Quality Control

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Outline

1. Introduction, control charts
2. Variables control charts (X-bar chart)
3. Attributes control charts (p and np charts)

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Introduction

- An industrial process is a sequence of transformations adding value and variability to the final product.
- A quality product should satisfy the specifications imposed by the consumers.
- To increase the quality of our production, we should decrease its variability.

Introduction

- We will identify a certain characteristic as the quality of the product and analyse its variability.
- Sources of variability of the final product:
    - Raw materials
    - Production methods
    - Manpower
    - Adjustment of machines
    - Weather conditions
    - Measurement erros

Variability causes

- Assignable causes. Important source of variability (improperly adjusted machines, operator errors, or defective raw materials).
  - There are few.
  - Each single one is an important source of variability.
  - Are occasionally present.
  - Easy to detect since they appear at an specific stage of the process.

Variability causes

- Chance causes. Natural variability (background noise) due to causes that are an inherent part of the process.
  - There are many, and some might be unknown.
  - Each of them causes only small variability.
  - Inherent part of the process.
  - Difficult (or impossible) to get rid of.

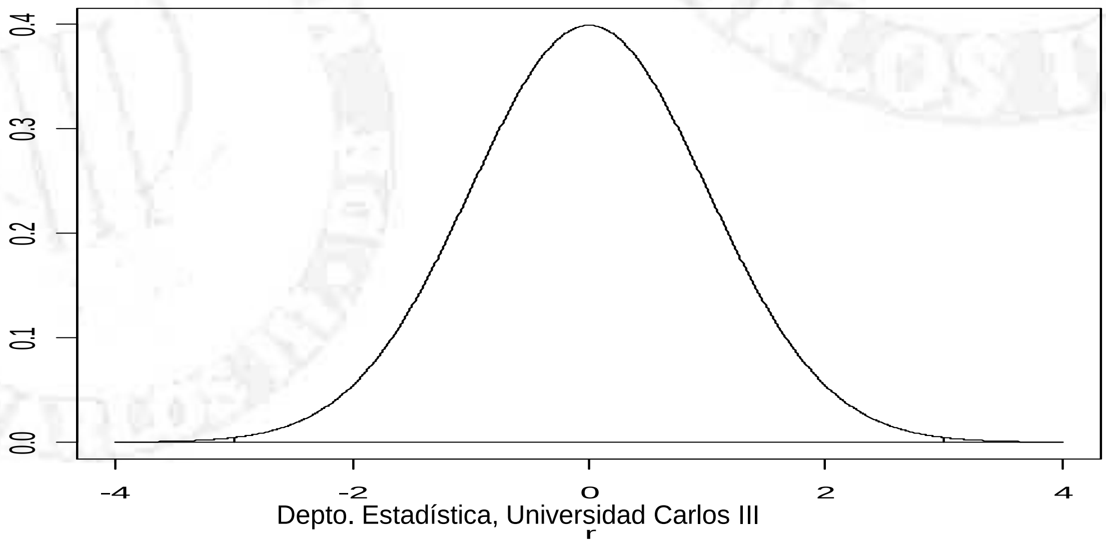
Introduction

- Out of control. Some assignable cause has occured and the quality characteristic of the produced items is substantially modified.

- In-control state. Only chance causes have occured, the quality characteristics has a small variability.

Introduction

It is commonly assumed that the quality characteristic follows a normal distribution N(μ,σ). In the in-control state, the quality characteristic of 99’73% of the produced items lies in an interval of width 6σ, ($\mu \pm 3\sigma$).



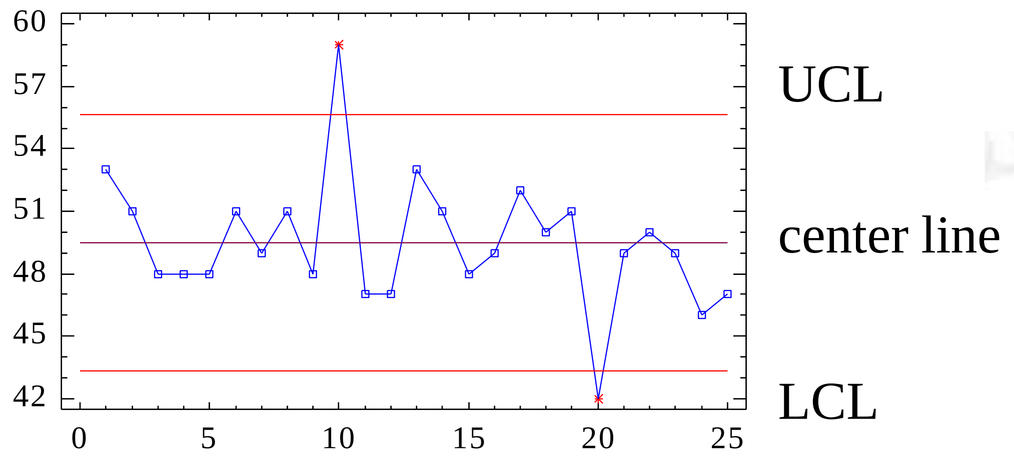
Control charts

Basic elements of a control chart:

