Bivariate data analysis

Categorical data - creating data set

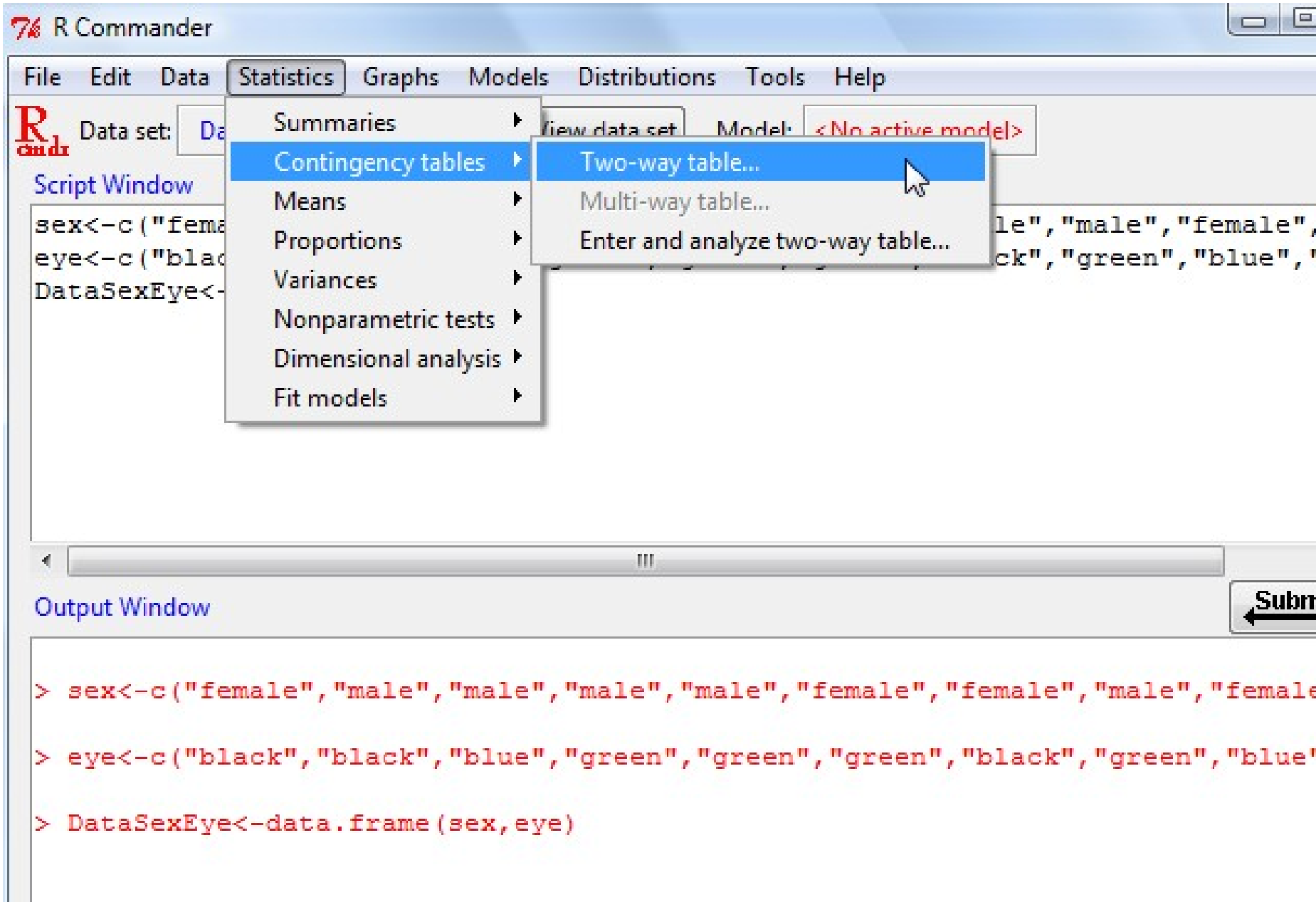
Upload the following data set to R Commander

sex	female	male	male	male	male	female	female	male	female	female
eye	black	black	blue	green	green	green	black	green	blue	blue

- Method 1: Type the table in the Notepad, save it and import to Rcmdr
- Method 2: Introduce directly in the Script Window

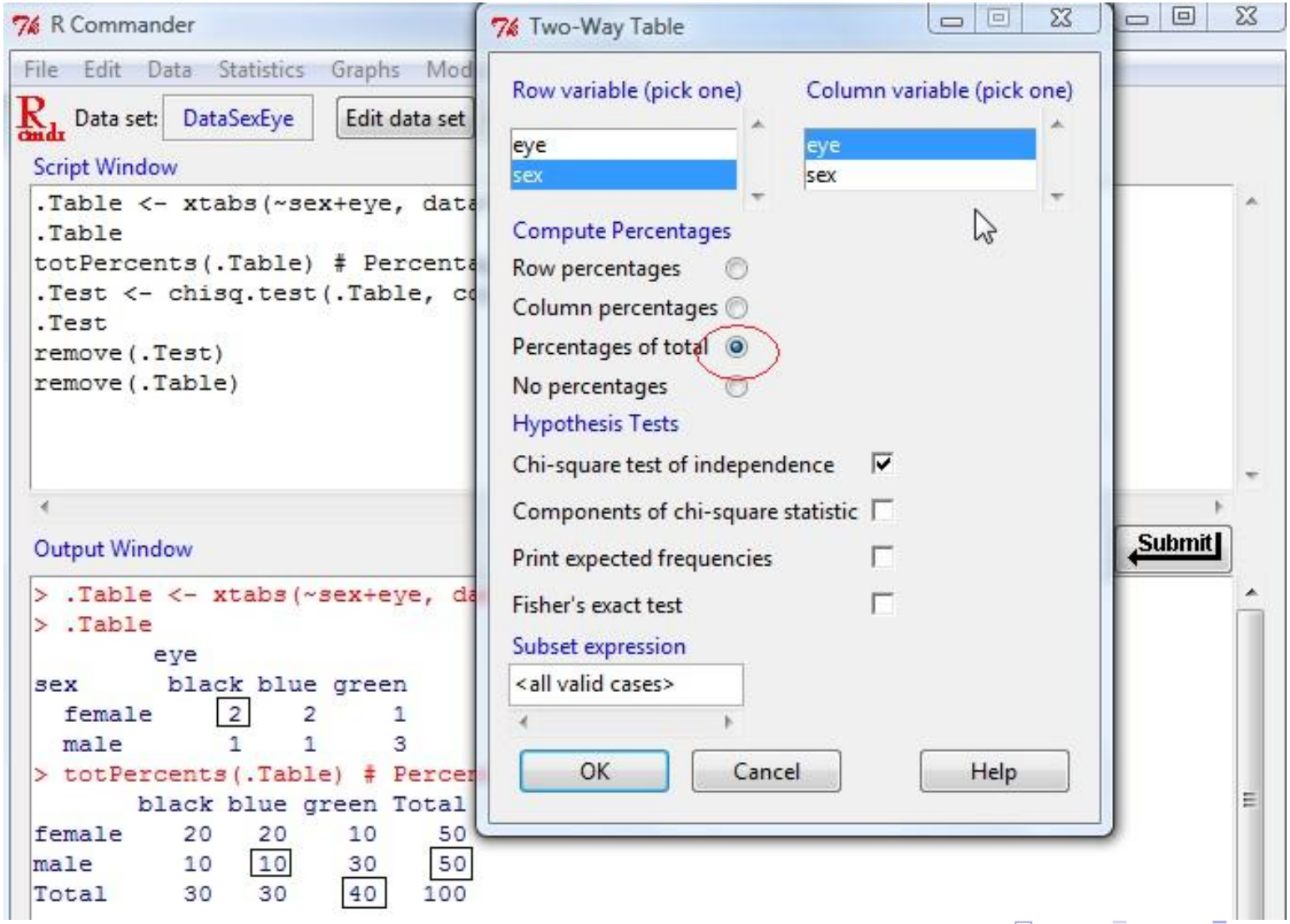
```
eye = c("black","black","blue","green","green",
"green","black","green","blue","blue")

sex = c("female","male","male","male","male",
"female", "female","male","female","female")

DataSexEye = data.frame(sex,eye)
```

Categorical data - contingency table



Categorical data - contingency table cont.

- How many of the sampled people are female with black eyes? (2)
- What % of the sampled people are male with blue eyes? (10%)
- What % of the sampled people are male? (50%)
- What % of the sampled people have green eyes? (40%)



