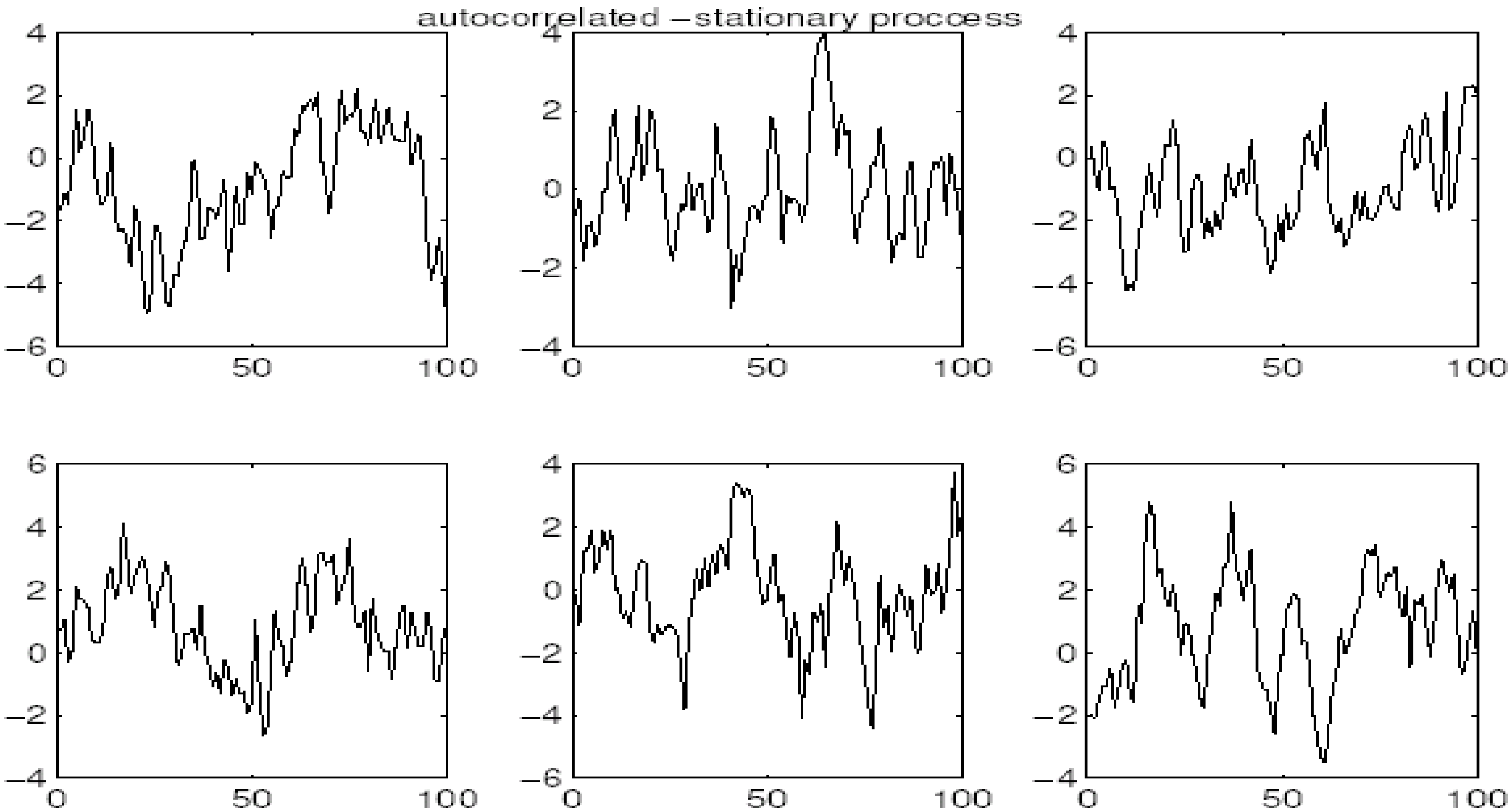
INTRODUCTION



INTRODUCTION



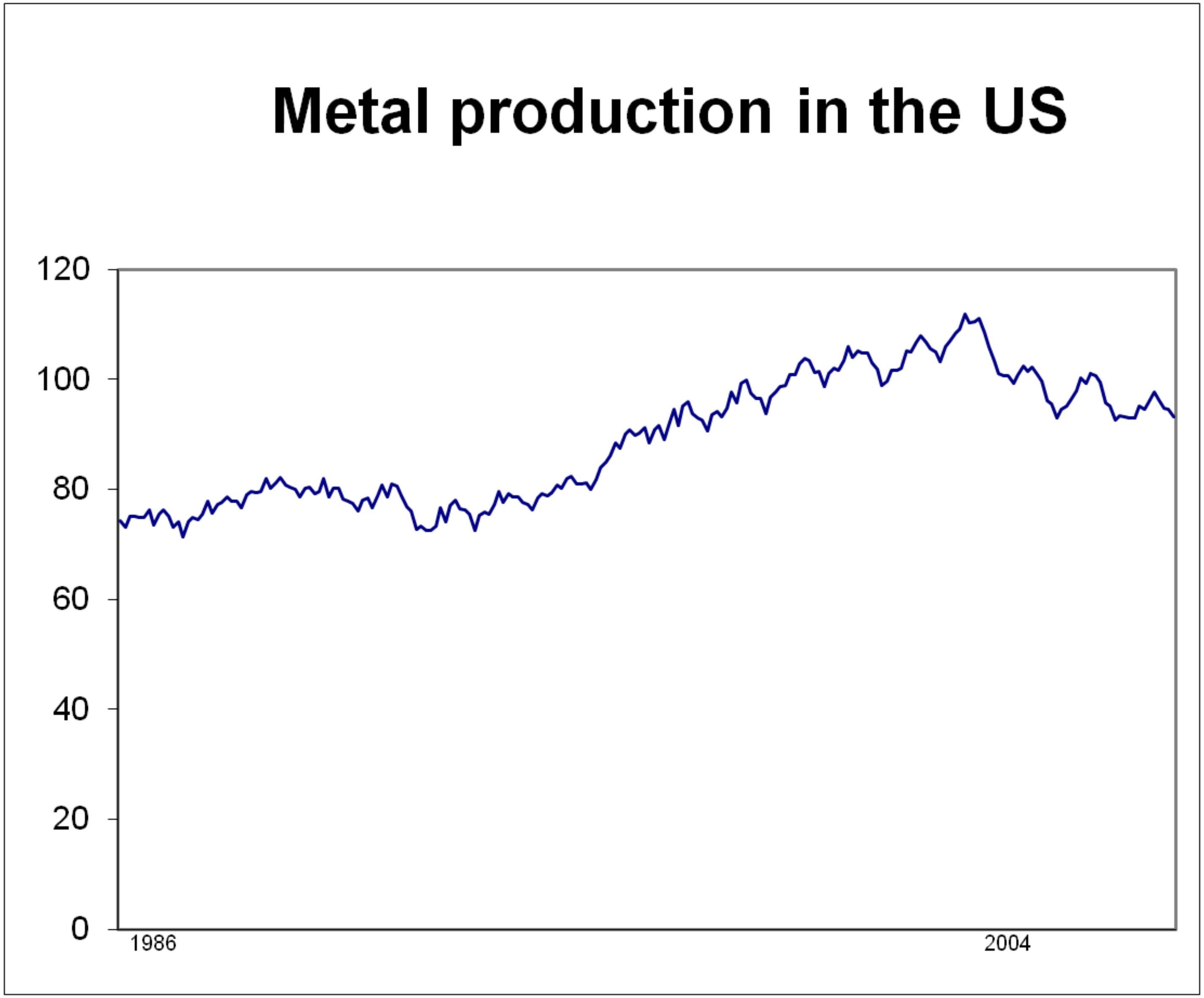
