

Final exam for Probability

Master in Statistics for Data Science

November 6, 2018

Name: _____

IMPORTANT: Justify all answers.

- P1. (1.5 points) A gambler has a fair coin and a two-headed coin in his pocket. He selects one of the coins at random. When he flips it, it shows heads.
- (0.75 points) What is the probability that it is the fair coin?
 - (0.5 points) Suppose that he flips the same coin a second time and, again, it shows heads. Now what is the probability that it is the fair coin?
 - (0.25 points) Suppose that he flips the same coin a third time and it shows tails. Now what is the probability that it is the fair coin?
- P2. (2 points) The Zero-Truncated Poisson (ZTP) distribution is the conditional probability distribution of a Poisson random variable, given that the value of the random variable is not zero (discrete distribution whose support is formed by the positive integers).
- (0.25 points) Determine the probability mass function of a ZTP random variable with parameter λ .
 - (0.5+0.25 points) Determine the mean and variance of a ZTP random variable with parameter λ .
 - (0.5 points) The number of items that a client buys in a supermarket follows a Poisson distribution with parameter $\lambda = 10$. Only when the client has bought some item will he will go through the checkout line. What is the probability that the number of items in a shopper's basket at the supermarket checkout line is greater than 5?
 - (0.5 points) Write a piece of code to simulate 1000 observations of a ZTP random variable with parameter $\lambda = 10$. Use your simulations to approximate the answer to part c).
- P3. (2 points) Each of the members of a 7-judge panel independently makes a correct decision with probability 0.7. Assume the panel's decision is made by majority rule and it is mandatory for all judges to take a decision (guilty or not guilty).
- (0.5 points) What is the probability that the panel makes the correct decision?
 - (0.5 points) Given that 4 of the judges agreed (the other 3 did also agree in the other decision), what is the probability that the panel made the correct decision?
- Assume now that each member of the panel can make the decision that the individual is guilty or not, but she can also reject to make a decision. As before, with probability 0.7 she will make the correct decision, but with probability 0.1 she will reject to make any decision.
- (0.5 points) What is the probability that 4 judges make the correct decision, one rejects to make any decision, and two make the wrong decision?

Assume now that the panel is selected at random from a group of 50 judges. Out of them, 35 will take the correct decision, while 15 will take the wrong one.

- d) (0.5 points) What is the probability that the panel makes the correct decision?
- P4. (2.5 points) IQ scores are commonly assumed to be normally distributed. An extensive Scottish survey¹ suggests that the mean IQ score for 11-year-old girls is 100.64 and its standard deviation is 14.1, while the mean IQ score for 11-year-old boys is 100.48 and its standard deviation is 14.9. The test was ran on 39343 girls (49.6%) and 40033 boys. Assume that the joint distribution of the IQs of a couple of twins is bivariate normal with correlation 0.8 regardless of their gender.
- a) (0.25 points) What percentage of (11-year-old Scottish) girls have an IQ greater than 115?
- b) (0.25 points) If 10 girls are selected at random, what is the probability that at least 6 of them have an IQ greater than 115?
- c) (0.25 points) What IQ is exceeded by 75% of girls?
- d) (0.25+0.25 points) Compute the mean and variance of the IQ in the previous population.
- e) (0.25 points) What is the probability that an individual selected at random from that population has an IQ greater than 115?
- f) (0.25 points) If an individual selected at random has an IQ greater than 115. What is the probability that it is a girl?
- g) (0.5 points) Consider now a couple of twins (one sister and one brother). What is the distribution of their average IQ?
- h) (0.25 points) What is the probability that the average IQ from part g) is greater than 115?
- P5. (2 points) From past experience, a professor knows that the test score of a student taking her final examination is a random variable with mean 75.
- a) (0.5 points) Give an upper bound for the probability that a student's test score will exceed 85.

Suppose, in addition, that the professor knows that the variance of a student's test score is equal to 25.

- b) (0.5 points) What can be said about the probability that a student will score between 65 and 85?
- c) (0.5 points) How many students would have to take the examination to ensure, with probability at least 0.9, that the class average would be within 5 of 75 (between 70 and 80)? Do not use the central limit theorem.
- d) (0.5 points) How many students would have to take the examination to ensure, with probability at least 0.95, that the class average would be within 1 of 75 (between 74 and 76)? Use the central limit theorem.

¹I.J. Deary, G. Thorpe, V. Wilson, J.M. Starr, L.J. Whalley, (2003) Population sex differences in IQ at age 11: the Scottish mental survey 1932, *Intelligence*, **31** (6), pp. 533–542