- Center line. Mean level of the characteristic.
- Upper Control Limit (UCL).
- Lower Control Limit (LCL).

Control charts

A control chart monitors some sample characteristic associated with the quality of the production.



Outline

1. Introduction, control charts
2. Variables control charts (*X*-bar chart)
3. Attributes control charts (*p* and *np* charts)

X-bar chart

- Monitors sample means.
- If $\bar{X} \sim N(\mu, \sigma/\sqrt{n})$, the X-bar control chart is

$$\mathrm{UCL} = \mu + 3\frac{\sigma}{\sqrt{n}}$$

$$\text{center line } = \mu$$

$$\mathrm{LCL} = \mu - 3\frac{\sigma}{\sqrt{n}}$$

X-bar chart

- Parameter estimation, k samples of size n
  - 1st sample: $X_{11}, X_{12}, \ldots, X_{1n}$
  - i-th sample: $X_{i1}, X_{i2}, \ldots, X_{in}$
  - k-th sample: $X_{k1}, X_{k2}, \ldots, X_{kn}$

- For each individual sample

$$\overline{X}_i = \frac{\sum_{j=1}^{n} X_{ij}}{n}\,; \quad S_i^2 = \frac{\sum_{j=1}^{n} (X_{ij} - \overline{X}_i)^2}{n-1}$$

X-bar chart

- Grand mean.

$$\bar{\bar{X}} = \frac{\sum_{i=1}^{k} \bar{X}_i}{k}$$

The grand mean is an unbiased estimator of the process mean, but E[S_i]=$c_4\sigma$. Therefore, we must transform S_i in order to obtain an unbiased estimator of σ,

$$\hat{\sigma} = \frac{\bar{S}}{c_4} \quad \text{where} \quad \bar{S} = \frac{\sum_{i=1}^{k} S_i}{k}$$

X-bar chart

- The X-bar chart is:

$$\text{UCL} = \mu + 3\frac{\text{E}[S_j]}{c_4\sqrt{n}}$$

$$\text{center line} = \mu$$

$$\text{LCL} = \mu - 3\frac{\text{E}[S_j]}{c_4\sqrt{n}}$$

$$\text{UCL} = \bar{\bar{x}} + 3\frac{\bar{s}}{c_4\sqrt{n}}$$

$$\text{center line} = \bar{\bar{x}}$$

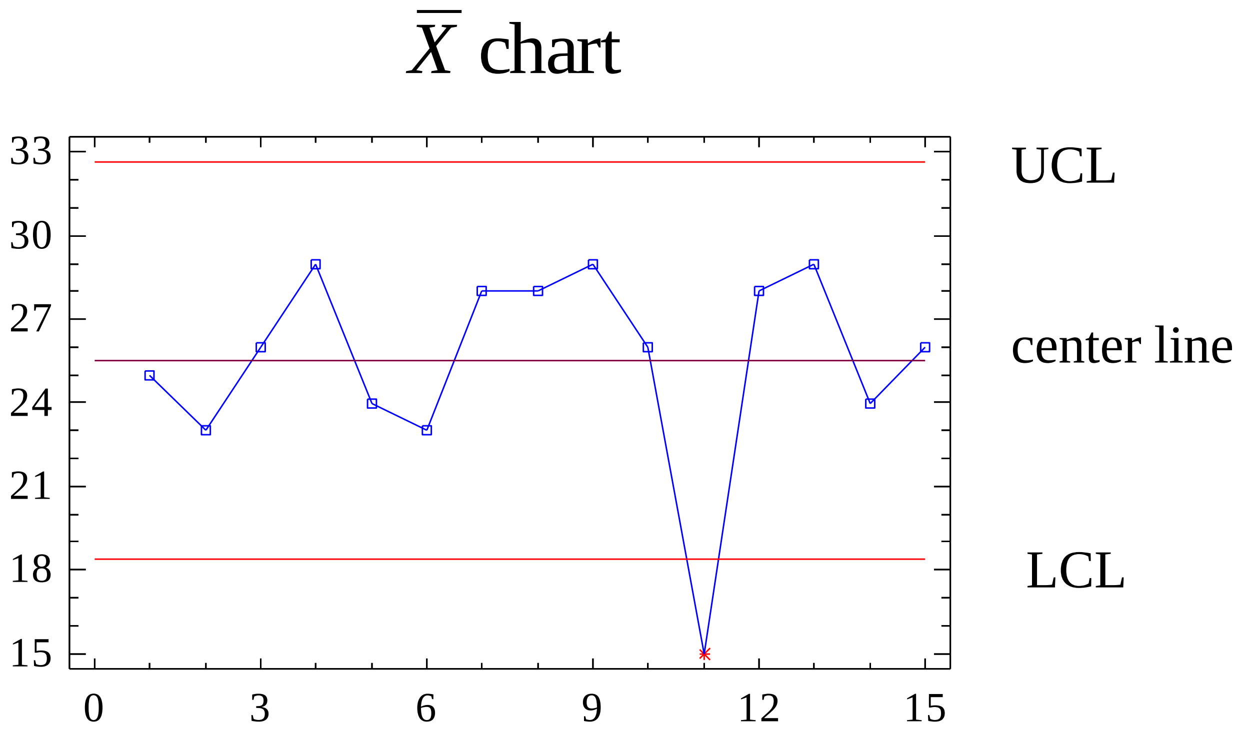
$$\text{LCL} = \bar{\bar{x}} - 3\frac{\bar{s}}{c_4\sqrt{n}}$$

X-bar chart

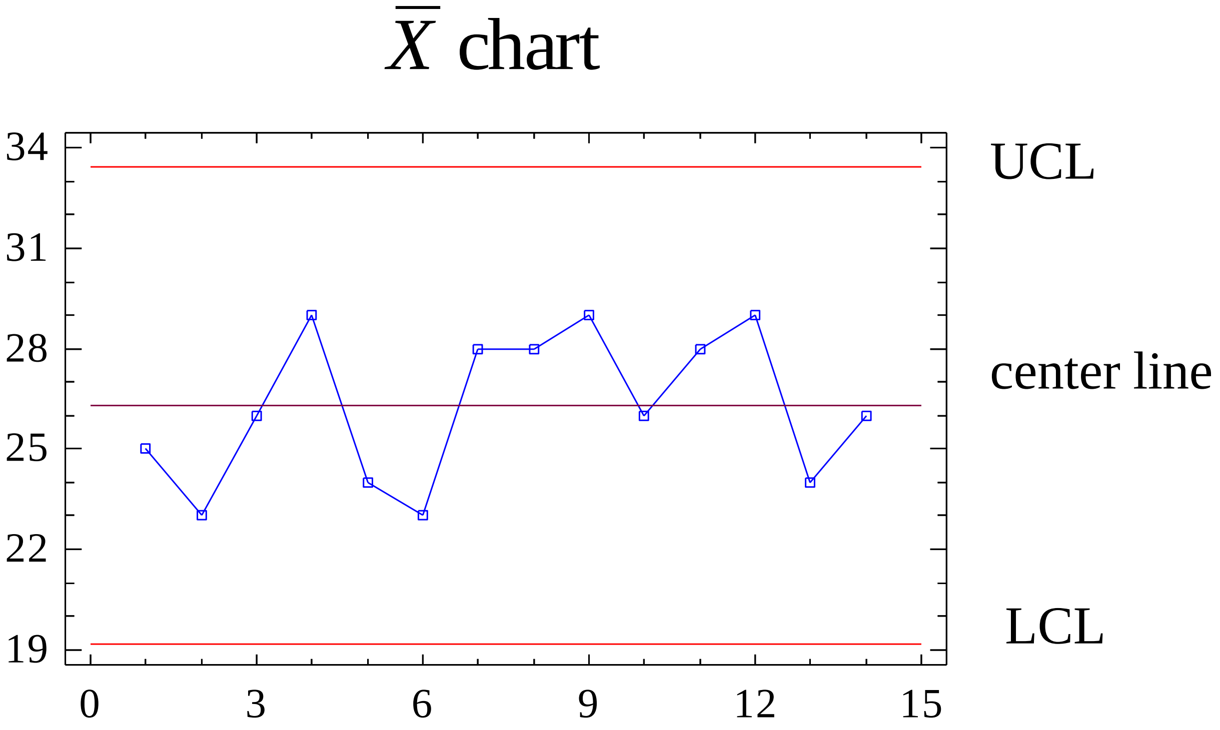
Estimating the control limits:

- The upper and lower control limits are estimated out of a set of trial samples.
- If the sample characteristic of any of the trial samples is out of the control limits, the corresponding sample is deleted and the control limits are estimated.
- The estimated control limits are used to monitor the sample characteristing in an on-going process.