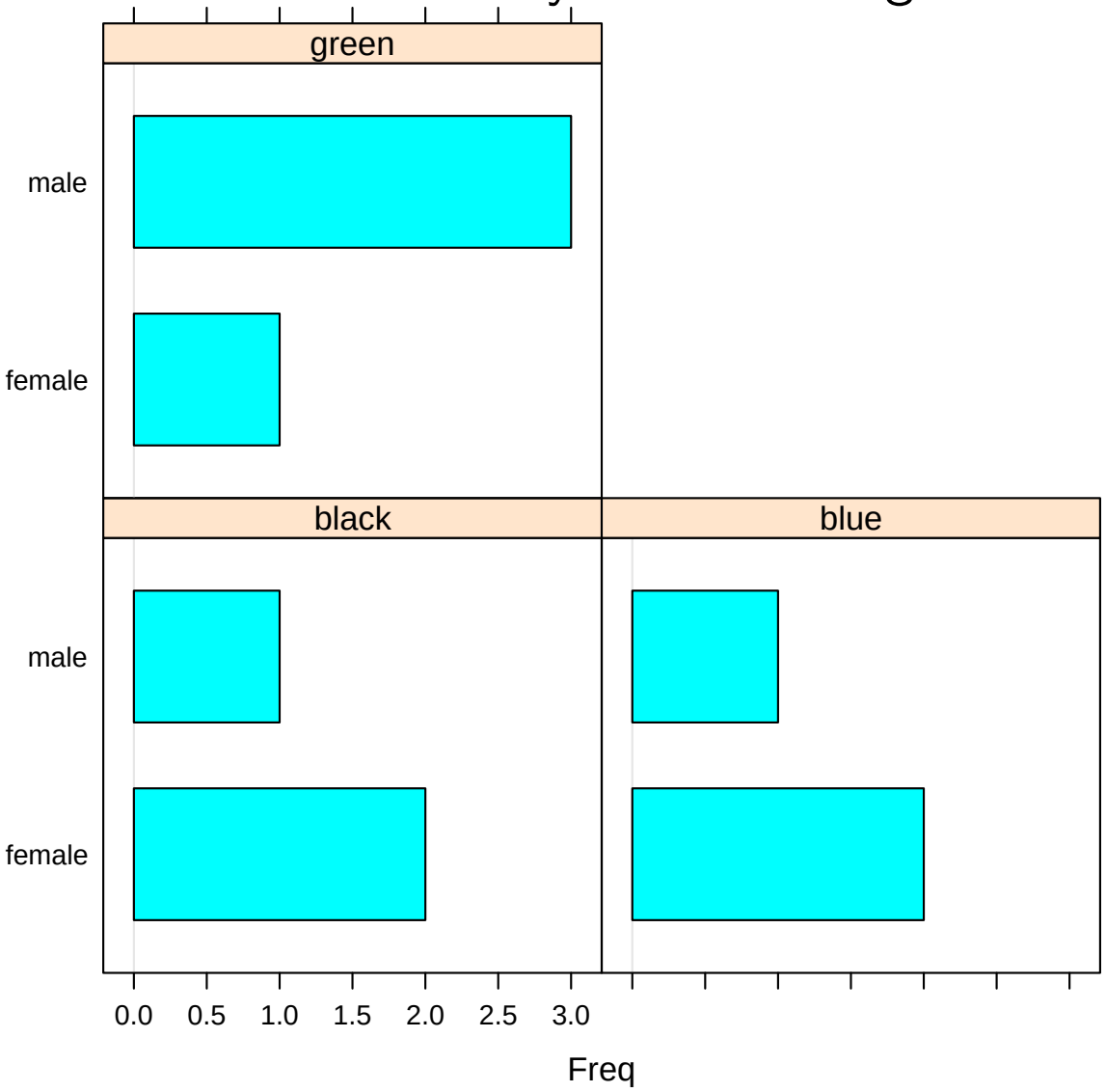
Categorical data - barchart

- Load the library lattice, then create `barchart` grouping the data by `sex`

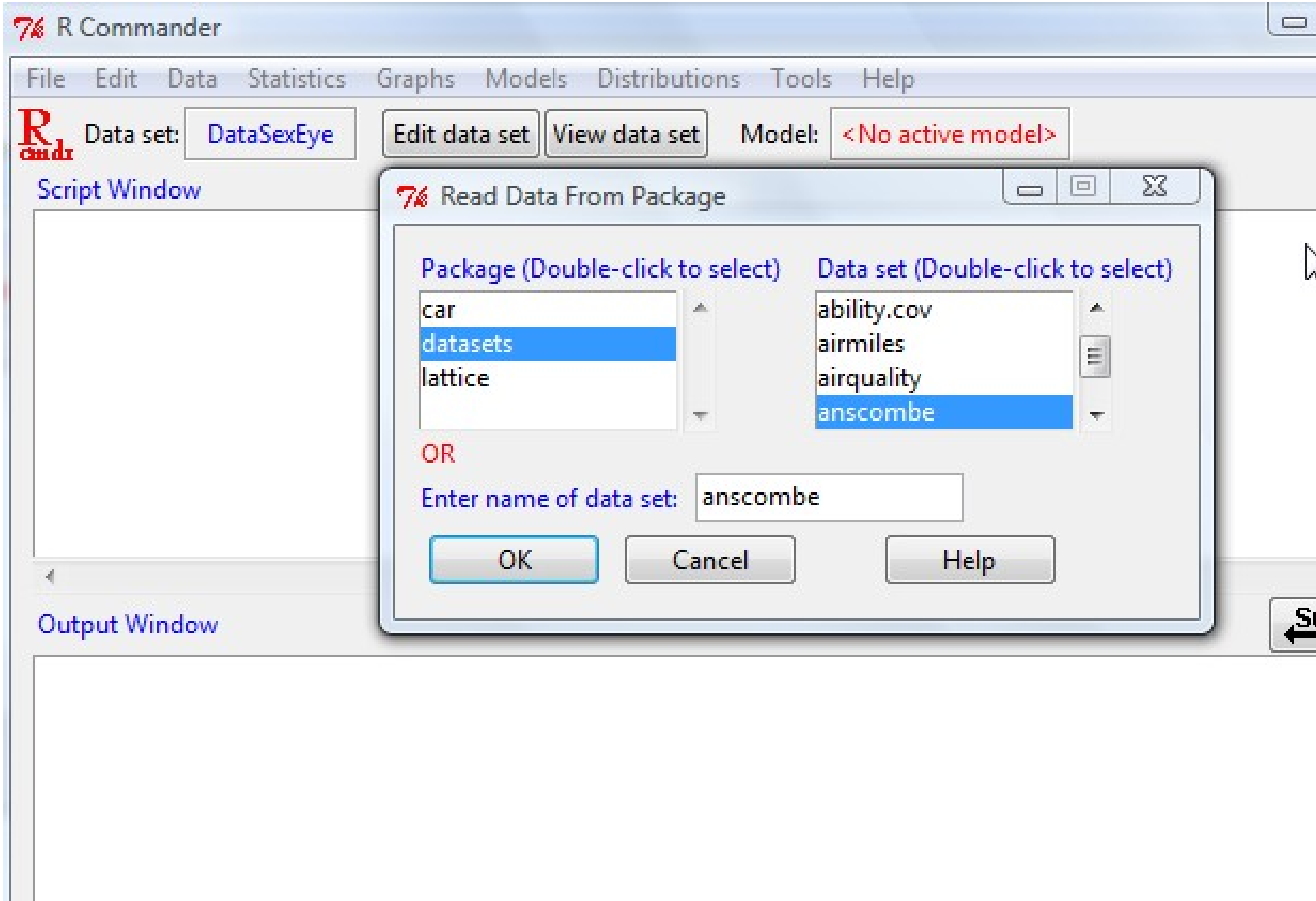
```
library(lattice)

barchart(DataSexEye, groups=DataSexEye$sex)
```

Categorical data - barchart cont.

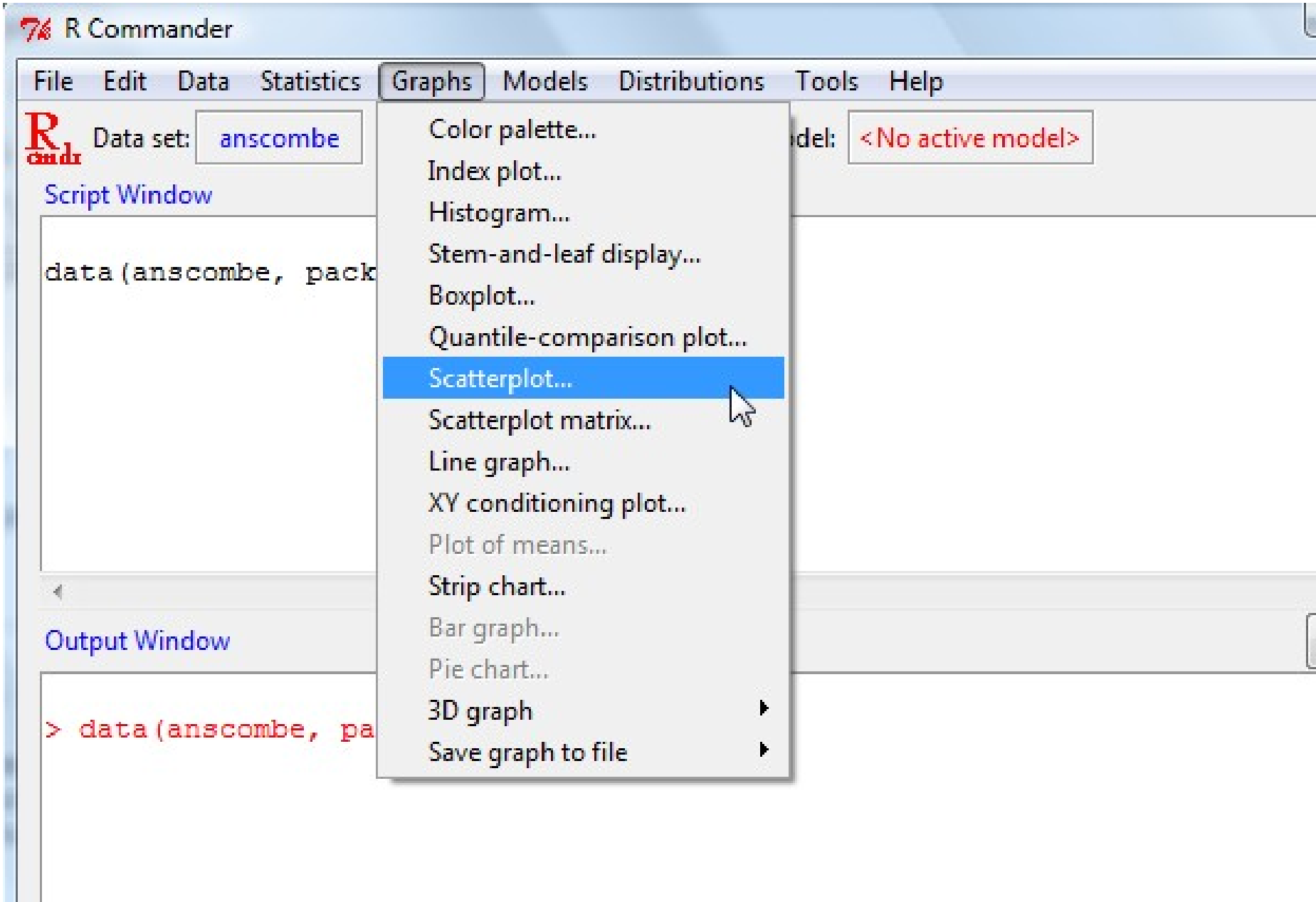
- Are there more females or males with blue eyes? (females)
- What is the most common eye color among males? (green)