INTRODUCTION

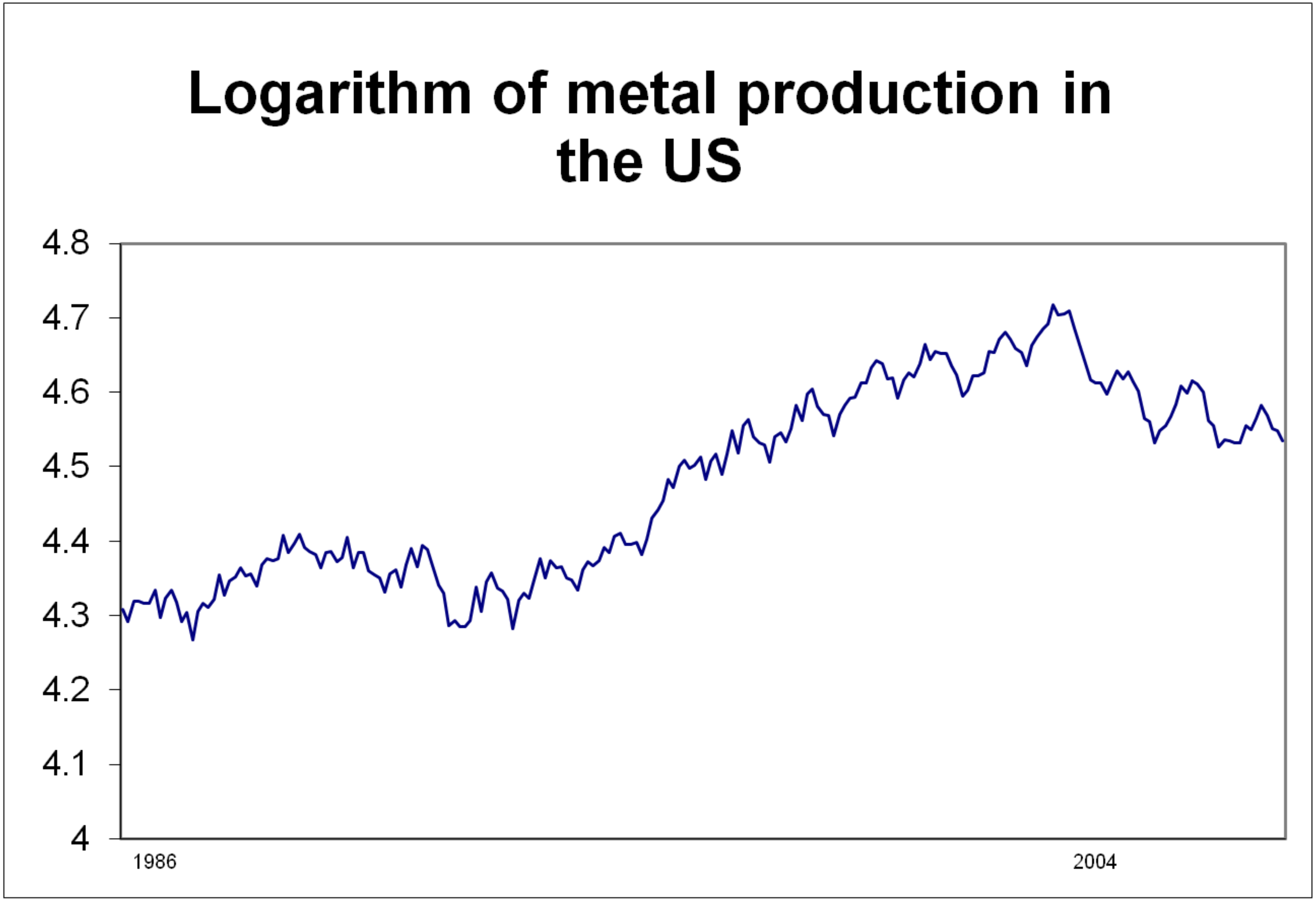


INTRODUCTION

Transformations to achieve stationarity

- Constant variance: log/level plus outlier correction.
- Stationary in mean: differencing.


