Variance reduction techniques

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1. **Pricing European options** European options may only be exercised at expiration. That is, if we buy today a European call option with strike price $k = 100$ and maturity $t_m = 1$ year, in one year (at maturity) we have the right to buy the asset for the fixed strike price $k = 100$, which we will do if the asset price at that precise moment is greater than k. If the price of the asset at that moment is below k, we will not exercise the option. Suppose we want to price a European call option with the initial asset price $S(0) = 100$, strike price $k = 100$, risk-free rate $r = 0.02$, volatility $\sigma = 0.25$, and maturity $t_m = 1$ year. The risk-neutral pricing process is

$$S(t) = S(0)\exp\left((r - \sigma^2/2)t + \sigma Z\sqrt{t}\right),$$

where $Z \sim \mathrm{N}(0,1)$. The option will only be exercised if the asset's price at maturity is greater than 100, so its payoff will be $\max\{S(t_m) - k, 0\}$, while the price is the expected payoff. Observe that the price is to be payed today, so it must be given in today's price of money, and we can use the risk-free rate to determine today's price of 100 monetary units in one year, which is $100\exp(-r) = 98.01987$. Use $MC = 10000$ simulations to price the option and give the variance of your estimator.

2. Use antithetic sampling in problem 1. and compare the results with those of the previous problem.

3. Use $-\max\{(r - \sigma^2/2)t_m + \sigma Z\sqrt{t_m}, 0\}$ as control variate in problem 1. If $t_m = 1$, by direct integration, it can be shown

$$\mathbb{E}\max\{(r - \sigma^2/2) + \sigma Z, 0\} = \frac{\sigma}{\sqrt{2\pi}}\exp\left(-\frac{(-r + \sigma^2/2)^2}{2\sigma^2}\right) + (r - \sigma^2/2)P(Z > -r/\sigma + \sigma/2),$$

while the computation of its variance and covariance with the asset price is better approximated by simulations. Compare your results with those of the previous problems.

4. Use stratified sampling (100 strata with equal probabilities through the support of the normal rv) in problem 1. and compare the results with those of the previous problems.

5. Repeat problem 1. with strike price $k = 200$ and keeping the remaining parameters unchanged. Use importance sampling with the density mass function of a normal rv with mean 1 and standard deviation 1 as instrumental and compare the results.