Numerical data - load anscombe data set from R library

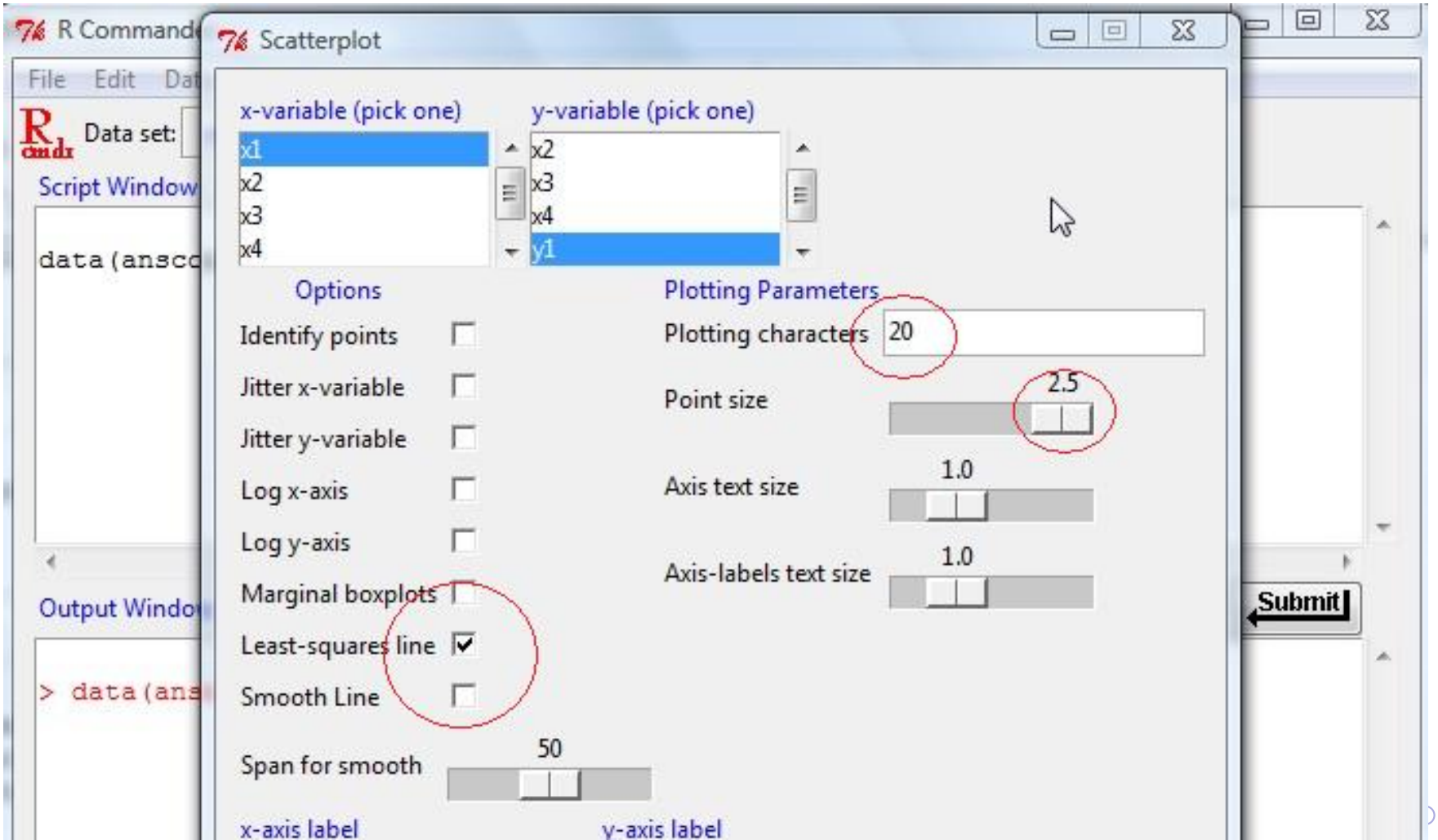


Numerical data - scatterplot of $y1$ versus $x1$

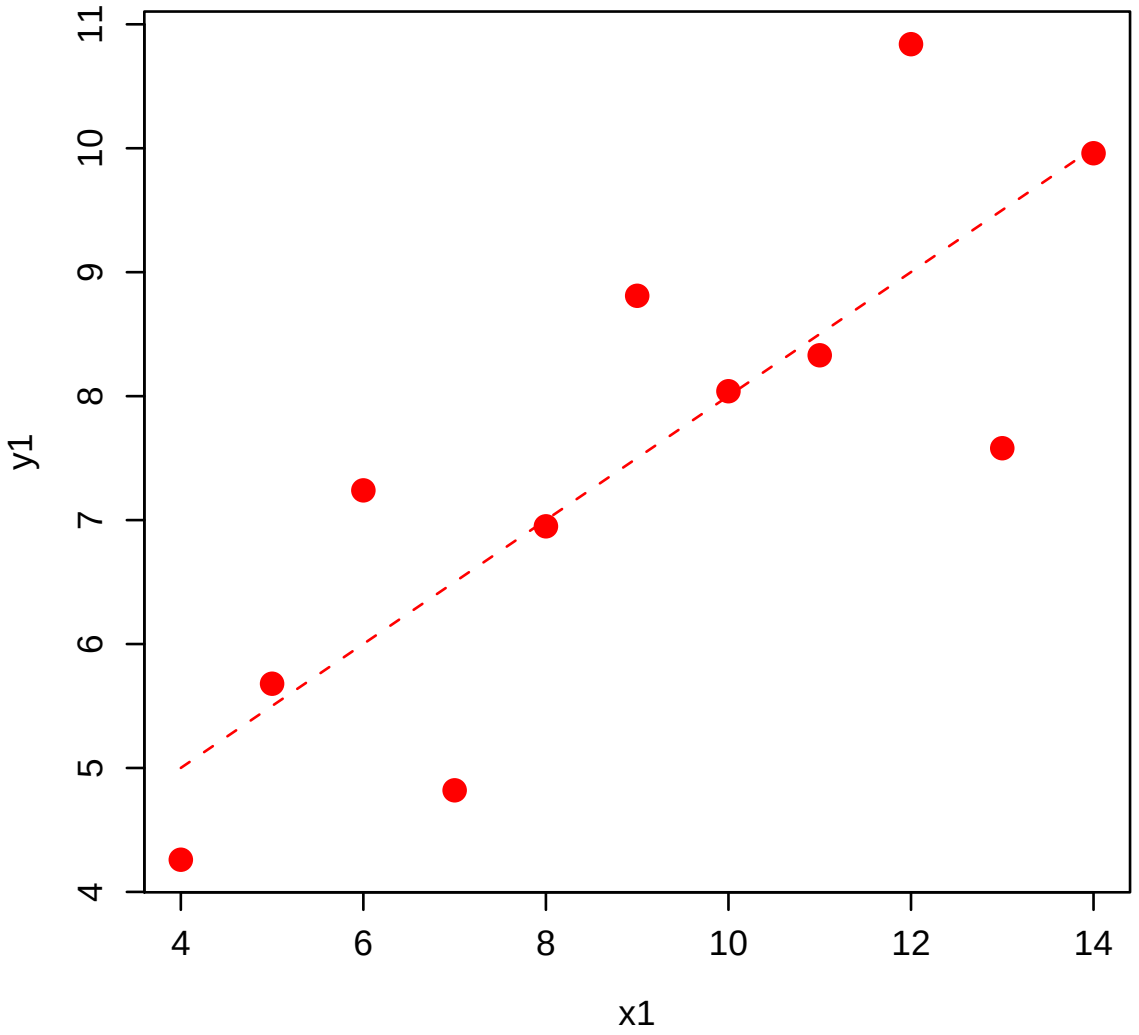


Numerical data - scatterplot of $y1$ versus $x1$ cont.

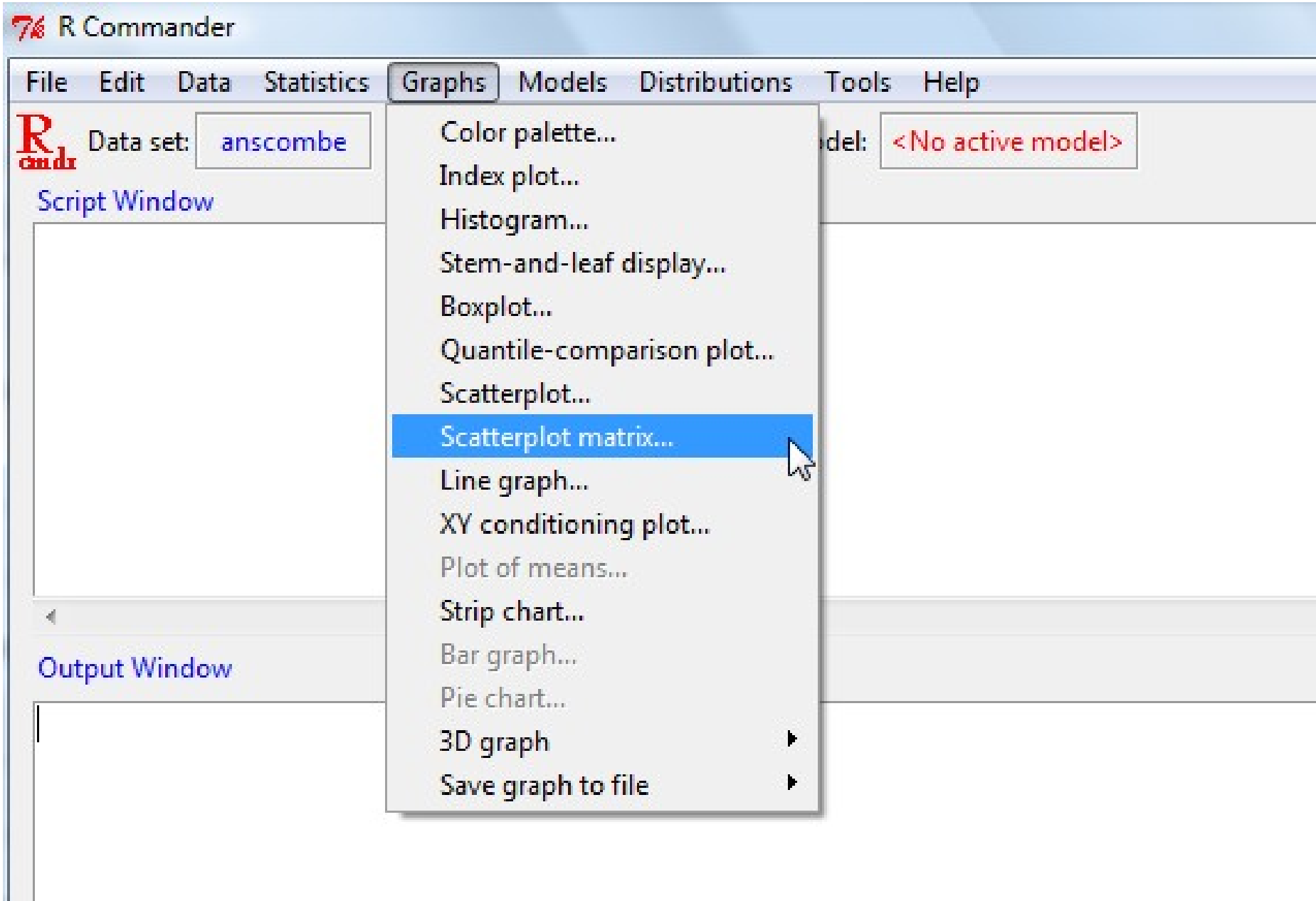
- Uncheck all but the Least-squares line
- Plotting characters 20 corresponds to bullets
- Increase the Point size to 2.5



Numerical data - scatterplot of $y1$ versus $x1$ cont.

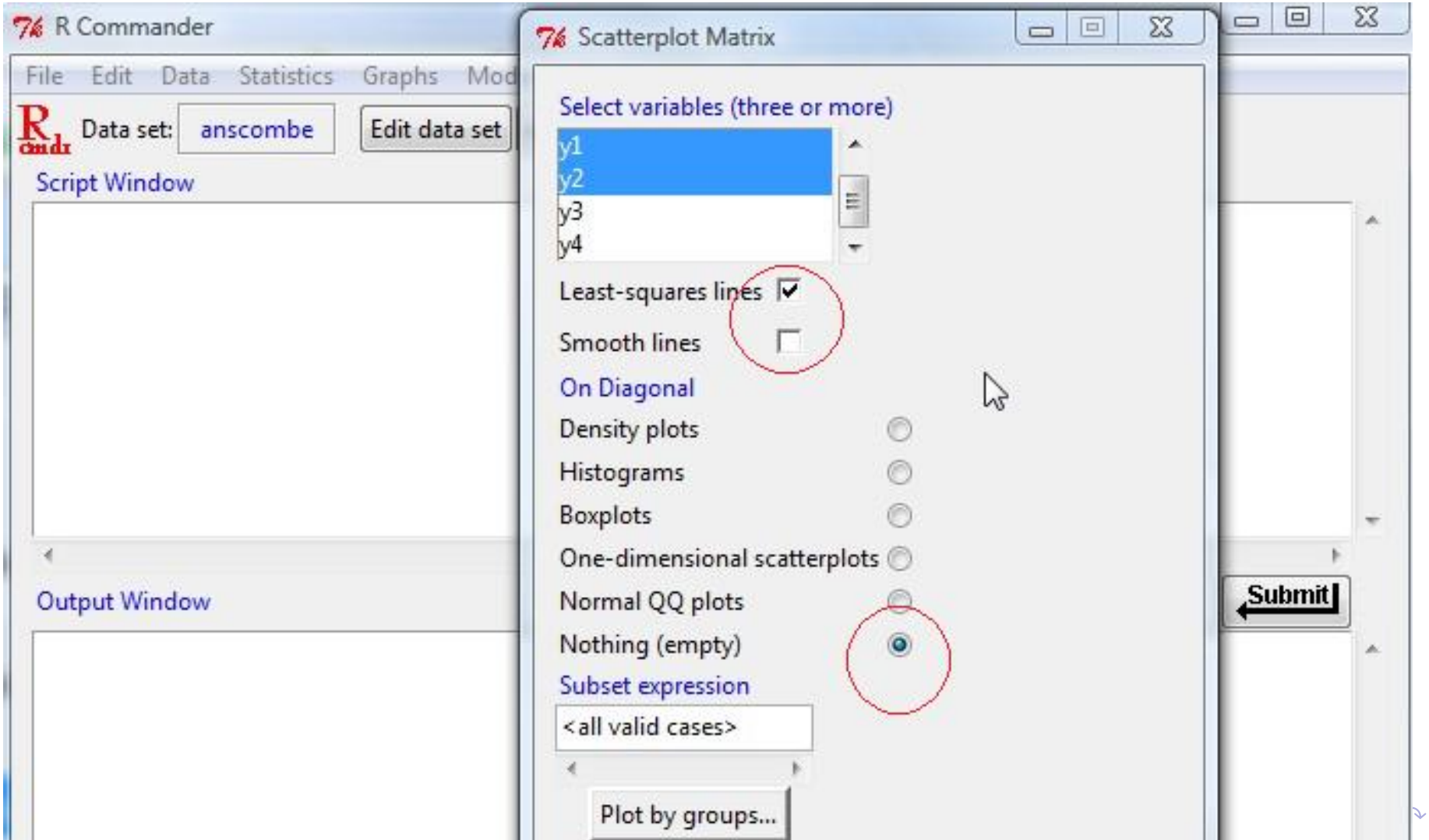


Numerical data - scatterplot matrix (only x_1, x_2, y_1, y_2)

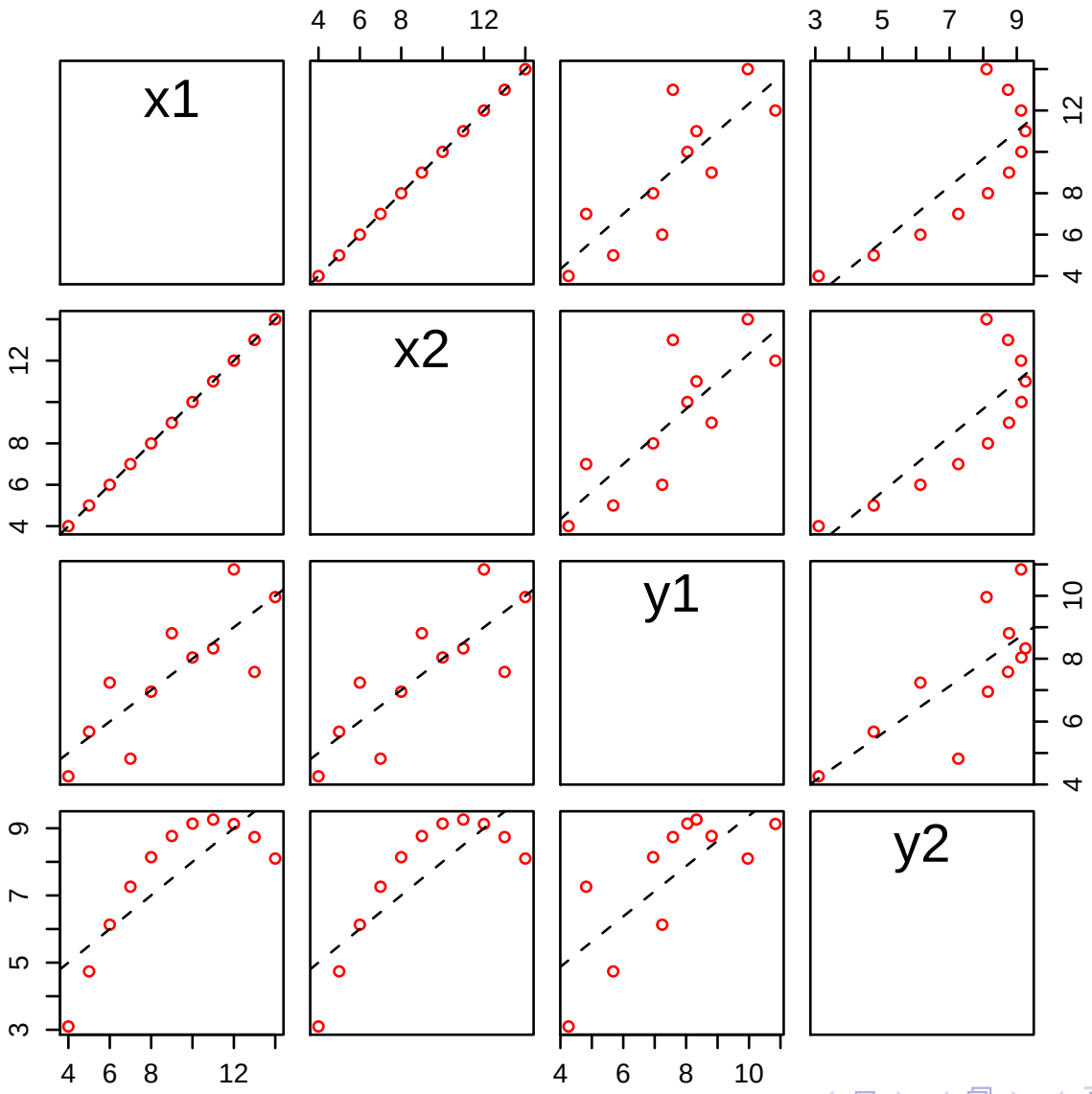


Numerical data - scatterplot matrix (only x_1, x_2, y_1, y_2) cont.