Metal production in the US
120
100
80
60
40
20
0
1986
2004


Logarithm of metal production in the US
4.8
4.7
4.6
4.5
4.4
4.3
4.2
4.1
4
1986
2004

Differencing

- Let B be the backward operator $B^j z_t = z_{t-j}$

- We shall use the operators:
  - Regular difference
  - Seasonal difference

$$\nabla = 1 - B$$

$$\nabla_s = 1 - B^s$$

If z_t is a deterministic trend ($z_t = a + bt$)

$$\nabla z_t = b$$

$$\nabla^2 z_t = 0$$

where

$$\nabla^2 z_t = \nabla(\nabla z_t)$$

- In general, ∇^{d} will reduce a polynomial of degree d to a constant.

- Example: for a quarterly series z_t, $\nabla_4 z_t = z_t - z_{t-4}$ will cancel a constant, but it will also cancel other deterministic periodic functions.

- Homogeneous difference equation

$$\nabla_4 z_t = (1 - B^4) z_t = z_t - z_{t-4}$$

- Characteristic equation: $r^4 - 1 = 0$
- Solution is given by $r = \sqrt[4]{1}$ (four roots on the unit circle)

$$r_1 = 1, r_2 = -1, r_3 = i, r_4 = -i$$

  - Two real roots and two complex conjugates with modulus 1 and frequency $\omega = \pi/2$.

Therefore,

$$\nabla_4 = (1 - B)(1 + B)(1 + B^2)$$

1. One in the zero frequency (trend)
2. One in the twice-a-year seasonality $\omega = \pi$
3. Associated with once-a-year seasonality $\omega = \pi / 2$

To see this just recall that the complementary function to the difference equation $z_t = bz_{t-1} + cz_{t-2}$ will eventually be of the form:

$$z_t = (r^t)[A\cos\theta t + B\sin\theta t]$$

where r is a positive constant and θ is an angle measured in radians. A and B are arbitrary constants to enable the solution to satisfy any starting point of z_t.

Hence, when the two solutions of the characteristic equations are i and $-i$, $\sin\theta = 1$ and $\theta = \frac{\pi}{2}$.

"Building block" is the white noise process

$$a_t \succ Niid(0, \sigma^2)$$

Implications:

$$E(a_t) = E(a_t / a_{t-1}, a_{t-2}, a_{t-3}, \ldots) = 0$$

$$E(a_t, a_{t-j}) = Cov(a_t, a_{t-j}) = 0$$

$$Var(a_t, a_{t-j}) = Var(a_t / a_{t-1}, a_{t-2}, \ldots) = \sigma_a^2$$

General ARMA(p,q) processes:

$$\phi(B)z_t = c + \theta(B)a_t$$

- z_t is the observable time series.
- a_t is sequence of white noise.
- c is the constant term.

- Autoregressive polynomial

$$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p$$

- Moving-average polynomial

$$\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q$$

- The AR and MA polynomials are assumed to have no common factor

- Stationarity implies that all zeros of $\phi(B)$ are restricted to lie outside the unit circle and in this case:

$$c = (1 - \phi_1 - \cdots - \phi_p)\mu$$

where μ is the mean of the series.

- Overall behavior of the series remains the same over time.

- Overall behavior of the series remains the same over time.
- In real world, however, time series data often exhibits a drifting behavior. Nonstationarity can be modeled allowing some of the zeroes in $\phi(B)$ to be equal to one. Thus,

$$\phi(B)(1-B^{d})z_t = c + \theta(B)a_t$$

- This is known as the ARIMA (p,d,q) model.

