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Electricity price forecasting through transfer function models

FJ Nogales[1]* and AJ Conejo[2]

[1]*Universidad Carlos III de Madrid, Madrid, Spain; and* [2]*Universidad de Castilla-La Mancha, Ciudad Real, Spain*

Forecasting electricity prices in presentday competitive electricity markets is a must for both producers and consumers because both need price estimates to develop their respective market bidding strategies. This paper proposes a transfer function model to predict electricity prices based on both past electricity prices and demands, and discuss the rationale to build it. The importance of electricity demand information is assessed. Appropriate metrics to appraise prediction quality are identified and used. Realistic and extensive simulations based on data from the PJM Interconnection for year 2003 are conducted. The proposed model is compared with naïve and other techniques.

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Introduction

During the last decade, electricity markets spread throughout the world seeking lower electricity prices while maintaining service quality (Sheblé, 1999; Ilic *et al*, 1998; Shahidehpour *et al*, 2002). Electricity markets are based on either a pool framework or bilateral contracts. In most markets, whatever the arrangement, market clearing is conducted once a day and provides hourly electricity prices (market clearing prices).

To accurately forecast these prices is critical for producers, consumers and retailers. In order to maximize its profits (that depend on future prices), any producer should self-schedule its units deriving in this manner its optimal bidding strategy in the day-ahead market. To optimally self-schedule its units, the producer needs accurate forecasts of prices before bidding time. Analogously, both consumers and retailers need price forecasts to self-schedule their respective consumptions and therefore to derive their respective bidding strategies in the market before bidding time. Many of these problems can be modelled as mathematical programmes. An overview of mathematical programming problems in electricity markets can be found in Conejo and Prieto (2001).

In this paper, we address the day-ahead electricity price forecasting problem whose time framework is illustrated in Figure 1. We assume that the market price forecasts for day d are required on day $d-1$, typically at hour h_b (around 10 am). On the other hand, data concerning results for day $d-1$ are available on day $d-2$ at hour h_c (around noon). Therefore, the actual forecasting of market prices for day d can take place between hour h_c of day $d-2$ and hour h_b of day $d-1$. For this reason, to forecast prices for day d, price data upto hour 24 of day $d-1$ are considered known.

Reported techniques to forecast electricity prices include ARIMA models: Fosso *et al* (1999) and Contreras *et al* (2003); dynamic regression models: Nogales *et al* (2002); neural network models: Ramsay and Wang (1998), Szkuta *et al* (1999), Hong and Hsiao (2002), Zhang *et al* (2003), Rodríguez and Anders (2004); wavelet transform models: Kim *et al* (2002); one-factor diffusion models: Barlow (2002); and mathematical programming models: Hogan *et al* (1996). In respect to forecasting quality, Tashman (2000) presents a useful discussion and review on out-of-sample forecast evaluation.

This paper describes the building of time-series models to forecast electricity prices. These models relate electricity demands with electricity prices throughout the time and are known as *transfer function models* in the engineering literature and *dynamic econometric models* in the economics literature. To assess the importance of demand information, demand data are in turn, taken and not taken into account to build the model. Additionally, three benchmark models are used to appraise the behaviour of the proposed technique.

Appropriate metrics to assess prediction quality for the proposed models are identified and used. Detailed computational simulations are performed using data for the PJM Interconnection (2004) in the US, a well-established electricity market. Data for the simulation span year 2003.

In most markets throughout the world, time-series of electricity prices present generally the following characteristics:

Correspondence: FJ Nogales, Department of Statistics, Universidad Carlos III de Madrid, Avda. de la Universidad, 30 28911 – Leganes, Spain.

E-mail: FcoJavier.Nogales@uc3m.es

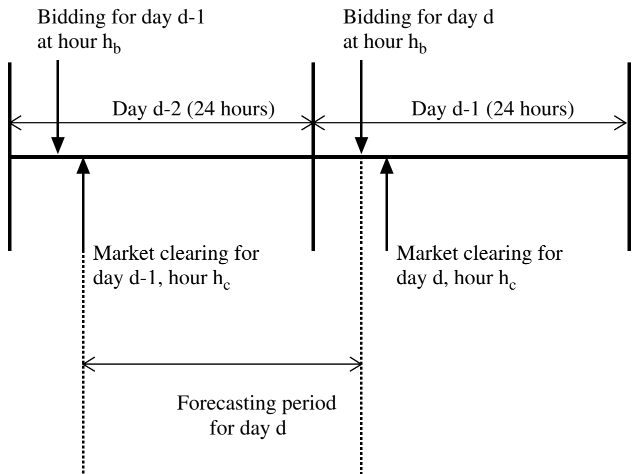

Figure 1 Time framework to forecast electricity prices for day *d*.