- Check Least-squares line



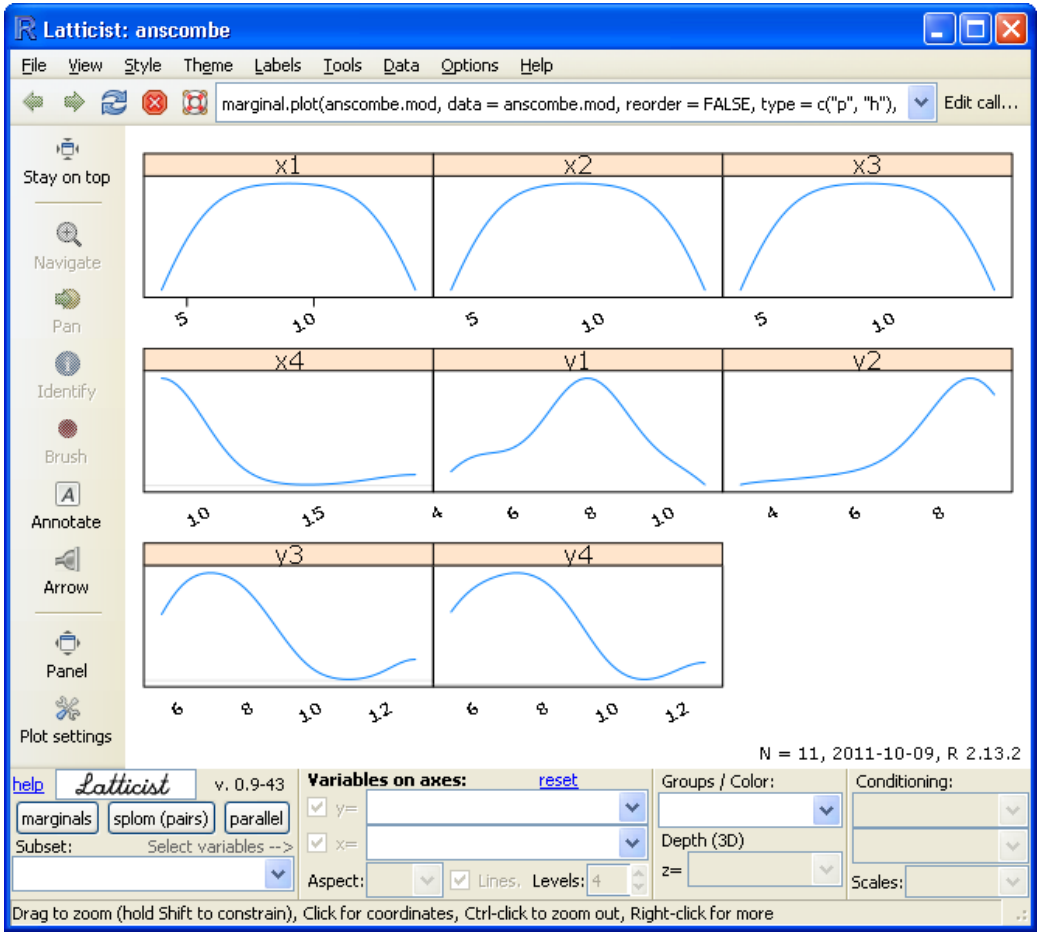
Numerical data - scatterplot matrix (only x_1, x_2, y_1, y_2) cont.



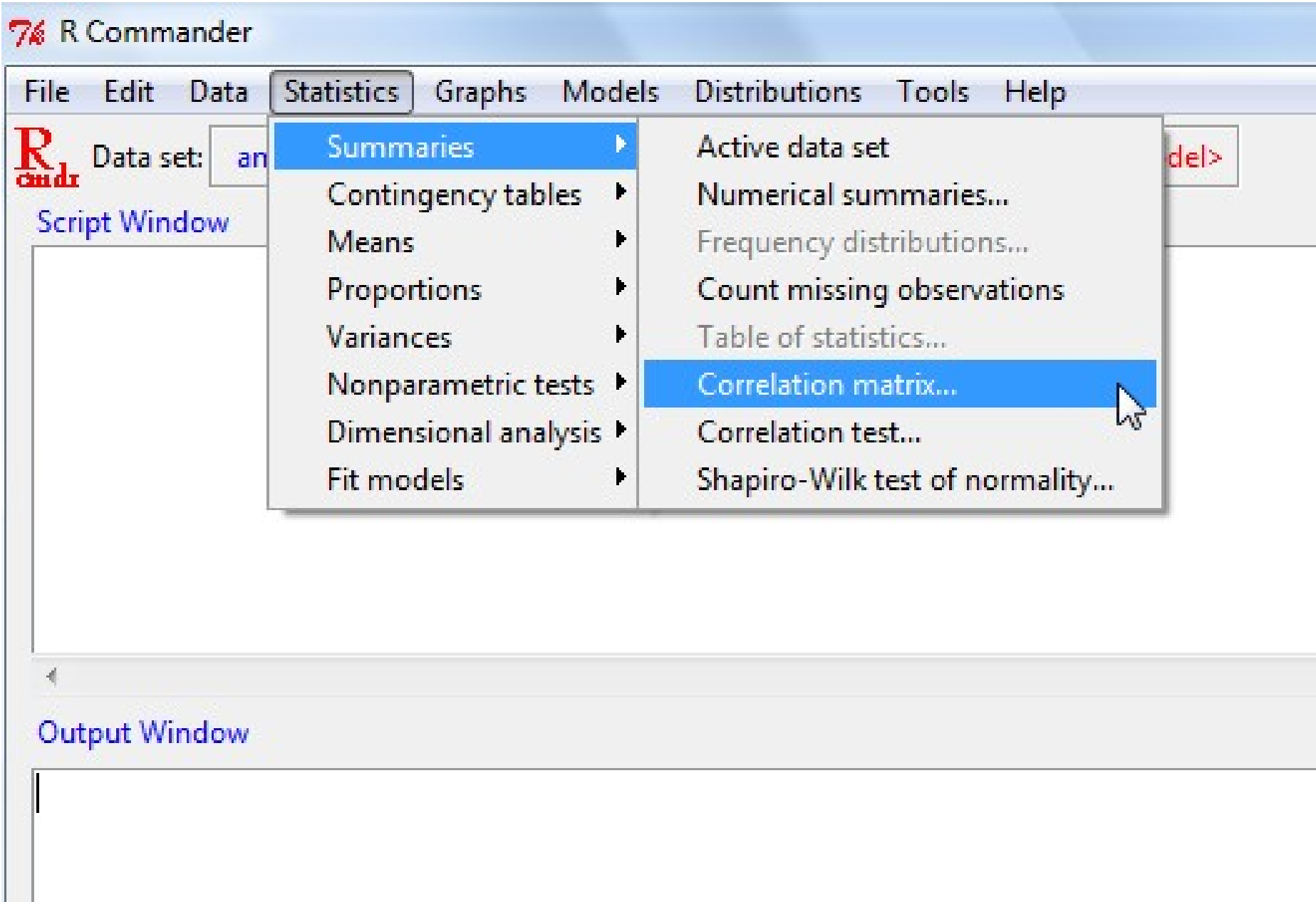
latticist environment

You can create interactive graphics:

```
data(anscombe, package="datasets")
library(latticist)
latticist(anscombe)
```

