

FORECASTING GROWTH WITH TIME SERIES MODELS

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ABSTRACT

This paper compares the structure of three models for estimating future growth in a time series. It is shown that a regression model gives minimum weight to the last observed growth and maximum weight to the observed growth in the middle of the sample period. A first-order integrated ARIMA model, or I(1) model, gives uniform weights to all observed growths. Finally, a second-order integrated ARIMA model gives maximum weights to the last observed growth and minimum weights to the observed growths at the beginning of the sample period. The forecasting performance of these models is compared using annual output growth rates for seven countries.

KEY WORDS ARIMA models; integrated processes; regression; stationary processes

INTRODUCTION

An important problem in modeling economic time series is forecasting the future growth of a given time series. Assuming that a linear model is appropriate for the data, the procedures most often used are as follows:

- (1) Detrend the observed data by regressing the observations on time, and use the residuals from this regression to build a stationary time series model. The series is forecasted by adding the values of the deterministic future trend and the forecast of the stationary residual.
- (2) Differentiate the series, test for unit roots and if the series is assumed to be integrated of order one (I(1)) build a stationary ARMA model in the first difference of the series. Typically models built in this way include a constant for many economic time series.
- (3) Differentiate twice the series and build the ARMA model on the second difference of the process that is assumed to be I(2). Then in most cases the I(2) model does not include a constant term.

The decision between these three procedures should be made by testing the number of unit roots in the time series model. However, the available tests are not very powerful, specially for short time series, and therefore it is important to understand the consequences of using these models.

Let Z_t , be the time-series data and let us call $b_t = Z_t - Z_{t-1}$ the observed growth at time t . It is shown in this paper that the estimate of future growth by the three procedures can be written as

$$\hat{\beta}_t = \sum \omega_i b_i$$

where the coefficients ω_i are a weighting function, that is, $\omega_i > 0$ and $\sum \omega_j = 1$. The next section of this paper proves that linear regression gives minimum weights to the last observed growth and maximum weights to the observed growth in the middle of the sample. The third section shows that an I(1) model with a constant term gives a uniform weight throughout the sample, that is, $\omega_i = n^{-1}$. The fourth section shows that an I(2) model gives maximum weight to the last observed growth and minimum to the oldest values. The fifth section compares these models in forecasting annual output growth for seven countries in the period 1960-91. The final section contains some conclusions.

REGRESSION ON TIME

Let us call Z_t the observed time series and let us assume for simplicity that the sample size is $n = 2m + 1$. Let $t = \{-m, \dots, 0, \dots, +m\}$. Then the least squares estimator of the slope in the regression on time

$$Z_t = \beta_0 + \beta_1 t + u \quad (1)$$

is given by

$$\hat{\beta}_1 = \frac{\sum tZ_t}{\sum t^2} = \left(2 \sum_{i=1}^m i^2\right)^{-1} \sum_{t=1}^m t(Z_t - Z_{-t}) \quad (2)$$

Calling $b_t = Z_t - Z_{t-1}$, for $t = -m+1, \dots, m$, the observed growth at each period, we note that

$$Z_t - Z_{-t} = \sum_{j=-t+1}^{j=t} b_j$$

and, after some straightforward manipulations that are shown in Appendix 1, the estimate of the slope can be written as

$$\hat{\beta}_1 = \sum_{j=1}^m \omega_j (b_j + b_{1-j}) \quad (3)$$

where the weights are given by

$$\hat{\omega}_j = \frac{3(j+m)(m-j+1)}{2(2m+1)m(m+1)} \quad j = 1, \dots, m \quad (4)$$

and add up to one. Therefore the estimated growth $\hat{\beta}_1$ is a weighted mean of all the observed growths b_j , such that the maximum weights are given to b_1 and b_0 that correspond to the observed growth in the middle of the sample period, and the minimum weights are given to b_m and b_{1-m} , the first and last observed growth.