1. nonconstant mean and variance,
2. high frequency,
3. high volatility,
4. presence of outliers,
5. daily and weekly seasonality,
6. calendar effect on weekends and holidays.

These characteristics make price time-series inherently hard to forecast in comparison with electricity demand and, in general, in comparison with diverse econometric variables.

Moreover, oligopolistic behaviour by major players in electricity markets (a rather common situation) embodies non-random effects in the electricity price formation, which make price prediction still more complicated.

As a result, the application of time-series models to predict electricity prices has to be carried out with particular care; since, in general, the straightforward application of standard prediction software is of no use.

Within the above framework, this paper provides both a novel application and an in-depth analysis of a particular time-series technique, the transfer function, which happens to behave efficaciously to predict electricity prices. Conejo *et al* (2005) provides a comparative overview of different prediction techniques for electricity prices, and identifies the transfer function technique as a promising procedure; however, no detailed analysis of this technique is carried out. This paper provides such detailed analysis.

The rest of this paper is organized as follows. The following section describes the building of a transfer function model. The next section describes the prediction models considered in this paper. The section thereafter presents detailed simulation results, with appropriate accuracy metrics, based on data from the PJM Interconnection and year 2003. Finally, the last section provides some relevant conclusions.

Transfer function models

In this section, models that relate electricity demand with electricity price throughout the time are developed. These models are known in the engineering literature as *transfer function models* whereas in the economics literature are known as *dynamic econometric models*. The interest of these models for forecasting purposes depends on the instantaneous/non-instantaneous relationship between demand and price. If this relationship is instantaneous, forecasting prices requires demand information. However, since the demand is not known, a univariate model is needed for it, which reduces the relevance of demand information. On the other hand, if the price–demand relationship is not instantaneous and a variation in the demand affects the price *b* hours later, then the demand is an advanced indicator of the price. In this case, knowledge of the demand may substantially improve the predictions for the price. The proposed models in this paper incorporate the possible, not instantaneous relationship between price and demand.

General structure and building steps

Throughout the paper, p_t denotes the electricity price in hour t (\$/MWh) and d_t denotes the corresponding electricity demand (MWh). As these series are not generally stationary, $x_t = f_1(d_t)$ and $y_t = f_2(p_t)$ denote the transformations to obtain stationary series for demand and price, respectively.

The transfer function models represent the relationship between these two series as

$$y_t = c + v(B)x_t + \eta_t \tag{1}$$

where B is the backshift operator: $Bz_t = z_{t-1}$, $B^2 z_t = z_{t-2}, \ldots,$ $B^k z_t = z_{t-k}$. The function v is modelled as

$$v(B) = v_0 + v_1 B + v_2 B^2 + \cdots \tag{2}$$

and is denominated the *transfer function*. The coefficients v_i in this function describe the dynamic relationship between the demand and price series and are denominated *impulse response factors*.

The number of factors in $v(B)$ can be infinity. Therefore, it is convenient to use a simpler representation. Moreover, this representation should embody the double seasonality structure appearing in the demand and price series (of order 24 and 168, respectively). Then, we propose the following model for the transfer function:

$$v(B) = \frac{w_{m_1,m_2,m_3}(B)}{\delta_{a_1,a_2,a_3}(B)} \tag{3}$$

$$\begin{aligned} w_{m_1,m_2,m_3}(B) = {} & w_0 + w_1 B + \cdots + w_{m_1} B^{m_1} \\ & + w_{24} B^{24} + \cdots + w_{24m_2} B^{24m_2} \\ & + w_{168} B^{168} + \cdots + w_{168m_3} B^{168m_3} \end{aligned} \tag{4}$$

$$\begin{aligned}\delta_{a_1,a_2,a_3}(B) = &(1-\delta_0 B - \cdots - \delta_{a_1} B)\\ &\times(1-\delta_{24}B^{24} - \cdots - \delta_{24a_2}B^{24a_2})\\ &\times(1-\delta_{168}B^{168} - \cdots - \delta_{168a_3}B^{168a_3})\end{aligned} \quad (5)$$

and m_1, m_2, m_3 and a_1, a_2, a_3 are integer numbers (small) to be determined.