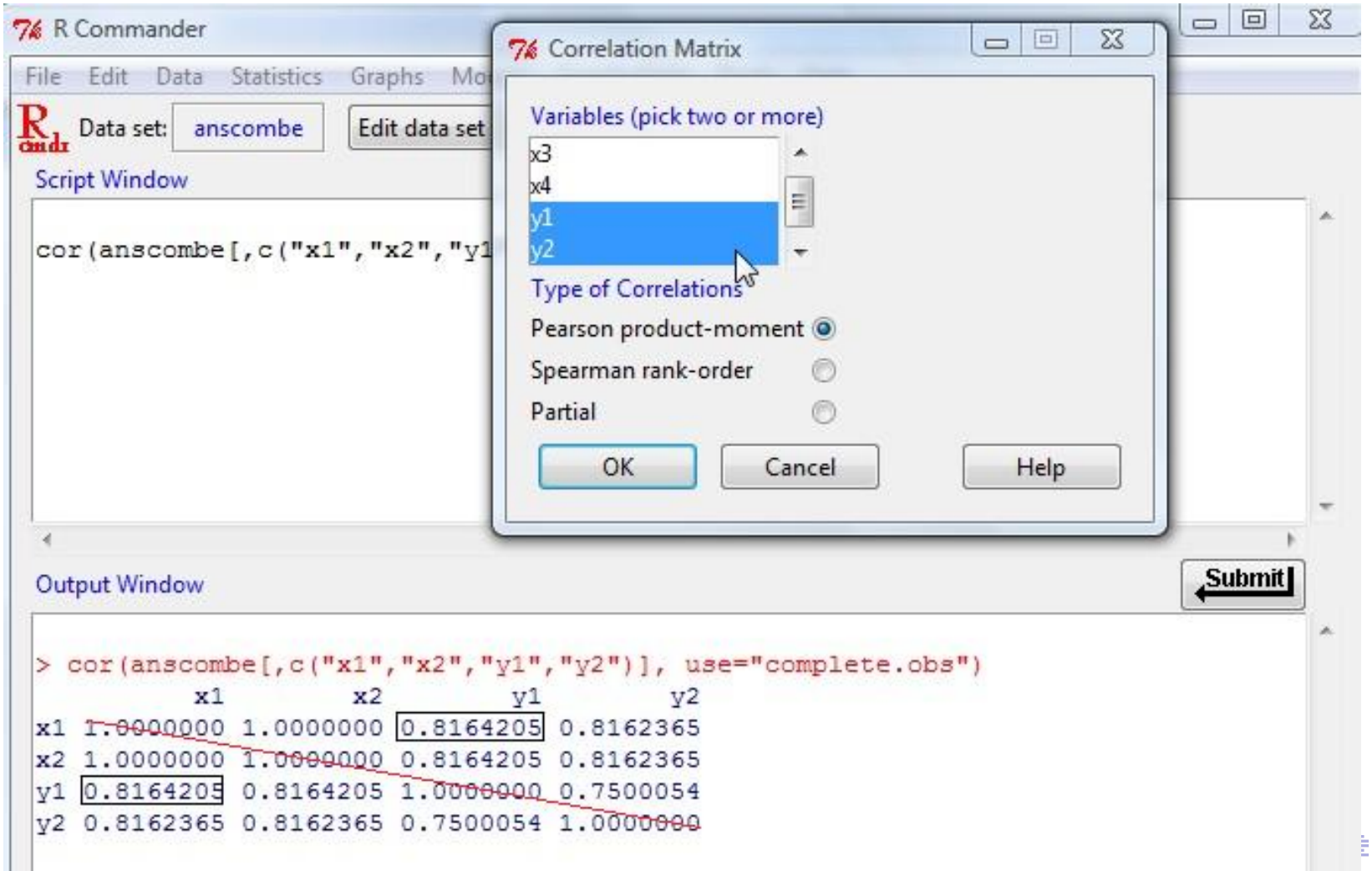


Numerical data - correlation matrix



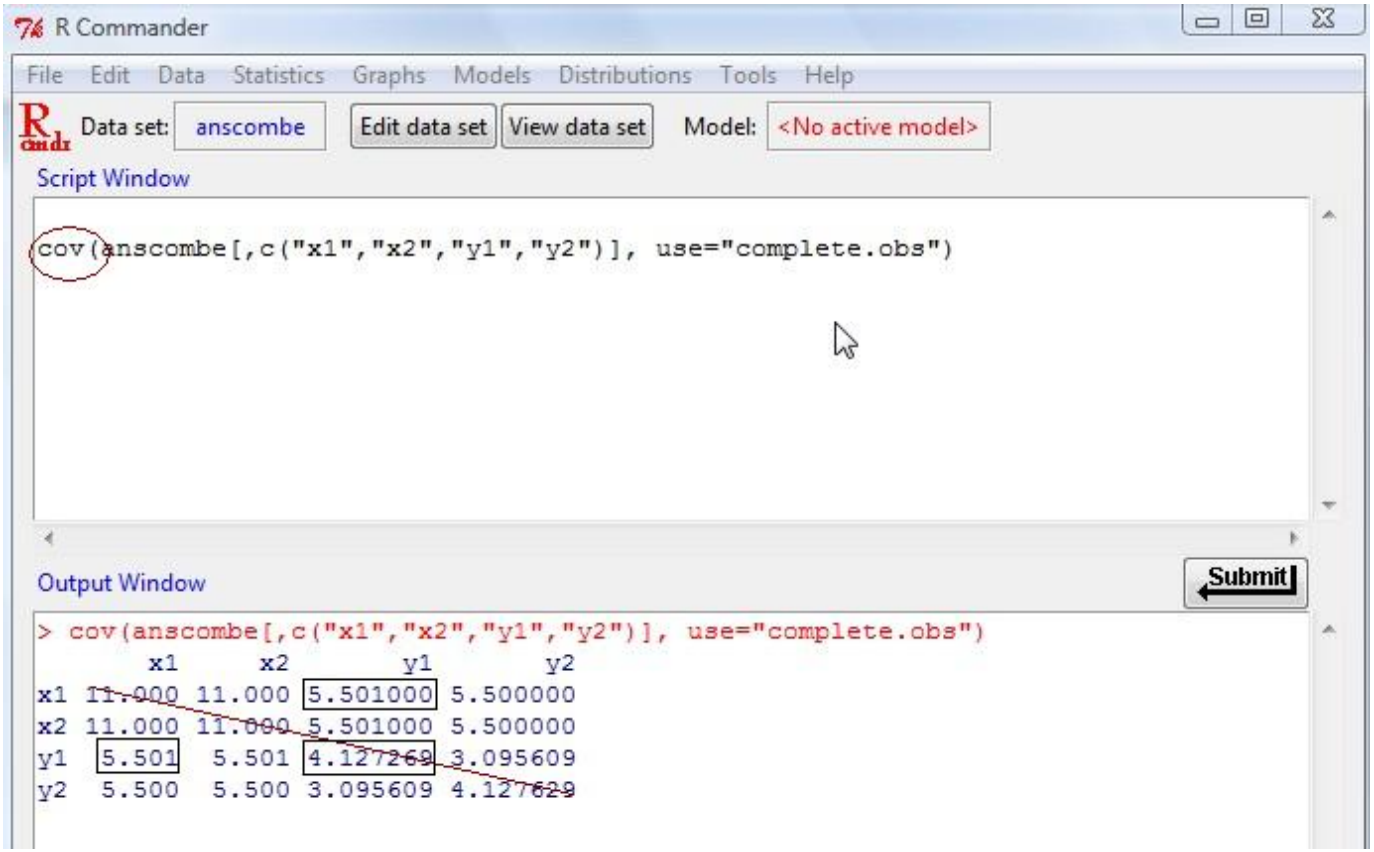
Numerical data - correlation matrix (only x_1, x_2, y_1, y_2) cont.

- Matrix is symmetrical with values on the diagonal $= 1$
- $cor(x_1, y_1) = cor(y_1, x_1) = 0.8164205$

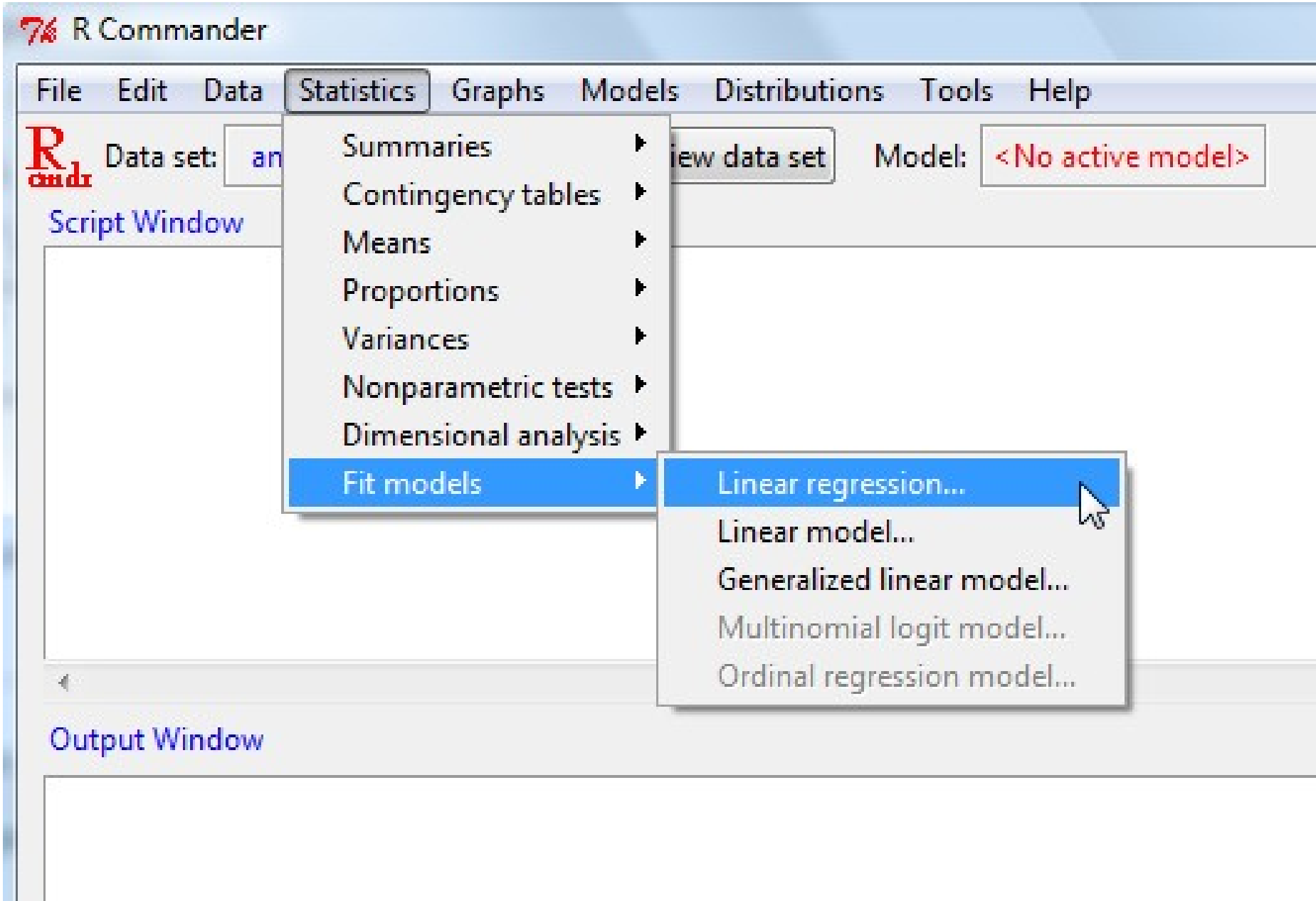


Numerical data - covariance matrix (only x_1, x_2, y_1, y_2)

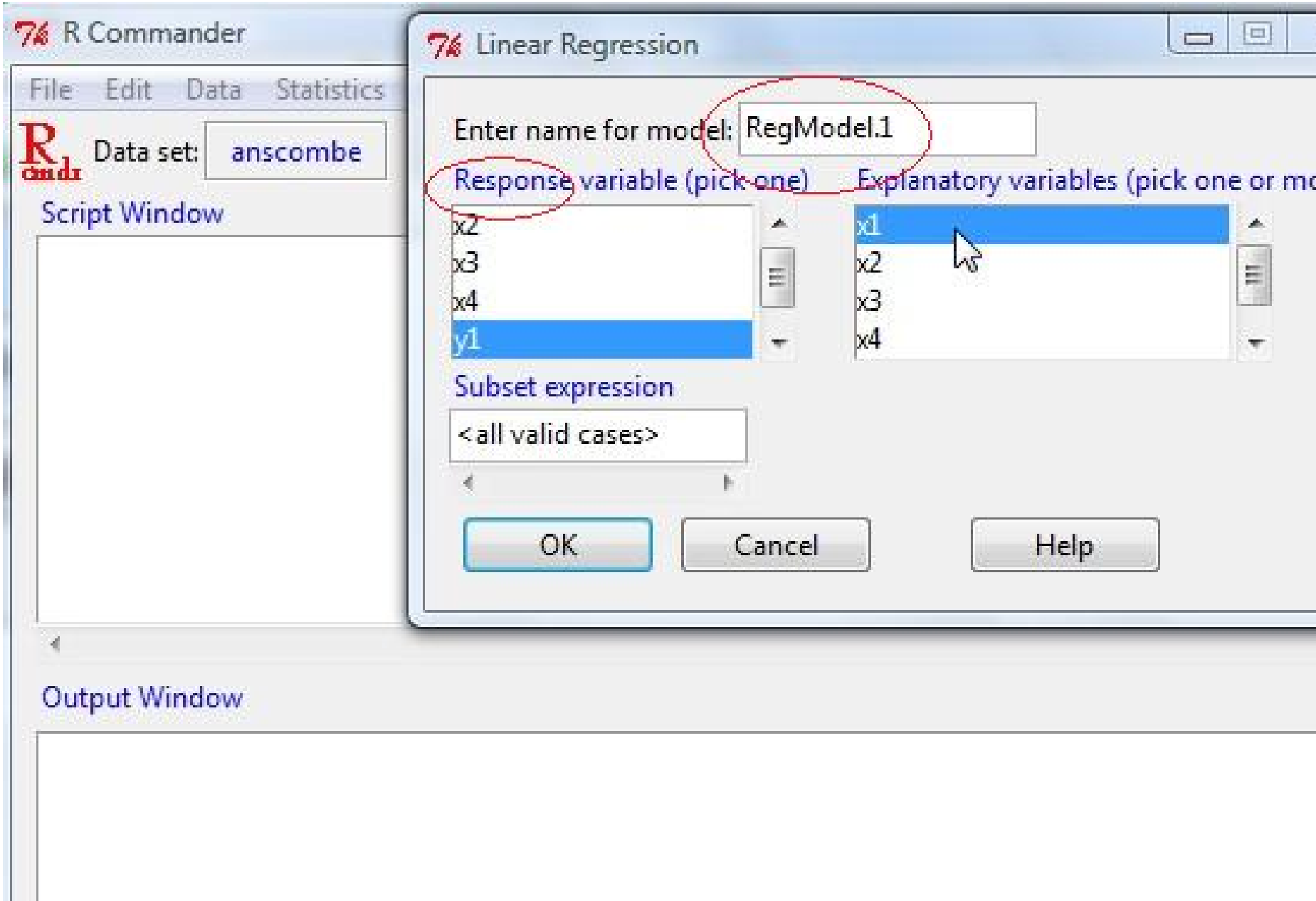
- Replace cor by cov in the last command in the Script Window
- $cov(x_1, y_1) = 5.501$
- Matrix is symmetrical with values on the diagonal = variances, eg, $cov(y_1, y_1) = var(y_1) = 4.127269$