The estimator (3) has an interesting interpretation. On the assumption that the linear model (1) holds, the $2m$ values b_t ($t = -m+1, \dots, m$) are unbiased estimates for β . These estimates are

correlated and have covariances

$$\begin{aligned}\text{Cov}(b_t, b_{t+1}) &= E[(b_t - \beta)(b_{t+1} - \beta)] = E[(u_t - u_{t-1})(u_{t+1} - u_t)] = -\sigma^2 \\ \text{Cov}(b_t, b_{t+j}) &= 0 \quad j > 1\end{aligned}$$

Therefore, the covariance matrix of these $2m$ estimates is the Toeplitz matrix:

$$\mathbf{V} = \begin{bmatrix} 2\sigma^2 & -\sigma^2 & 0 & \dots & 0 \\ -\sigma^2 & 2\sigma^2 & . & & \\ . & . & . & & \\ . & . & . & . & -\sigma^2 \\ 0 & \dots & \dots & -\sigma^2 & 2\sigma^2 \end{bmatrix} \quad (5)$$

It is easy to show (Newbold and Granger, 1974) that given a vector $\hat{\theta}$ of unbiased estimators of a parameter θ with covariance matrix \mathbf{V} , the best (in the mean squared sense) linear unbiased estimator of θ given by

$$\hat{\theta}_T = (1' \mathbf{V}^{-1} 1)^{-1} (1' \mathbf{V}^{-1} \hat{\theta}) \quad (6)$$

where $1' = (1 \ 1 \ \dots \ 1)$. Now, the inverse of the Toeplitz matrix (5) has been studied by Shaman (1969), who obtained the exact inverse of a first-order moving average process. As \mathbf{V} can be interpreted as the covariance matrix of a non-invertible ($\theta = 1$) first-order moving average process, then $\mathbf{V}^{-1} = \{v_{ij}\}$ will be given by

$$v_{ij} = \frac{i(2m-j+1)}{2m+1} \quad j > i, \quad i = 1, \dots, 2m$$

and $v_{ij} = v_{ji}$. Therefore

$$\mathbf{V}^{-1} = \frac{1}{(2m+1)} \begin{bmatrix} 2m & 2m-1 & 2m-2 & \dots & 1 \\ 2m-1 & 2(2m-1) & 2(2m-2) & \dots & 2 \\ 2m-2 & 2(2m-2) & 3(2m-2) & \dots & 3 \\ \vdots & \vdots & \vdots & & \\ 2 & 4 & 6 & \dots & 2m-1 \\ 1 & 2 & 3 & \dots & 2m \end{bmatrix} \quad (7)$$

It is proved in Appendix 1 that the estimator (3) can also be obtained by applying (6) to the unbiased but correlated estimates b_t .

Suppose now that an ARMA model is fitted to the residuals of the regression model (1). Then the equation for the h -steps-ahead forecast is

$$\hat{Z}_t(h) = \hat{\beta}_0 + \hat{\beta}_1 h + \hat{n}_t(h)$$

where $\hat{n}_t(h)$ is the forecast of the zero mean stationary process fitted to the residuals. As for a stationary process the long-run forecast converges to the mean, $\hat{n}_t(h) \rightarrow 0$, and the parameter $\hat{\beta}_1$ is the long-run estimated growth of the time series.

FORECASTING GROWTH WITH AN I(1) MODEL

The ARIMA approach in modelling time series with trend is to differentiate the data and then fit a stationary ARMA process. Assuming that a difference is enough to obtain a stationary series,

that is, the series is integrated of order one or I(1), the fitted model is

$$\nabla Z_t = \beta + n_t \quad (8)$$

where n_t follows an ARMA model

$$n_t = \sum \psi_i a_{t-i} \quad (9)$$

The process $\{a_t\}$ is a Gaussian white-noise process and the series $\{\psi_i\}$ converge, so that n_t is a zero mean stationary process. Calling \mathbf{V} to the covariance matrix of n_t , the estimate of β in model (8) is given by the generalized least squares estimator

$$\hat{\beta} = (1' \mathbf{V}^{-1} 1)^{-1} (1' \mathbf{V}^{-1} b) \quad (10)$$

where the vector \mathbf{b} has components $b_t = \nabla Z_t$. Assuming that n_t is stationary and invertible it is well known (see Fuller, 1976) that $\bar{b} = 1/n \sum b_t$ is asymptotically unbiased for β with variance σ^2/n .