As indicated previously, the price can be related to the demand including a certain delay $b \geqslant 0$. To model this effect, factor B^b is introduced:

$$v(B) = \frac{w_{m_1,m_2,m_3}(B)}{\delta_{a_1,a_2,a_3}(B)} B^b$$

It should be noted that if the relationship between x_t and y_t is instantaneous, then $b = 0$.

The part in y_t not explained by x_t, that is, η_t, is denominated the *perturbation* series and, normally, it is considered to be an ARMA process.

The construction of a transfer function model includes identification, estimation and diagnosis, as explained below:

Step 0: Several transformations (eg differentiation and/or Box–Cox) are applied to the original series to obtain stationary ones. Then, a time-series model is identified for y_t.

Step 1: Coefficients (m_1, m_2, m_3), (a_1, a_2, a_3) and b in the transfer function are determined as well as the coefficients in the perturbation series η_t.

Step 2: All parameters in the model, w_i and δ_i, are estimated, as well as the parameters in the perturbation series η_t.

Step 3: A diagnosis check is used to validate model assumptions. If the hypotheses of the model are validated, the procedure concludes and the model is ready for forecasting; otherwise, the procedure continues in Step 1 to refine the model.

In the following, each step of the above scheme is detailed:

In Step 0, to stabilize the variance, the following Box–Cox transformation is proposed:

$$y_t = \frac{p_t^{\lambda} - 1}{\lambda} \quad (6)$$

$$x_t = \frac{(d_t/100)^{\lambda} - 1}{\lambda} \quad (7)$$

where λ is the power coefficient of the transformation. This value is estimated by minimizing the root of the mean squared error of the original price data. The same value of λ is applied to the demand data.

For the PJM market, the selected value for λ is 0.5 for all the models considered in the paper; that is, the squared root stabilizes the variance. In addition to the selection of the power coefficient value, the other issue to consider is that the direct un-transformation of the forecasts is biased. We have used the un-transformation method provided in Guerrero (1993) to adjust for bias when un-transforming forecasts and confidence intervals for them.

In Step 1, the proposed model for the perturbation term η_t is considered to be the same as the one for the variable y_t. The final model for η_t might be simpler than that for y_t, because x_t explains part of the dynamics of y_t. To identify the function in (3), the following model is first estimated:

$$y_t = c + (v_0 + v_1 B + v_2 B^2 + \cdots + v_{200} B^{200}) x_t + \eta_t \quad (8)$$

Then, the delay structure of the impulse response factors is analysed. A high number of factors in (8) is due to seasonality of orders 24 and 168. In successive trials, the observation of the residuals obtained in Step 3 (observed values minus predicted ones) helps to refine the structure of the model.

In Step 2, parameter estimation is based on maximizing a conditional likelihood function for the available data, as described in Hillmer and Tiao (1979). Instead of the exact likelihood function, a conditional function should be used, because long lag models are considered, that is, models that contain large delay operators and/or parameters with high order. It should be noted that the exact likelihood function method may originate numerical instability. Moreover, given the number of observations used (more than 1400), both functions produce similar results.

In Step 3, once a model is estimated and before accepting it, several diagnosis tests are required to validate model assumptions:

1. Tests for randomness are used to check if the residuals (actual prices minus fitted prices, as estimated in Step 1) satisfy normality and incorrelation assumptions (based on Ljung–Box statistics).
2. By means of cross-correlations, the unidirectional representation of the model is verified, that is, x_t influences y_{t+k}, for $k \geqslant 0$, but not the other way.
3. By means of cross-correlations, the incorrelation between x_t and η_t is verified.

If these hypotheses are validated, the model can be used to forecast prices. Otherwise, the residuals contain a certain structure that should be studied to refine the model in Step 1.

Prediction models

In this section, several prediction models that explain the price are shown. The first model is based on the transfer function methodology described in the previous section. To verify the influence of the demand data, demand is eliminated from this model, which results in one additional model. Finally, to assess the performance of the proposed models, we considered two benchmark models. As a result, four prediction models are considered for electricity price forecasting.

General model: M1

Next, a model that explains the price as a function of the demand is presented. This model has been developed following the methodology shown in the section on General structure and building steps.

Model 1:

$$v(B) = \frac{w_0 + w_1 B + w_2 B^2 + w_{24} B^{24} + w_{168} B^{168}}{(1 - \delta_{24} B^{24})(1 - \delta_{168} B^{168})} \quad (9)$$

with

$$\begin{aligned}&(1 - \phi_1 B)(1 - \phi_{24} B^{24})(1 - \phi_{168} B^{168})\eta_t \\ &\quad = (1 - \theta_1 B - \theta_{168} B^{168})(1 - \theta_{24} B^{24} - \theta_{48} B^{48})(1 - \theta_{168} B^{168}) a_t\end{aligned} \quad (10)$$

where a_t is the white noise.