Simple linear regression - y1 versus x1

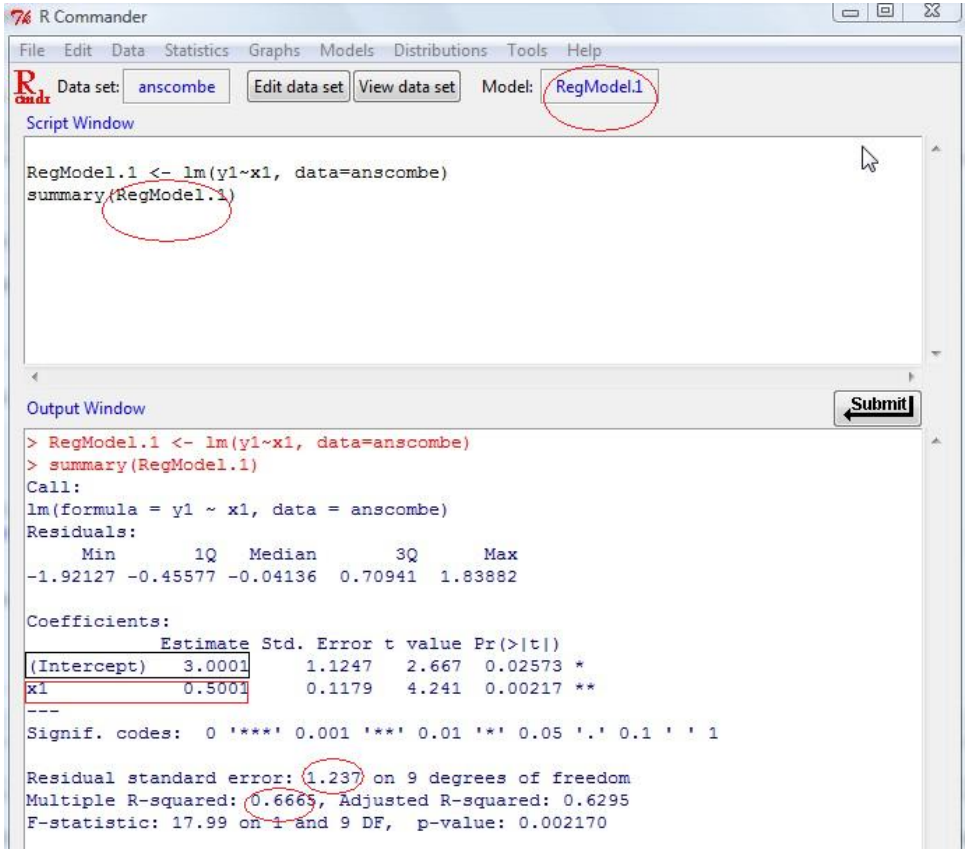


Simple linear regression - y1 versus x1 cont.



Simple linear regression - y1 versus x1 cont.

- Intercept estimate: $a = 3.0001$
- Slope estimate: $b = 0.5001$
- Residual standard deviation: $s_R = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \widehat{y}_i)^2}{n-2}} = 1.237$
- R-squared: $R^2 = 0.6665 \quad \Rightarrow \quad cor(x, y) = \sqrt{0.6665}$



Regression Diagnostics: Tools for Checking the Validity of a Model (I)

- Determine whether the proposed regression model provides an adequate fit to the data: plots of standardized residuals.
- The plots assess visually whether the assumptions are being violated.
- Determine which (if any) of the data points have x values that have an unusually large effect on the estimated regression model (**leverage points**).
- Determine which (if any) of the data points are *outliers*: points which do not follow the pattern set by the bulk of the data.

Regression Diagnostics: Tools for Checking the Validity of a Model (II)

- If leverage points exist, determine whether each is a *bad leverage point*. If a bad leverage point exists we shall assess its influence on the fitted model.
- Examine whether the assumption of constant variance of the errors is reasonable. If not, we shall look at how to overcome this problem.
- If the data are collected over time, examine whether the data are correlated over time.
- If the sample size is small or prediction intervals are of interest, examine whether the assumption that the errors are normally distributed is reasonable.

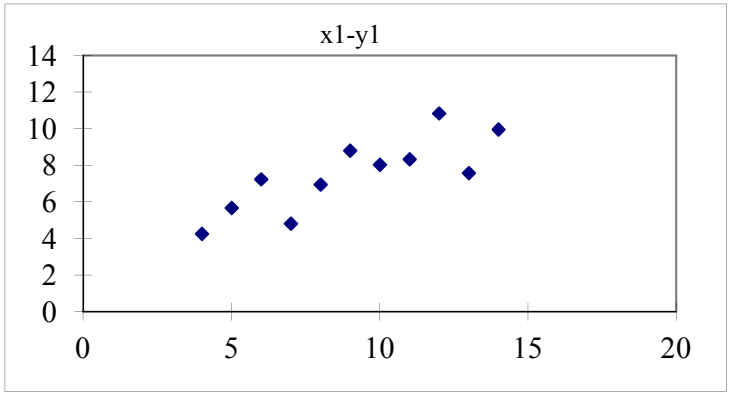
Sources:
Edward R. Tufte, *The Visual Display of Quantitative Information* (Cheshire, Connecticut: Graphics Press, 1983), pp. 14-15.

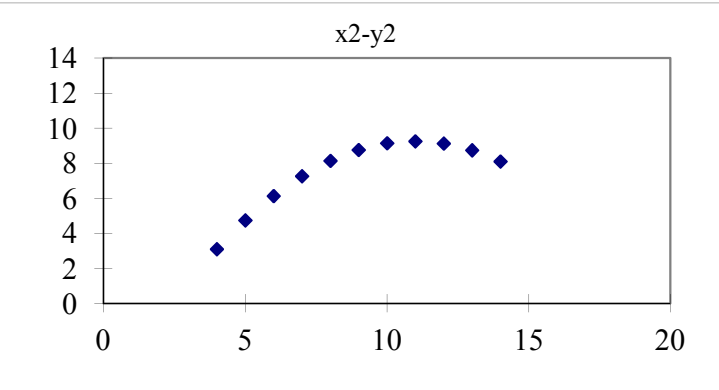
F.J. Anscombe, "Graphs in Statistical Analysis," *American Statistician*, vol. 27 (Feb 1973), pp. 17-21.

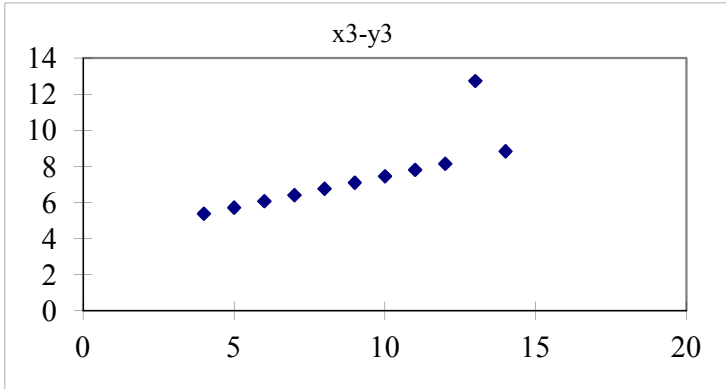
Anscombe's Data											
Observation	x1	y1		x2	y2		x3	y3		x4	y4
1	10	8,04		10	9,14		10	7,46		8	6,58
2	8	6,95		8	8,14		8	6,77		8	5,76
3	13	7,58		13	8,74		13	12,74		8	7,71
4	9	8,81		9	8,77		9	7,11		8	8,84
5	11	8,33		11	9,26		11	7,81		8	8,47
6	14	9,96		14	8,1		14	8,84		8	7,04
7	6	7,24		6	6,13		6	6,08		8	5,25
8	4	4,26		4	3,1		4	5,39		19	12,5
9	12	10,84		12	9,13		12	8,15		8	5,56
10	7	4,82		7	7,26		7	6,42		8	7,91
11	5	5,68		5	4,74		5	5,73		8	6,89
				Summary Statistics							
N	11	11		11	11		11	11		11	11
mean	9,00	7,50		9,00	7,50091		9,00	7,50		9,00	7,50
SD	3,16	1,94		3,16	1,94		3,16	1,94		3,16	1,94
r	0,82			0,82			0,82			0,82	

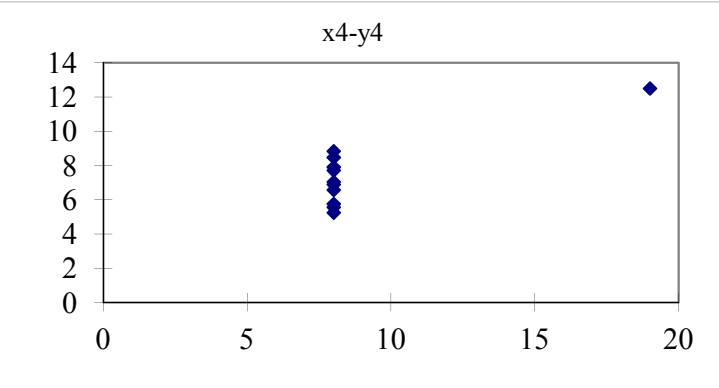
Use the charts below to get the regression lines via Excel's Trendline feature.