- Some special cases of ARIMA(p,d,q):
  - AR(1)
  - MA(1)
  - ARMA(1,1)
  - IMA(1,1)
  - ARIMA(0,1,1)(0,1,1)

- AR(1) to MA(∞) by recursive substitution.

$$z_t = \phi z_{t-1} + a_t$$

$$z_t = \phi(\phi z_{t-2} + a_{t-1}) + a_t = \phi^2 z_{t-2} + \phi a_{t-1} + a_t$$

$$z_t = \phi^k z_{t-k} + \phi^{k-1} a_{t-k+1} + \cdots + \phi^2 a_{t-2} + \phi a_{t-1} + a_t$$

- If $|\phi| < 1$, then

$$z_t = \sum_{j=0}^{\infty} \phi^j a_{t-j}$$

which is a MA(∞)

- Some considerations:

- ARMA models are not unique (variety of possible representations.
- Parsimony is always a desirable property.
- AR representations are easiest to estimate (OLS) and to be interpretated.

- For simplicity, we will assume zero mean and a starting point m. The ARIMA (p,d,q)

$$\phi(B)z_t = \theta(B)a_t$$

- Can be rewritten as:

$$D_\phi z = D_\theta a + \omega$$

where, $z = (z_m,\ldots,z_t)'$ $a = (a_m,\ldots,a_t)'$

$$\omega = (w_m,\ldots,w_{m-r-1},0,\ldots,0)'$$

$w_m,\ldots,w_{m-r-1}$ are r=max(p,q) initial values.

PROPERTIES

$$D_\phi = \begin{bmatrix} 1 & & & & & \\ -\phi_1 & \cdots & & & & \\ \cdots & \cdots & \cdots & & & \\ -\phi_p & & \cdots & \cdots & & \\ \cdots & & & \cdots & \cdots & \\ \cdots & \cdots & -\phi_p & \cdots & -\phi_1 & 1 \end{bmatrix}_{(t-m+1)(t-m+1)}$$

$$D_\theta = \begin{bmatrix} 1 & & & & & \\ -\theta_1 & \cdots & & & & \\ \cdots & \cdots & \cdots & & & \\ -\theta_q & & \cdots & \cdots & & \\ \cdots & & & \cdots & \cdots & \\ \cdots & \cdots & -\theta_q & \cdots & -\theta_1 & 1 \end{bmatrix}_{(t-m+1)(t-m+1)}$$

- ARMA weights

$$z = D_\phi^{-1} D_\theta a + D_\phi^{-1} \omega$$

Where,

$$D_\phi^{-1} D_\theta = \begin{bmatrix} 1 & & & \\ \psi_1 & & & \\ \dots & & & \\ \psi_{t-m} & \dots & \psi_1 & 1 \end{bmatrix}$$

Properties

- The ψ-weights can be obtained by equating coefficients of powers of B from the relations:

$$\phi(B)\psi(B) = \theta(B)$$

where

$$\psi(B) = 1 + \psi_1 B + \psi_2 B^2 + \ldots$$

PROPERTIES

- The ARIMA (p,d,q) model can then be rewritten in the MA form as:

$$z_t = a_t + \sum_{h=1}^{t-m} \psi_h a_{t-h}$$

Properties

- In the same way,

$$D_\theta^{-1} D_\phi z - D_\theta^{-1}\omega = a$$

- Where,

$$D_\theta^{-1} D_\phi = \begin{bmatrix} 1 & & & \\ -\pi_1 & & & \\ \dots & & & \\ -\pi_{t-m} & \dots & -\pi_1 & 1 \end{bmatrix}$$

- With $$\theta(B)\pi(B)=\phi(B).$$

- The AR form of the model is then,

$$z_t = \sum_{h=1}^{t-m} \pi_h z_{t-h} + a_t$$

PROPERTIES

- These two expressions are of fundamental importance in understanding the nature of the model.
- The MA form with the ψ weights, shows how the observation z_t is affected by current and past shocks or innovations.
- The AR form with the π weigths, indicates how the observation is related to its own past values.