It should be noted that the parameters of the numerator in (9) describe the initial effects of the demand, that is, the price at hour t is related, following a decay pattern, to the values of demands at hours t, $t-1$, $t-2$, $t-24$ and $t-168$. The denominator in (9) characterizes the decay pattern of these effects in the price. Eq. (10) establishes the relationship between past prices and actual prices as an ARMA process with double seasonality. Moreover, in this model no delay factor is considered ($b=0$). Therefore, we conclude that an instantaneous relation between demand and price does exist.

No demand model: M2

In order to verify the influence of the demand data, demand modelling is eliminated from Model 1, which results in Model 2.

Model 2:

$$\begin{aligned}&(1 - \phi_1 B)(1 - \phi_{24} B^{24})(1 - \phi_{168} B^{168}) y_t \\ &\quad = c + (1 - \theta_1 B - \theta_{168} B^{168})(1 - \theta_{24} B^{24} - \theta_{48} B^{48}) \\ &\quad \times (1 - \theta_{168} B^{168}) a_t\end{aligned} \quad (11)$$

Benchmark models: M3 and M4

To assess the performance of the proposed models with respect to straightforward ones, we considered two benchmark models. The first one is a naïve model for which price forecasts equal real prices corresponding to one day before.

Model 3:

$$y_t = y_{t-24} + a_t \quad (12)$$

where a_t is the white nose

For the second benchmark model, we have adapted the standard exponentially weighted moving average (frequently used in practice without seasonality) to include the seasonality of orders 24 and 168, respectively. The motivation of this model is in the following forecast function:

$$\begin{aligned}\hat{y}_{t+h} &= \alpha y_t + (1-\alpha)\hat{y}_t + \beta y_{t-24} + (1-\beta)\hat{y}_{t-24} \\ &\quad + \gamma y_{t-168} + (1-\gamma)\hat{y}_{t-168}\end{aligned} \quad (13)$$

where $0<\alpha$, β, $\gamma<1$ are the smoothing parameters. This function expresses past predictions as weighted combinations of past prices and past predictions of orders 1, 24 and 168, respectively. This forecast function coincides with the forecast function of the following time-series model.

Model 4:

$$\begin{aligned}&(1 - B)(1 - B^{24})(1 - B^{168}) y_t \\ &\quad = (1 - \theta_1 B)(1 - \theta_{24} B^{24})(1 - \theta_{168} B^{168}) a_t\end{aligned} \quad (14)$$

where a_t is the white noise.

Numerical results

In this section, detailed simulation results are provided. First, appropriate metrics to appraise prediction quality are identified. Then, results from realistic and extensive simulations based on data from the PJM Interconnection and year 2003 are presented. Finally, the proposed transfer function model is compared with other models.

Accuracy metrics

To appraise the accuracy of the proposed models, we use metrics that capture the error for each hour in the considered forecast period. For clarity, the measures used in this paper are defined below.

Note that $p_{t,d}$ denotes the electricity price in hour t of day d and $\hat{p}_{t,d}$ its forecast value.

The percentage error and the quadratic percentage error incurred at hour t of day d are computed as

$$e_{t,d} = \frac{|\hat{p}_{t,d} - p_{t,d}|}{p_{t,d}} \quad \text{and} \quad e2_{t,d} = \frac{(\hat{p}_{t,d} - p_{t,d})^2}{p_{t,d}}$$

respectively.

The mean percentage error and the mean quadratic percentage error in day d are defined, respectively, as

$$\text{emean}_d = \frac{1}{24}\sum_{t=1}^{24} e_{t,d} \quad \text{and} \quad \text{emean2}_d = \frac{1}{24}\sum_{t=1}^{24} e2_{t,d}$$

Similarly, the median percentage error and the maximum percentage error in day d are defined, respectively, as

$$\begin{aligned}&\text{emedian}_d = \text{median}\{e_{1,d}, \ldots, e_{24,d}\} \quad \text{and} \\ &\text{emax}_d = \text{maximum}\{e_{1,d}, \ldots, e_{24,d}\}\end{aligned}$$

We have selected the median as a measure of the *mean* error due to the high number of outliers that price series present.