Data


x1-y1
14
12
10
8
6
4
2
0
0
5
10
15
20


x2-y2
14
12
10
8
6
4
2
0
0
5
10
15
20

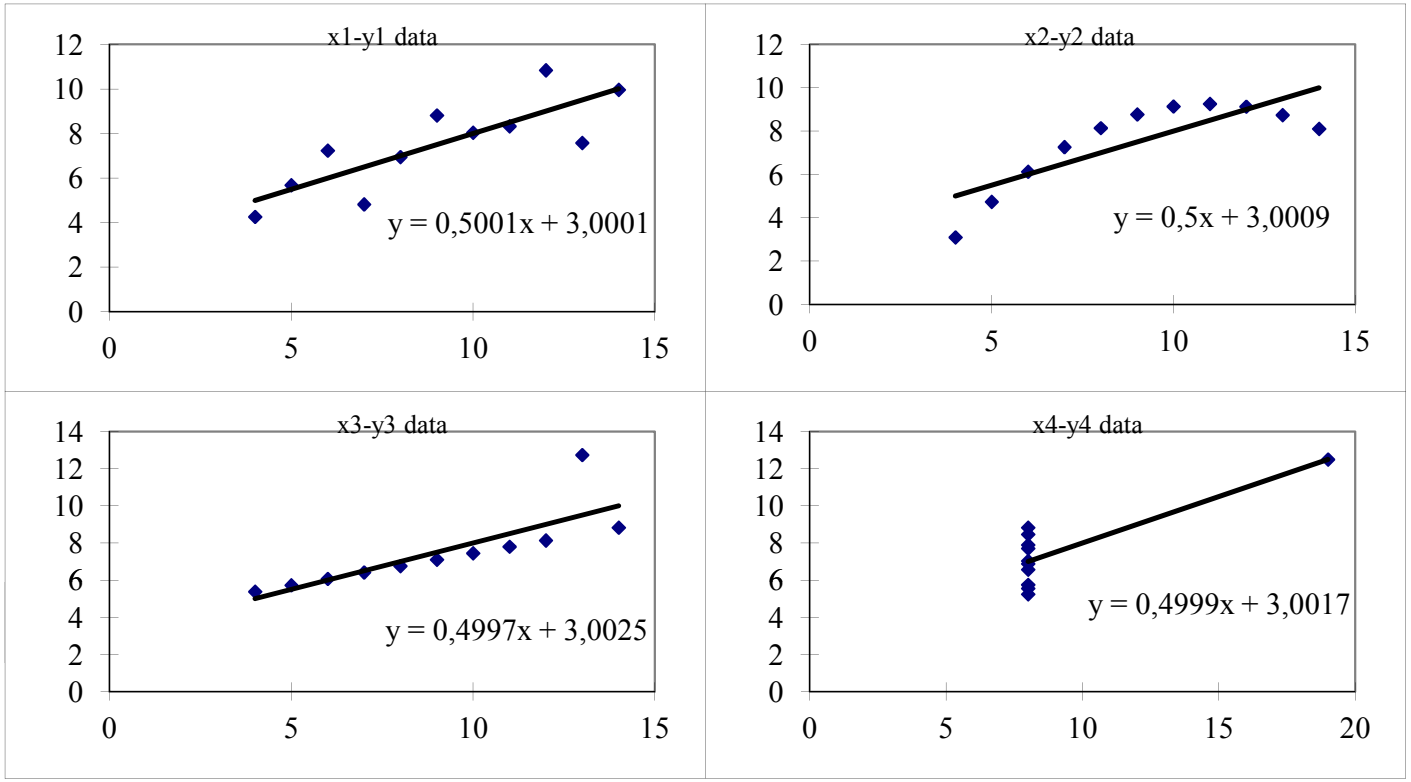

x3-y3
14
12
10
8
6
4
2
0
0
5
10
15
20


x4-y4
14
12
10
8
6
4
2
0
0
5
10
15
20

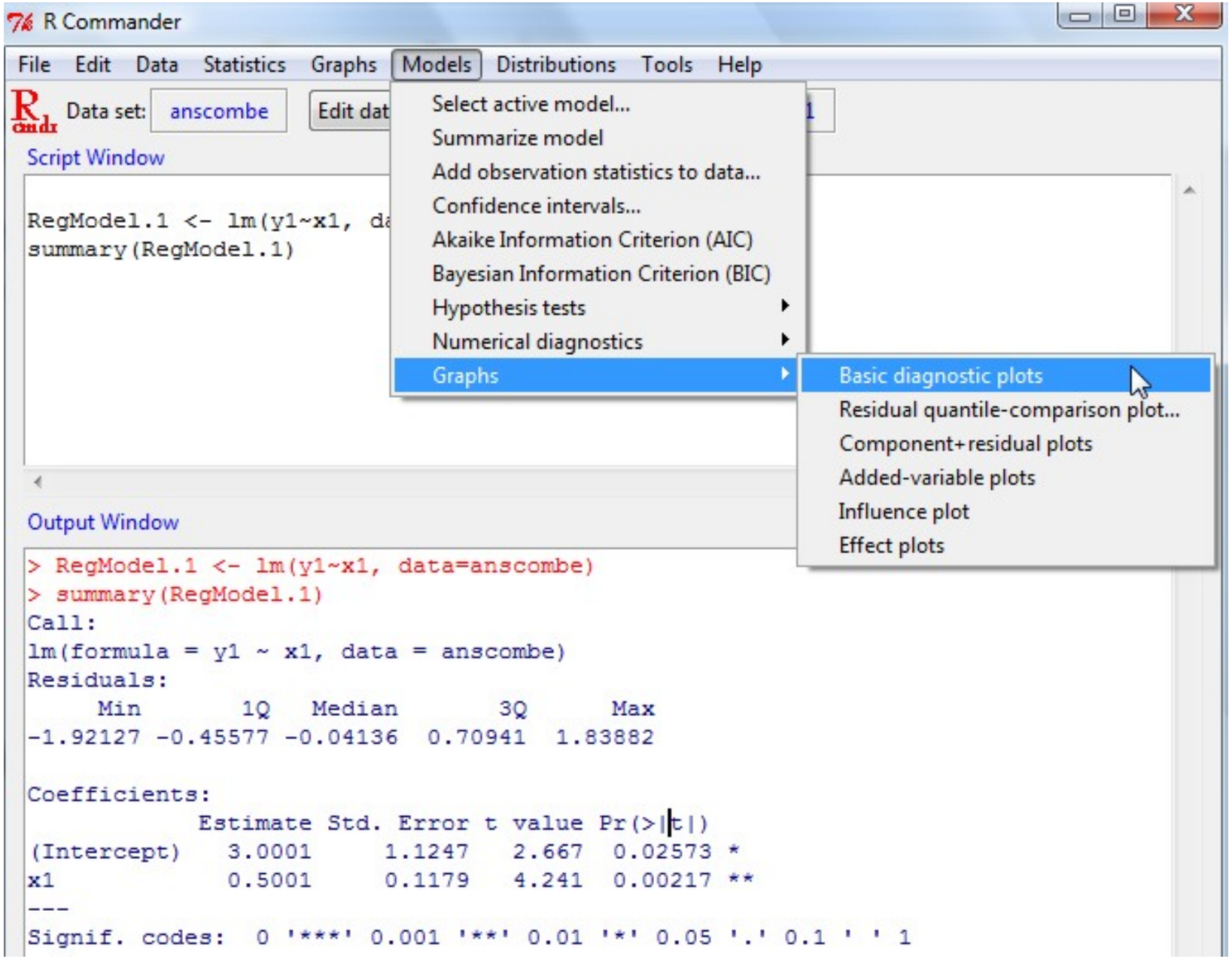
Regression Results

LINEST OUTPUT		x1-y1			x2-y2			x3-y3			x4-y4	
slope	intercept	0,50	3		0,50	3		0,50	3		0,50	3
SE	SE	0,12	1,12		0,12	1,13		0,12	1,12		0,12	1,12
R^2	RMSE	0,67	1,24		0,67	1,24		0,67	1,24		0,67	1,24
F	df	17,99	9		17,97	9		17,97	9		18,00	9
Reg SS	SSR	27,51	13,76		27,50	13,78		27,47	13,76		27,49	13,74

Data

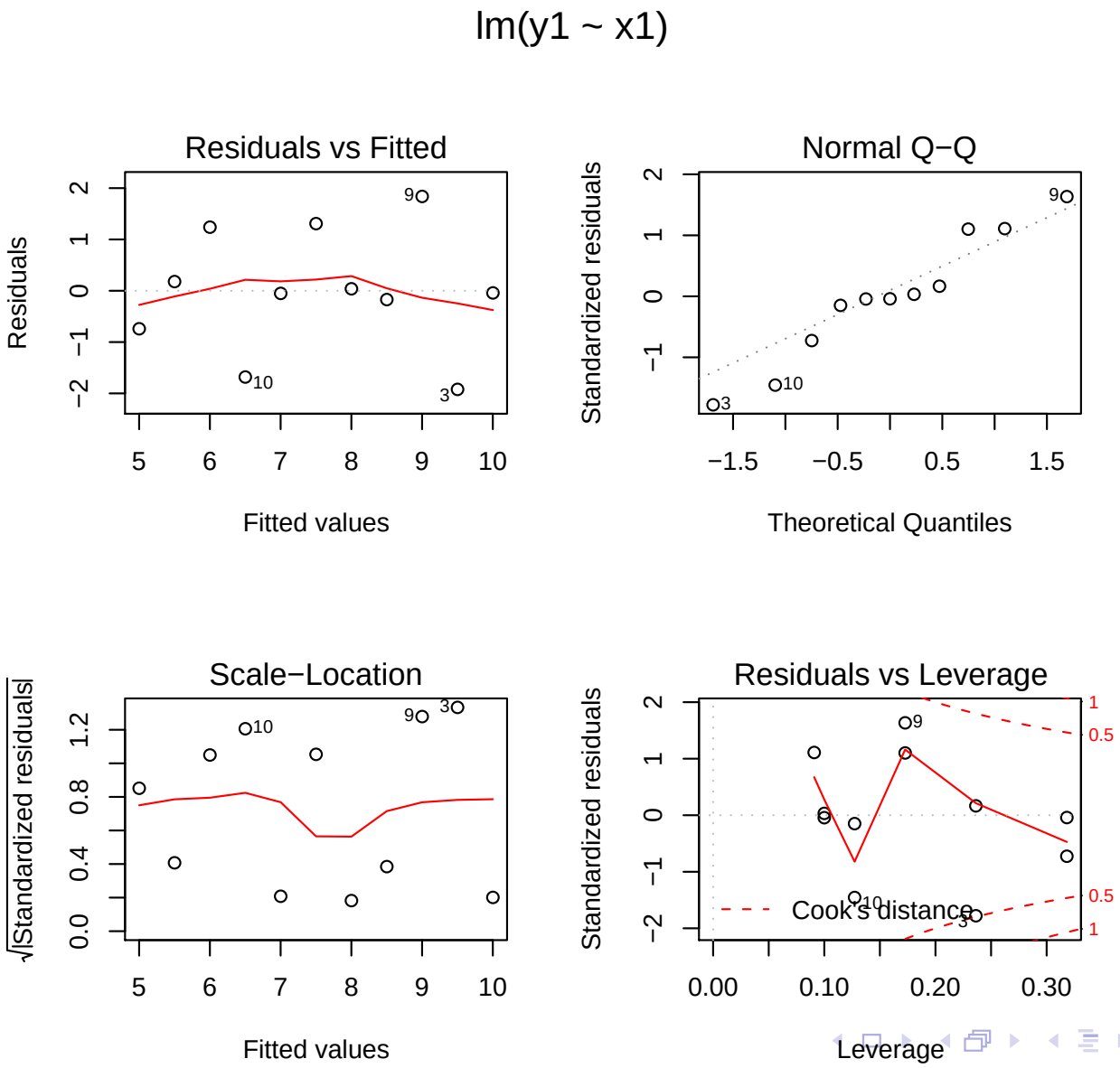

x1-y1 data
y = 0,5001x + 3,0001
x2-y2 data
y = 0,5x + 3,0009
x3-y3 data
y = 0,4997x + 3,0025
x4-y4 data
y = 0,4999x + 3,0017

Simple linear regression - residual plot (method 1)



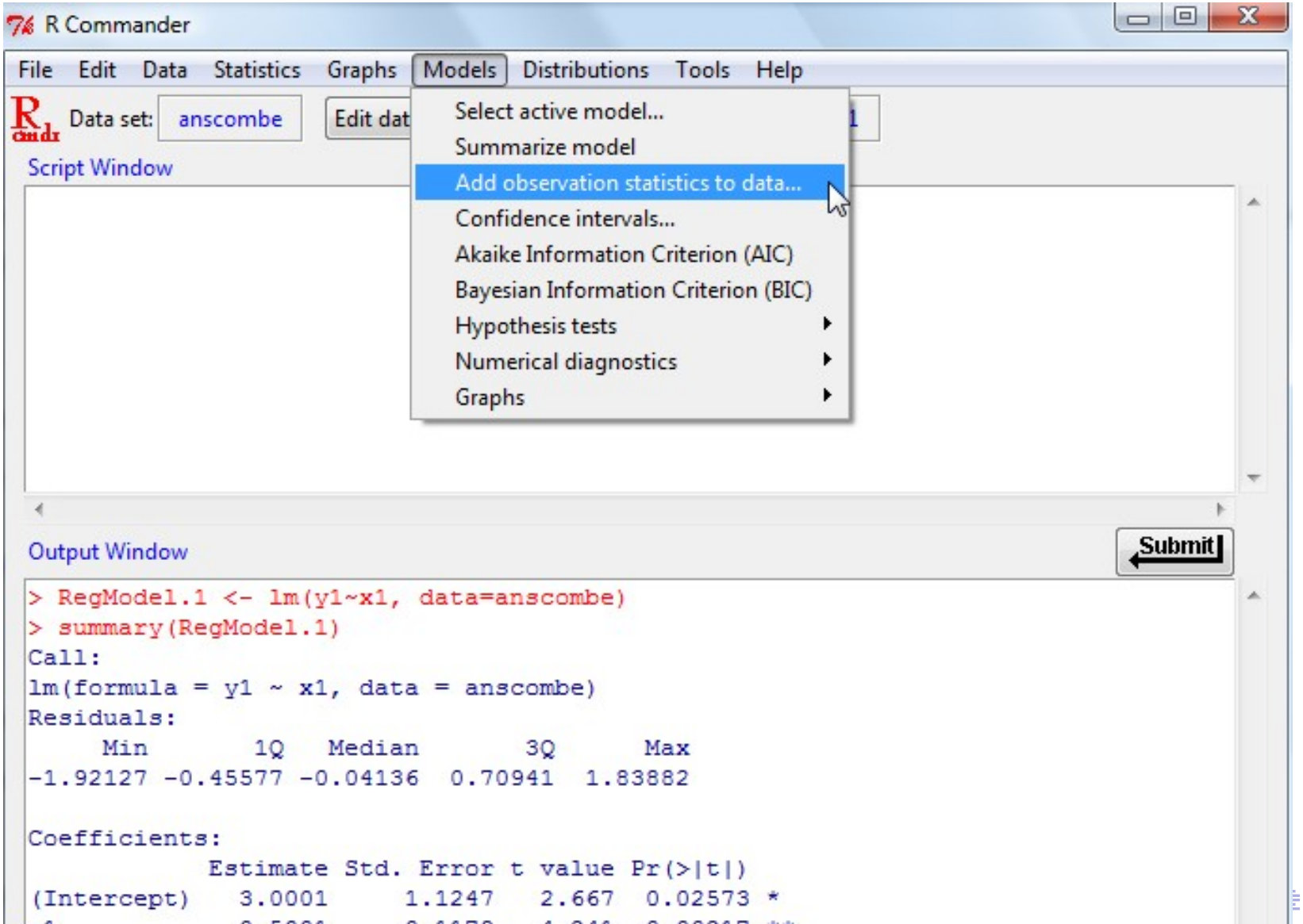
Simple linear regression - residual plot (method 1) cont.