Examples : AR(1) model

- MA representation:

$$z_t = a_t + \phi_1 a_{t-1} + \phi^2 a_{t-2} + \ldots.$$

- AR representation:

$$z_t = \phi z_{t-1} + a_t$$

Examples : MA(1) model

- MA representation:

$$z_t = a_t - \theta a_{t-1}$$

- AR representation:

$$z_t = -\theta z_{t-1} - .... - \theta^j z_{t-j} - ... - a_t$$

Examples : IMA(1,1) model

- MA representation:

$$z_t = a_t + (1-\theta)(a_{t-1} + a_{t-2} + ...)$$

- AR representation:

$$z_t = (1-\theta)z_{t-1} + \theta(1-\theta)z_{t-2} + \theta^2(1-\theta)z_{t-3} + ... + a_t$$

Stationarity condition

- Consider,

$$z_t = a_t + \sum_{h=1}^{t-m} \psi_h a_{t-h}$$

- Since the innovations are assumed normally distributed, it follows that the observations are also normally distributed.
- It is seen that, if in the characteristic equation:

$$\psi_k = A_1 r_1^k + \ldots + A_{p0} r_{p0}^k$$

- (Where the A's denote polinomials in k and the r's are the distinct zeroes of $\phi(B)$)
- Then as $|r_j| < 1, \quad j = 1, \ldots, po$ we have that

$$t - m \to \infty,$$

$$E(z_t) \to 0, \quad \operatorname{cov}(z_t, z_{t+k}) \to \sigma_a^2 \left(\sum_{h=0}^{\infty} \psi_h \psi_{h+k} \right)$$

Stationarity condition

- So that z_t will be stationary in this asymptotic sense.
- This is the stationarity condition of an ARMA model, and is equivalent to require that the zeroes of $\phi(B)$ are lying outside the unit circle.

- Let us denote,

$$\gamma(k) = \text{cov}(z_t, z_{t+k}) = \text{cov}(z_t, z_{t-k}) = \gamma(-k)$$

- For an alternative expressions of the autocovariance function in terms of the ARMA parameters,

$$z_{t-k}(z_t - \phi_1 z_{t-1} - \ldots - \phi_p z_{t-p}) =$$
$$= z_{t-k}(a_t - \theta_1 a_{t-1} - \ldots - \theta_q a_{t-q})$$

LAG K AUTOCOVARIANCE

- By taking expectations on both sides and using the MA form, we obtain, for $k \geq 0,$

$$\gamma(k) = \sum_{h=1}^{p} \phi_h \gamma(k-h) + g_k$$

- Where,

$$g_k = \begin{cases} -\sigma_a^2 \sum_{h=0}^{q-k} \psi_h \theta_{h+k} & k = 0, \pm 1, \pm 2, \ldots \\ 0 & k > q \end{cases}$$

AUTOCORRELATION FUNCTION

- The autocorrelation function is defined as

$$\rho(k) = \frac{\gamma(k)}{\gamma(0)}, \quad k = 0, \pm 1, \pm 2, \ldots$$

- By substitution of $\gamma(k)$, we obtain that

$$\rho(k) = \sum_{h=1}^{p} \phi_h \rho(k-h) + \frac{g_k}{\gamma(0)} \quad k \geq 0$$

AUTOCORRELATION FUNCTION

- When $\phi(B)=0$, in a MA(q) model, then,

$$\rho(k)=\begin{cases} -\theta_q(1+\theta_1^2+\ldots\theta_q^2)^{-1}, & k=q \\ 0, & k>q \end{cases}$$

- That is, the autocorrelation function cuts off after lag q.

PARTIAL AUTOCORRELATION FUNCTION

- <u>Intuition</u>

- AR(1):

$$Z_{t-3} \Rightarrow Z_{t-2} \Rightarrow Z_{t-1} \Rightarrow Z_t$$

- AR(2):

$$Z_{t-3} \Rightarrow Z_{t-2} \Rightarrow Z_{t-1} \Rightarrow Z_t$$

PARTIAL AUTOCORRELATION FUNCTION

- The autocorrelation function only takes into account that Z_t and Z_{t-2} are related in both cases.