When n is large, the expected growth h periods ahead is given by

$$\beta_T(h) = E[Z_{T+h} - Z_{T+h-1}]$$

and it will be estimated by

$$\hat{\beta}_T(h) = \hat{\beta} + \hat{n}_t(h)$$

where $\hat{n}_t(h)$ is the h -steps-ahead forecast of the stationary process n_t . As for h large the $\hat{n}_t(h)$ will go to zero, the mean value forecast, the long-run growth will be estimated by $\hat{\beta}$. As

$$\hat{\beta} = \frac{1}{n} \sum b_t = \frac{1}{n} (Z_n - Z_1) \quad (11)$$

the long-run growth will be estimated simply by using the first and last observed values. Also, this estimate can be interpreted as a weighted average with uniform weighting of the observed growths b_t .

FORECASTING GROWTH WITH AN I(2) MODEL

Some economic time series require differencing twice to obtain a stationary model. Then the series is called integrated of order two or I(2), and the model used is

$$\nabla^2 Z_t = n_t \quad (12)$$

where

$$n_t = \sum \psi_i a_{t-i} \quad (13)$$

and the process $\{a_t\}$ is a Gaussian white-noise process. The series $\{\psi_i\}$ converge so that n_t is a zero mean stationary and invertible process. The h -steps-ahead forecast from model (12) can be written

$$\hat{Z}_t(h) = \hat{\beta}_0^{(t)} + \hat{\beta}_1^{(t)} h + \hat{n}_t(h) \quad (14)$$

where $\hat{\beta}_0^{(t)}$ and $\hat{\beta}_1^{(t)}$ depend on the origin of the forecast and $\hat{n}_t(h)$ is the h -steps-ahead forecast of the zero mean stationary process. Again, as the forecast $\hat{n}_t(h)$ will go to zero, the long-run growth will be estimated by $\hat{\beta}_1^{(t)}$. To understand the structure of $\hat{\beta}_1^{(t)}$ let us consider first the simplest case in which n_t follows an MA(1) process, $n_t = (1 - \theta B)a_t$. Then the forecast for any lag h is given by

$$\hat{Z}_t(h) = \hat{\beta}_0(t) + \hat{\beta}_1^{(t)} h \quad (15)$$

because $\hat{n}_t(1)$ is a constant. Let us obtain the form of $\hat{\beta}_1^{(t)}$ as a function of the observed growths ∇Z_t . Assuming that the origin is $t = T - 1$, then we can obtain $\hat{\beta}_0^{(T-1)}$ in model (15) using the two forecasts $\hat{Z}_{T-1}(1)$ and $\hat{Z}_{T-1}(2)$ as follows:

$$\begin{aligned}\hat{Z}_{T-1}(1) &= Z_{T-1} + \nabla Z_{T-1} - \theta a_{T-1} = \hat{\beta}_0^{(T-1)} + \hat{\beta}_1^{(T-1)} \\ \hat{Z}_{T-1}(2) &= 2Z_{T-1}^{(1)} - Z_{T-1} = \hat{\beta}_0^{(T-1)} + 2\hat{\beta}_1^{(T-1)}\end{aligned}$$

and subtracting the first equation from the second,

$$\hat{\beta}_1^{(T-1)} = \nabla Z_{T-1} - \theta a_{T-1} = b_{T-1} - \theta(1 - \theta B)^{-1} \nabla b_{T-1}$$

which leads to

$$\hat{\beta}_1^{(T-1)} = (1 - \theta)[b_{T-1} + \theta b_{T-2} + \theta^2 b_{T-3} \dots] \quad (16)$$

that is, the forecasted future growth is an exponentially weighted average of past observed growths.

In general, it is easy to show that

$$\hat{\beta}_1^{(T-1)} = \sum a_j b_{T-j}$$

where the a_j coefficients depend on the moving-average structure of the process and behave like the $\pi(B) = \psi(B)^{-1}$ structure of weights.

FORECASTING INTERNATIONAL GROWTH RATES

In order to illustrate the performance of the three models compared in this paper we have applied them to forecast gross national product for seven countries in the period 1960–91. Many sophisticated models have been used to forecast international growth rates and turning points. See, for instance, Garcia-Ferrer *et al.* (1987), Zellner and Hong (1989), Min and Zellner (1993), and the references in these papers. Our objective here is not to build the 'best' model to forecast annual output growth, but to illustrate the forecast accuracy for different forecast horizons of the three models analysed in the paper.