X-bar chart



X-bar chart



X-bar chart

n	c_4
2	0.7979
3	0.8862
4	0.9213
5	0.9400
6	0.9515
7	0.9594
8	0.9650
9	0.9693
10	0.9727

Process capability

The process capability of a variable control chart whose quantitative characteristic X satisfies $\mathrm{Var}[X] = \sigma^2$ is given by

$$\text{Capability} = 6\sigma,$$

where σ is the standard deviation of X when the process is in the in-control state.

$$\text{Estimated capability} = 6\hat{\sigma} = 6\frac{\bar{s}}{c_4}.$$

Process capability

- Estimating the process capability:
  - Take k samples of size n (trial samples)
  - Build an X-bar control chart
  - Delete the samples that have been taken when the process was out of control (if any) and recalculate the control limits
  - (test data is normaly distributed)

Process Capability Ratio

The Process Capability Ratio compares the process capability with the product specifications

$$\text{PCR} = \frac{\text{Specifications}}{\text{Capability}} = \frac{\text{USL-LSL}}{6\sigma}$$

where USL (Upper Specification Limit) and LSL (Lower Specification Limit) constitute limit bounds for an admissible product.

- PCR > 1 few nonconforming units produced;
- PCR < 1 large number of nonconforming units produced;
- PCR ≈ 1 0.27% nonconforming units.

Average Run Length

If the process suffers a shift in its mean (or standard deviation) such that the probability of detecting it with the *X*-bar chart is *p*, the mean number of samples until the shift is detected is

- Average Run Length. ARL $= 1/p$

It is the mean of a Geometric random variable.

Analysis of Patterns of Control Charts

- In a control chart it is possible to detect nonrandom patterns.
  - Example: long sequence of subsequent points above or below the center line.
- Such arrangements of points are called violating runs.

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Attributes charts

Attributes control charts are designed to monitor quality characteristics that cannot be measured on a quantitative scale.

In a p chart, the proportion of items possessing a certain attribute p is monitored

- Proportion of nonconforming (defective) units
- Proportion of clients that claim

p chart

Sample size of the i-th sample: n_i

The number of nonconforming units in the i-th sample is d_i and follows a Binomial distribution with parameters n_i and p.

$$P(d_i = r) = \binom{n_i}{r} p^r (1-p)^{n_i - r} \quad \text{if} \quad r = 0,1,2,\ldots,n_i$$

p chart

for the sample fraction defective $\hat{p}_i = \dfrac{d_i}{n_i}$

- We know that: $\mathrm{E}[\hat{p}_i] = p$

$$\mathrm{Var}[\hat{p}_i] = \frac{p(1-p)}{n_i}$$

After the CLT, it holds:

$$\hat{p}_i \approx \mathrm{N}\left(p, \sqrt{\frac{p(1-p)}{n_i}}\right)$$

p chart

- The *p* chart is given by:

$$\text{UCL} = \mathrm{E}[\hat{p}_i] + 3\sqrt{\mathrm{Var}[\hat{p}_i]}$$

$$\text{center line} = \mathrm{E}[\hat{p}_i]$$

$$\text{LCL} = \mathrm{E}[\hat{p}_i] - 3\sqrt{\mathrm{Var}[\hat{p}_i]}$$

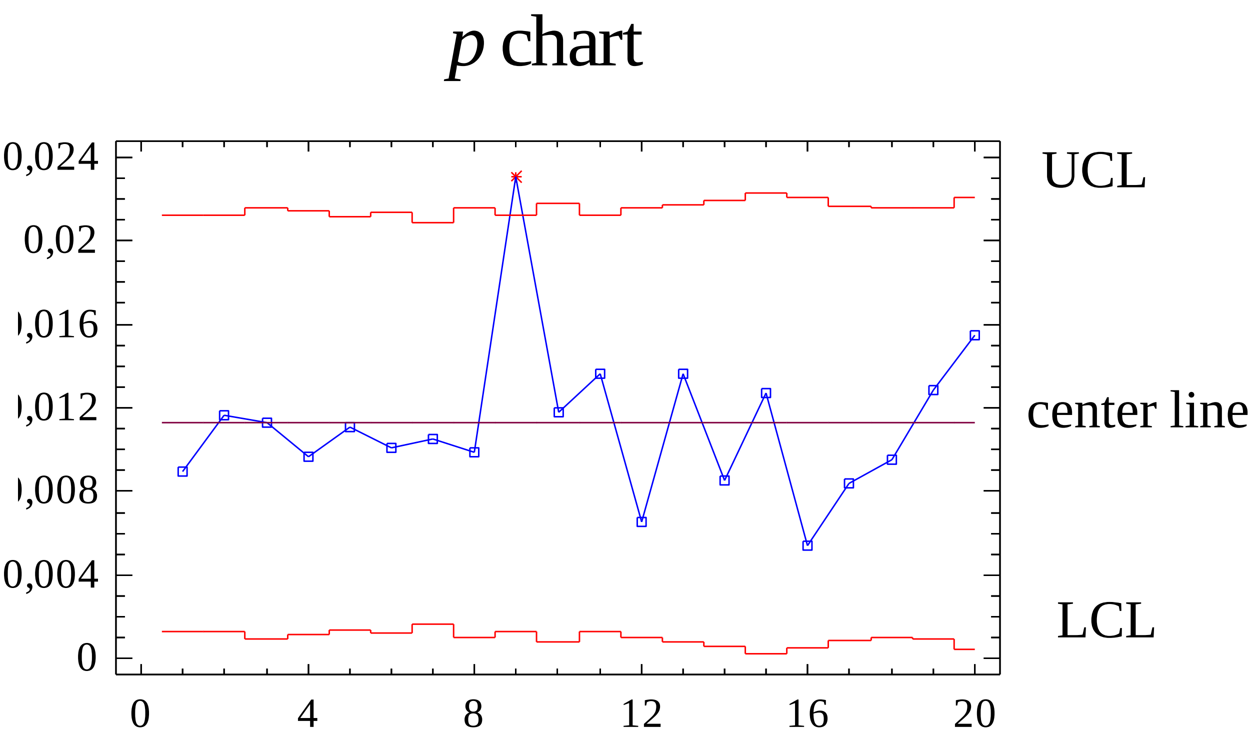
$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}}$$

$$\text{center line} = \bar{p}$$

$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}}$$

$$\text{where} \quad \bar{p} = \frac{\sum_{i=1}^{k} d_i}{\sum_{i=1}^{k} n_i}, \text{ average fraction defective}$$

p chart



p chart
0,024
0,02
),016
),012
),008
0,004
0
0
4
8
12
16
20
UCL
center line
LCL

p chart

Estimating the control limits:

- We take k samples (at least 20) of size n_i (commonly at least 30)
- The sample proportion of nonconforming units in each sample is computed
- p is estimated by the average fraction defective
- The control limits are computed
- If any sample fraction defective lies out of the control limits, the sample is deleted and limits recomputed

p chart

- Capability. Proportion of conforming units produced in the in-control state

$$\text{Capability} = 1 - p$$

$$\text{Estimated capability} = 1 - \overline{p}$$

np chart

The *np* chart is equivalent to the *p* chart.

It monitors the number of nonconforming units in a sample d_i (instead of its proportion).

It is useful when:

- The number of nonconforming units is more important than the proportion.
- The sample size is constant.

np chart

- Let $n_i = n$ constant.

After the CLT

$$d_i = n\hat{p}_i \approx \mathrm{N}\left(np, \sqrt{np(1-p)}\right)$$

$$\mathrm{UCL} = \mathrm{E}[d_i] + 3\sqrt{\mathrm{Var}[d_i]}$$

$$\text{center line} = \mathrm{E}[d_i]$$

$$\mathrm{LCL} = \mathrm{E}[d_i] - 3\sqrt{\mathrm{Var}[d_i]}$$

$$\mathrm{UCL} = n\overline{p} + 3\sqrt{n\overline{p}(1-\overline{p})}$$

$$\text{center line} = n\overline{p}$$

$$\mathrm{LCL} = n\overline{p} - 3\sqrt{n\overline{p}(1-\overline{p})}$$

np chart

- Capability. As for the *p* chart, the process capability is the proportion of conforming units produced in the in-control state

$$\text{Capability} = 1 - p$$

$$\text{Estimated capability} = 1 - \bar{p}$$