- But if we want to measure the direct relationship (without the intermediate Z_{t-1}), we found that it is zero for the AR(1) and different from zero for the AR(2) model

Partial autocorrelation function

- The partial autocorrelation function is, therefore, a measure of the linear relation among observations k-periods apart, independently of the intermediate values.

Partial autocorrelation function

- Consider first an stationary AR(p) model. The autoregressive coefficients are related to the autocorrelations by the Yule-Walker equations,

$$G_p \phi_p = \rho_p$$

- where, G_p is the (pxp) matrix,

Partial autocorrelation function

$$G_p = \begin{bmatrix} 1 & \rho(-1) & \dots & \rho(-p+2) & \rho(-p+1) \\ \rho(1) & 1 & & \vdots & \rho(-p+2) \\ \vdots & & \ddots & & \vdots \\ \rho(p-2) & & & 1 & \rho(-1) \\ \rho(p-1) & \rho(p-2) & \dots & \rho(1) & 1 \end{bmatrix}$$

Partial autocorrelation function

- Regarding this as system of p equations and p unknowns (the ϕ coefficients), the solution is, for p>1, the ratio of two determinants

$$\phi_p = |H_p| / |G_p|$$

where

$$H_p = \begin{bmatrix} 1 & \rho(-1) & \ldots & \rho(-p+2) & \rho(1) \\ \rho(1) & 1 & & \vdots & \rho(2) \\ \vdots & & \ddots & & \vdots \\ \rho(p-2) & & & 1 & \rho(p-1) \\ \rho(p-1) & \rho(p-2) & \ldots & \rho(1) & \rho(p) \end{bmatrix}$$

PARTIAL AUTOCORRELATION FUNCTION

- A pxp matrix, and $\phi_p = \rho(1)$ for p=1. This leads to define, for any stationary model

$$\varphi_k \begin{Bmatrix} \rho(k) & k=1 \\ |H_k| / |G_k| & k>1 \end{Bmatrix}$$

- Which is known as the Partial Autocorrelation Function.

PARTIAL AUTOCORRELATION FUNCTION

It has the property that, for a stationary AR(p) model,

$$\varphi_k = \begin{cases} \phi_k & k = p \\ 0 & k > p \end{cases}$$

In other words, φ_k vanishes for k>p when the model is AR(p).

PROPERTIES

	SAF	PAF
AR(P)	slow decay towards zero	p different from zero
MA(q)	q different from zero	slow decay towards zero
ARMA(p,q)	Slow decay towards zero	Slow decay towards zero

- AR(1) model

- Model:

$$z_t = \phi z_{t-1} + a_t$$

- Variance:

$$\sigma_z^2 = \phi^2 \sigma_z^2 + \sigma_a^2$$

$$\sigma_z^2 = \frac{\sigma_a^2}{1-\phi^2}$$

- Autocovariance function:

$$\gamma(k) = \begin{cases} \sigma_z^2 & k = 0 \\ \dfrac{\phi\sigma_a^2}{1-\phi^2} & k = 1 \\ \phi\gamma(k-1) & k > 1 \end{cases}$$

Examples

- Autocorrelation function:

$$\rho(k) = \begin{cases} 1 & k = 0 \\ \phi & k = 1 \\ \phi^k & k > 1 \end{cases}$$

- Partial autocorrelation:

$$\varphi(1) = \rho(1)$$

Examples

- <u>AR(2) model</u>

- Model:

$$z_t = \phi_1 z_{t-1} + \phi_2 z_{t-2} + a_t$$

- Variance:

$$\sigma_z^2 = \left(\frac{1-\phi_2}{1+\phi_2}\right) \frac{\sigma_a^2}{\left\{(1-\phi_2)^2 - \phi_1^2\right\}}$$