Finally, we define measures for the whole forecasting period. The mean absolute percentage error (MAPE) and the mean of the median percentage error (more robust in the presence of outliers) are defined, respectively, as

$$\mathrm{MAPE} = \frac{1}{D}\sum_{d=1}^{D} \mathrm{emean}_d \quad \text{and} \quad \mathrm{MAPE2} = \frac{1}{D}\sum_{d=1}^{D} \mathrm{emedian}_d$$

Finally, the maximum mean percentage error and the root of the mean squared error are defined, respectively, as

$$\mathrm{EMax} = \frac{1}{D}\sum_{d=1}^{D} \mathrm{emax}_d \quad \text{and} \quad \mathrm{RMSE} = \sqrt{\frac{1}{D}\sum_{d=1}^{D} \mathrm{emean2}_d}$$

respectively.

The rather standard metrics above provide a comprehensive picture of forecasting errors and allows characterizing and comparing different forecasting procedures.

In addition, a part from comparing real prices with predicted ones, it is often valuable to compute confidence intervals of the forecasts. These confidence intervals provide information on the potential variability of these forecasts. We have computed 95% confidence intervals for all models using the un-transformation proposed by Guerrero (1993) to correct bias.

Case study

The PJM electricity market is used to carry out predictions. This is a well-established electricity market in the US, whose day-ahead price information is publicly available and easily reachable in PJM (2004).

The four models presented in the section on Prediction models are evaluated for their out-of-sample forecasting performance as explained in the following. First, we select a fixed number of days for the estimation period and a fixed number of days for the forecasting horizon period. In our case, the estimation period contains 61 days (2 months), that is 1464 h, which are immediately previous to the forecasting period of 62 days: 1 July 2003 to 31 August 2003.

Using the estimation period, the four models are estimated and 24 h-ahead forecasts (confidence intervals included) are computed for all the models. The differentiation and transformation (square root) for the forecasts are reversed and the accuracy indices presented in the previous section are computed. Finally, the time window is moved forward by 24 h and the procedure is repeated to obtain updated forecasts until the last day of the prediction horizon (31 August 2003) is forecast.

Forecasts for Model 1 are based on true demand, because it is useful to separate electricity price models from demand models. In this manner, the impact of demand information on price predictions can be efficiently assessed by Model 2.

All models have been implemented in the SCA System (Liu and Hudak, 1994). This system estimates the models, obtains 24-h ahead forecasts for y_t and finally reverses the differentiation and Box–Cox transformations to obtain forecasts in original units, $\hat{p}_t$. The cases have been run on a Pentium IV computer with 1.5 Gb of RAM at 1.70 GHz. Running time for next-day forecasts, including estimation and forecasting of prices and confidence intervals, is under 30 CPU seconds for each model.

Table 1 Error forecasting measures in the PJM market for the period 1 July 2003 to 31 August 2003

Model	*MAPE*	*MAPE2*	*EMax*	*RMSE*
M1	0.109	0.095	0.278	0.836
M2	0.133	0.117	0.330	1.028
M3	0.157	0.137	0.394	1.230
M4	0.164	0.148	0.379	1.295

Table 1 presents summaries of the error measures presented in the previous section for forecasts calculated daily during the period 1 July 2003 to 31 August 2003.

For the sake of illustration, Figure 2 shows the evolution of $\mathrm{emedian}_d$ for Models 1, 2, 3 and 4. Observe that Model 1 clearly outperforms the two benchmark models (Models 3 and 4) and produces considerably smaller error of prediction. Moreover, the impact of demand information on price predictions can be assessed comparing Models 1 and 2. It can be observed that a small but significant improvement in prediction is attained in the model with demand when compared to the same model without demand. For the proposed model, the demand information improves the mean of the median percentage error in about 19%.

Finally, to provide information on the potential variability of these forecasts, confidence intervals for the proposed model (Model 1) are presented. Figure 3 shows 95% confidence intervals obtained for the first week of July 2003 (shadow area), together with the evolution of the actual price for the same period (solid line).

It should be noted that, actual price lies below lower forecasting limit in hours 98, 132, 134, 135, 136, 137 and 138, and lies above upper forecasting limit in hours 42, 110, 111, 112, 113 and 114. That is, 7.7% of the time the actual price is outside the corresponding confidence intervals but, as can be observed, the violation is very small.

In summary, observing Table 1 and analysing Figures 2 and 3, the following conclusions are drawn.