- Residuals versus fitted (top left plot)



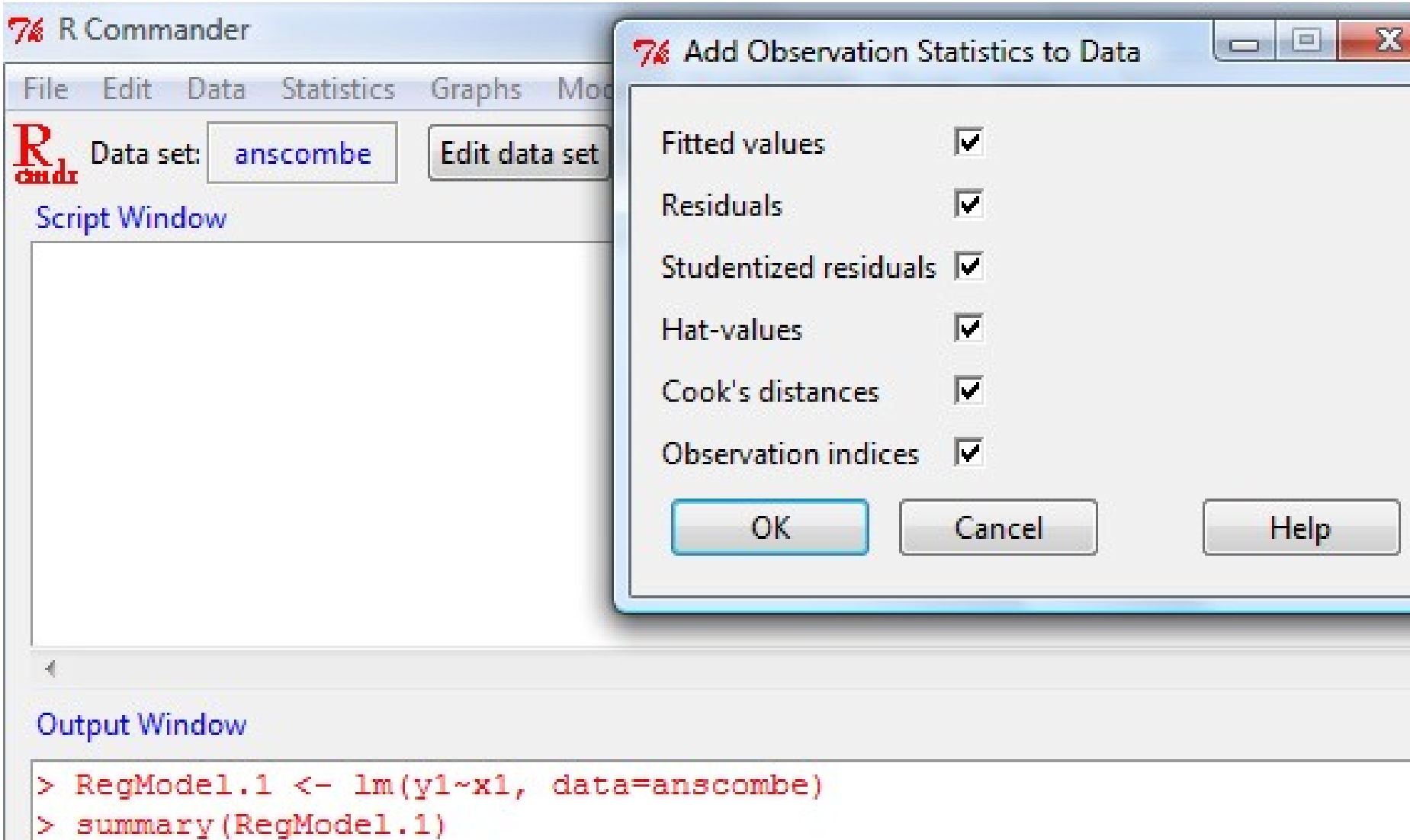
Simple linear regression - residual plot (method 2)

- Append the fitted values, residuals, standardized residuals etc to the existing data set



Simple linear regression - residual plot (method 2 cont.)

- Append the fitted values, residuals, studentized residuals etc to the existing data set



Simple linear regression - residual plot (method 2 cont.)

- Now the data set has new columns on the right with $\hat{y}$, r, etc

R Commander

File Edit Data Statistics Graphs Models Distributions Tools Help

Data set: anscombe | Edit data set | View data set | Model: RegModel.1

Script Window

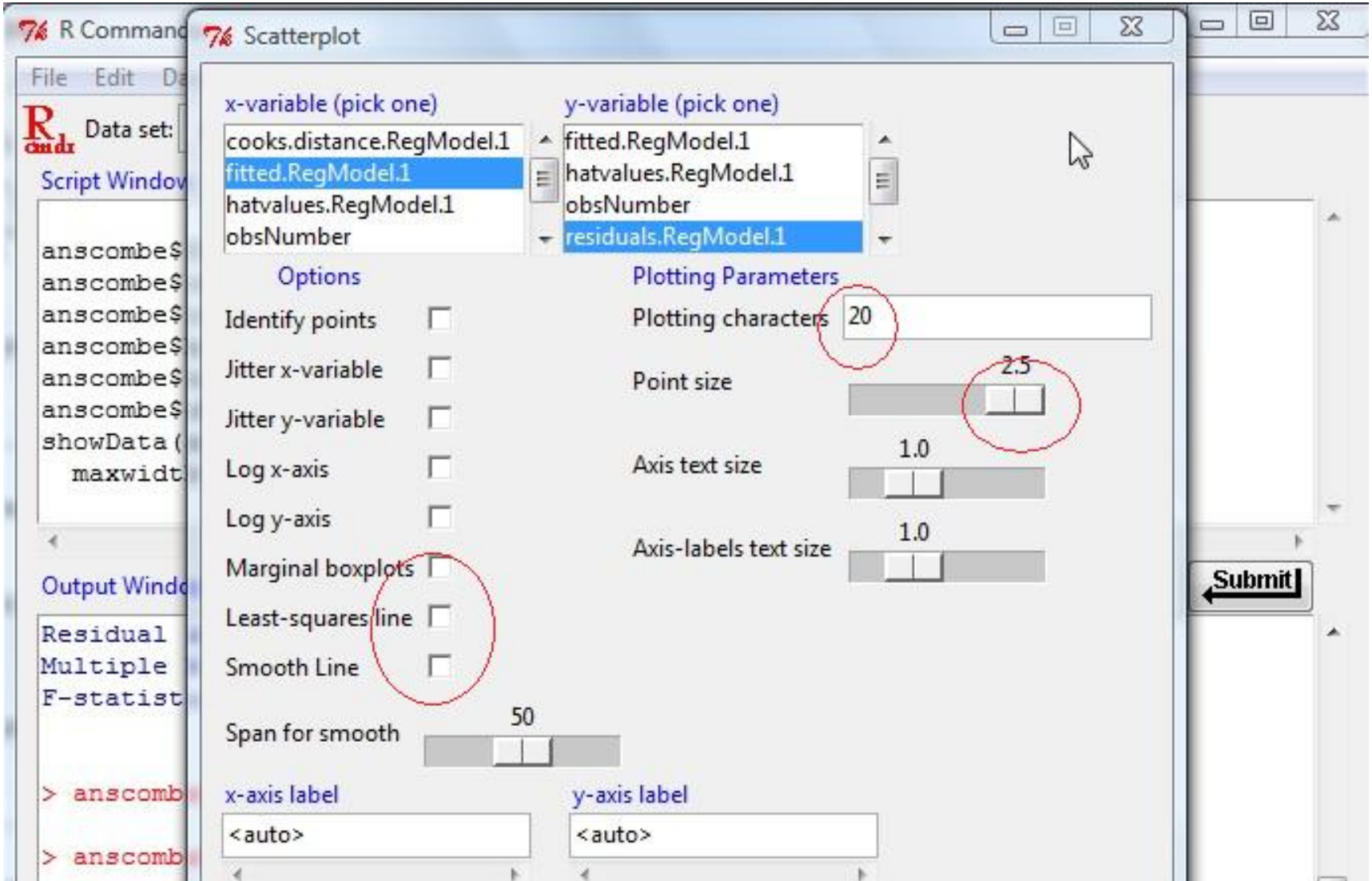
```
anscombe$fitted.RegModel.1 <- fitted(RegModel.1)
anscombe$residuals.RegModel.1 <- residuals(RegModel.1)
anscombe$rstudent.RegModel.1 <- rstudent(RegModel.1)
anscombe$hatvalues.RegModel.1 <- hatvalues(RegModel.1)
anscombe$cooks.distance.RegModel.1 <- cooks.distance(RegModel.1)
anscombe$obsNumber <- 1:nrow(anscombe)
showData(anscombe, placement='-20+200', font=getRcmdr('logFont'),
  maxwidth=80, maxheight=30)
```

anscombe

	x1	x2	x3	x4	y1	y2	y3	y4	fitted.RegModel.1	residuals.RegModel.1	rstude
1	10	10	10	8	8.04	9.14	7.46	6.58	8.001000	0.03900000	
2	8	8	8	8	6.95	8.14	6.77	5.76	7.000818	-0.05081818	
3	13	13	13	8	7.58	8.74	12.74	7.71	9.501273	-1.92127273	
4	9	9	9	8	8.81	8.77	7.11	8.84	7.500909	1.30909091	
5	11	11	11	8	8.33	9.26	7.81	8.47	8.501091	-0.17109091	
6	14	14	14	8	9.96	8.10	8.84	7.04	10.001364	-0.04136364	
7	6	6	6	8	7.24	6.13	6.08	5.25	6.000636	1.23936364	
8	4	4	4	19	4.26	3.10	5.39	12.50	5.000455	-0.74045455	
9	12	12	12	8	10.84	9.13	8.15	5.56	9.001182	1.83881818	
10	7	7	7	8	4.82	7.26	6.42	7.91	6.500727	-1.68072727	

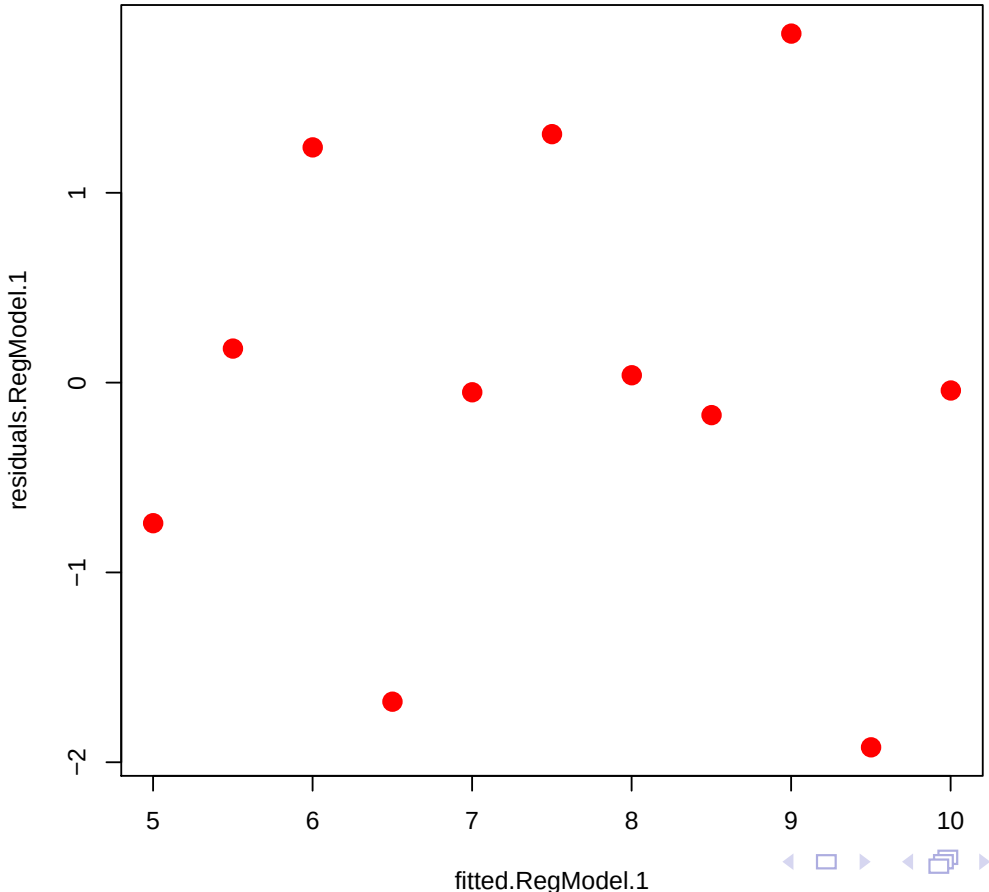
Simple linear regression - residual plot (method 2 cont.)

- Use the scatterplot option in the Graphs menu to plot residuals versus fitted



Simple linear regression - residual plot (method 2 cont.)

- Residuals versus fitted (cloud of points oscillates around the horizontal axis $y = 0$)
- There is no pattern, no heteroscedasticity $\Rightarrow$ regression model is appropriate



Simple linear regression - residual plot (method 2 cont.)

- Studentized Residuals ($\frac{r_i}{s_R}$) versus x_1 (cloud of points oscillates around the horizontal axis $y = 0$)
- There is no pattern, no heteroscedasticity $\Rightarrow$ regression model is appropriate