- Autocorrelation function:

$$\rho(k) = \begin{cases} \dfrac{\phi_1}{1-\phi_2} & k=1 \\ \phi_2 + \dfrac{\phi_1^2}{1-\phi_2} & k=2 \end{cases}$$

$$\rho_k = \phi_1 \rho_{k-1} + \phi_2 \rho_{k-2} \quad k > 2$$

MA(1) model

- Model:

$$z_t = a_t - \theta a_{t-1}$$

- Variance:

$$\sigma_z^2 = \sigma_a^2 + \theta^2 \sigma_a^2$$

$$\sigma_z^2 = \sigma_a^2 (1 + \theta^2)$$

- Autocovariance function:

$$\gamma(k) = \begin{cases} \sigma_z^2 & k = 0 \\ -\theta\sigma_a^2 & k = 1 \\ 0 & k > 1 \end{cases}$$

EXAMPLES

- Autocorrelation function:

$$\rho(k) = \begin{cases} 1 & k = 0 \\ \dfrac{-\theta}{1+\theta^2} & k = 1 \\ 0 & k > 1 \end{cases}$$

SPECIFICATION

- The class of ARMA(p,q) models is extensive.
- Guidelines are needed in selecting a member of the class to represent the time series data at hand

SPECIFICATION

- Box and Jenkins (1976) proposed an iterative model building strategy.

  - Tentative specification or identification of a model.
  - Efficient estimation of model parameters.
  - Diagnostic checking of fitted model for further improvement.

Specification

- Tentative specification. The aim is to employ statistics that:
  - 1. Can be readily calculated from the data.
  - 2. Allow the user to tentatively select a model .

SPECIFICATION

- Three methods:
  - The Sample Autocorrelation Function (SAF)
  - The Sample Partial Autocorrelation Function (SPAF)
  - Use of diagnostic tools (AIC,BIC) to find the best model (TRAMO-automatic).

- Sample Autocorrelation Function. The SACF of z_t are defined as

$$\hat{\rho}_k = C_k / C_0 \quad k = 1, 2, \ldots$$

with $C_j = \sum_{t=1}^{n-j} (z_t - \bar{z})(z_{t+j} - \bar{z})$

and $\bar{z}$ the sample mean of the n available observations

- Properties:

- 1.For stationary models,

$$\hat{\rho}_k \to \rho_k \quad n \to \infty$$

- 2. When there exists a unit root in the AR polynomial

$$\hat{\rho}_k \xrightarrow{p} 1$$

SPECIFICATION

- If the SACF of the original series is persistently close to 1 as k increases, one forms the first difference , ∇z_t , and studies its SACF to determine whether further differencing is called for.

SPECIFICATION

- Once stationary is achieved, a cutting off pattern after, say a lag "q", in the SACF will then lead to tentative specification of a MA(q) model.

- The Sample Partial Autocorrelation Function:

$$\hat{\varphi}_k \quad k = 1,2,\ldots$$

- Are obtained by replacing the ρ_k in

$$\varphi_k \begin{Bmatrix} \rho(k) & k = 1 \\ \left|H_k\right| / \left|G_k\right| & k > 1 \end{Bmatrix}$$

- by their sample estimates $\hat{\rho}_k$

Properties:

- 1.For stationary models,

$$\hat{\varphi}_k \to \varphi_k \quad n \to \infty$$

- 2. The $\hat{\varphi}_k$ are asymptotically normally distributed
- 3. For a stationary AR(p) model

$$Var(\hat{\varphi}_k) \cong n^{-1} \quad k > p$$

SPECIFICATION

- Properties 1, 2 and 3 make SPACF a convenient tool for specifying the order or a stationary AR model (cutting off pattern after lag p)

- Not valid for non-stationary models.

Weakness of the SACF and SPACF.

1. Subjective judgment is often required to decide on the order of differencing.
2. For stationary ARMA models, both SACF and SPACF tend to exhibit a gradual "tapering off" behavior, making the specification very difficult.

Diagnostic Tools

- The program (TRAMO), in an automatic way, specifies a set of possible models, estimate them and select the best one based on AIC and BIC criteria.

- However an accurate jugment is always necessary to interprect the output of TRAMO.