1. Naïve techniques are clearly outperformed by the procedure proposed, which allows concluding the practical interest of the technique developed. This can be observed in Figure 2 and Table 1.
2. Comparing rows 1 and 2 of Table 1, we conclude that demand information does improve predictions substantially. This is also concluded from Figure 2.
3. As illustrated in Figure 3, the confidence intervals for the forecasts are sufficiently accurate for practical applications.

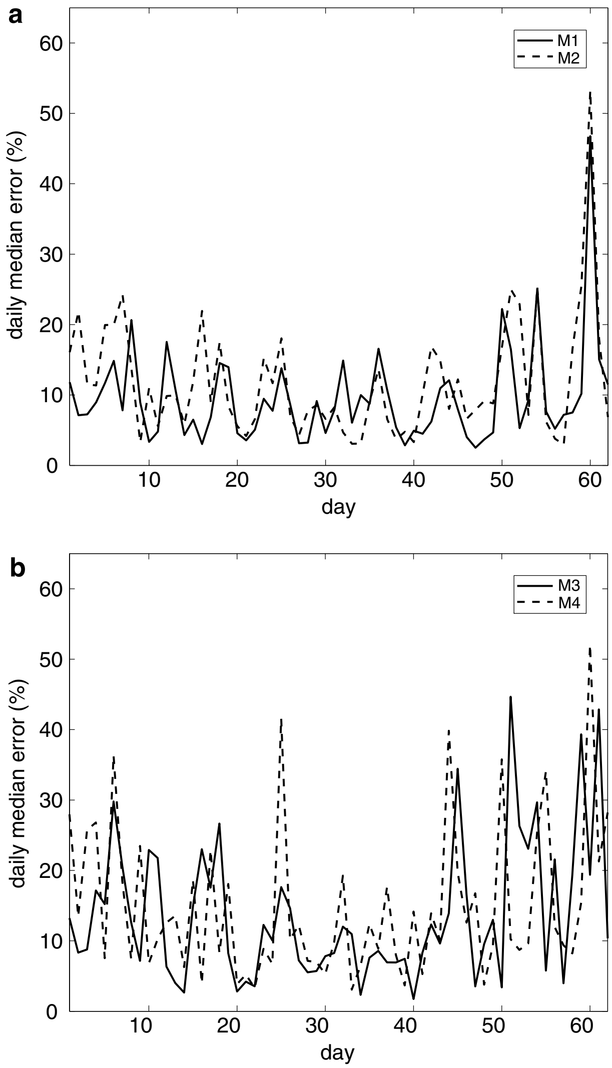

Figure 2 Modelling and demand effect: evolution of the median percentage error from 1 July, 2003 to 31 August, 2003. (a) Models 1 and 2 and (b) Models 3 and 4.

Conclusions

This paper analyses exhaustively a transfer function model to forecast day-ahead electricity prices. The effectiveness of this technique is assessed using different naïve techniques. The use of electricity demand series as an explicative variable improve predictions but not in a drastic manner. Exhaustive analysis using data from the PJM Interconnection reveal an appropriate forecasting functioning of the technique proposed. From a detailed analysis of the numerical results available in the technical literature, it can be concluded that the quality of predictions using the proposed technique is generally superior to the quality of predictions using alternative procedure such as standard time-series models (ARIMA) or neural networks.

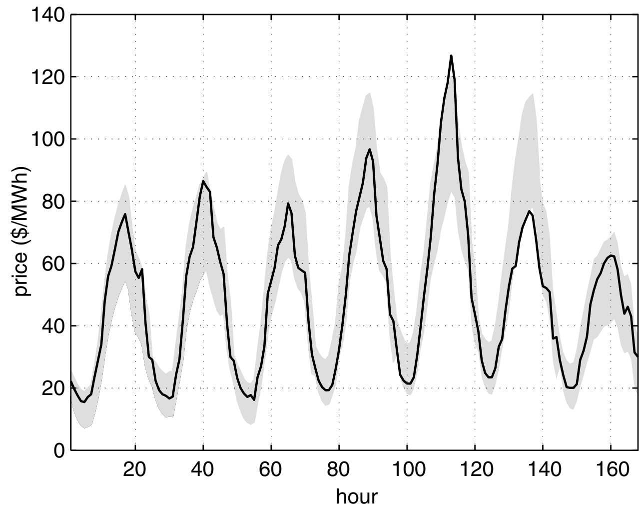

Figure 3 Evolution of actual prices (solid line) and confidence intervals (shadow) for price forecasts: first week of July 2003.

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