The data we use are given in Appendix 2 and represent gross national product in the period 1960–91 for the United States, Japan, and the five largest countries of the European Union (France, Germany, Italy, Spain, and the United Kingdom).

Four models are used for the logarithm of the gross national product. The first (M1) is the linear regression on time given by equation (1). The second (M2) is a random walk with drift, that is model (8) with $n_t = a_t$. The third is an IMA (2, 1) model, that is, model (12) with $n_t = (1 - \theta B) a_t$. The fourth is a random walk without drift on the rate of growth $\ln Y_t / Y_{t-1}$, where Y_t is gross national product, and it is equivalent to the third model with $\theta = 0$. Note that the forecast from any of these four models is a linear trend. In the first the slope is obtained by a given maximum weight to the observed growth in the middle of the sample; in the second the slope gives equal weight to all observed growths; in the third the weights decrease exponentially; and in the fourth only the last observed growth is taken into account to build the forecast. Therefore, M1 gives, relatively, minimum weight to the more recent data whereas M4 gives them the maximum weight.

We have used the period 1960–79 to fit the models and 1980–91 to check their forecasting performance. M1 has been fitted by least squares to the 20 points, the parameter β in M2 (see equation (8)) has been estimated with equation (11), and the parameter θ in M3 has been

estimated by maximum likelihood for the seven countries with the results given in Table I. M4 does not require any parameter estimation.

Table II presents the forecasting accuracy of the four models for three forecast horizons: one, two, and three steps ahead. The procedure used to build this table is as follows:

- (1) The fitted models were employed to generate twelve one-step-ahead forecasts, eleven two-step-ahead forecasts, and ten three-step-ahead forecasts for the years 1980–91. Models M1 and M2 were re-estimated to include all past data prior to the forecast origin. The parameter θ in M3 was always kept fixed to 0.7, the mean value for the seven countries (see Table I). We have checked that re-estimating the parameter θ with each new data improves very few results, but makes model M3 more expensive in computing time. In this way the updating of the forecast equation is very simple in all the models used. Finally, M4 does not need any parameter estimation.
- (2) The error of the three types of forecasts were computed for each of the seven countries. As an overall measure of accuracy we have used the mean squared error of the forecast. This measure has been computed for the three forecast horizons: one (S1), two (S2) and three (S3) steps-ahead.

It can be seen that for the one-step-ahead forecast I(2) models are the best in six of the seven countries. Only for the United States are the I(1) forecasts slightly better than those generated by the I(2) models. The I(2) models are also the best for two and three-steps-ahead forecasts for

Table I. Maximum likelihood estimation of the moving average parameter

	France	Germany	Italy	Japan	Spain	UK	USA
θ	0.62	0.84	0.83	0.52	0.3	0.95	0.93

Table II. Mean squared error of the one, (S1), two, (S2), and three, (S3), steps-ahead forecasts for the four models

	France					Germany			
	M1	M2	M3	M4		M1	M2	M3	M4
S1	0.0120	0.0003	0.0001	0.0001	S1	0.0066	0.0004	0.0003	0.0002
S2	0.0166	0.0012	0.0005	0.0006	S2	0.0093	0.0015	0.0011	0.0011
S3	0.0220	0.0026	0.0010	0.0011	S3	0.0125	0.0031	0.0024	0.0031
	Italy					Japan			
	M1	M2	M3	M4		M1	M2	M3	M4
S1	0.0084	0.0003	0.0002	0.0002	S1	0.0320	0.0005	0.0001	0.0002
S2	0.0120	0.0014	0.0011	0.0013	S2	0.0432	0.0019	0.0004	0.0007
S3	0.0162	0.0031	0.0025	0.0037	S3	0.0569	0.0043	0.0008	0.0014
	Spain					United Kingdom			
	M1	M2	M3	M4		M1	M2	M3	M4
S1	0.0252	0.0006	0.0002	0.0002	S1	0.0022	0.0007	0.0007	0.0005
S2	0.0340	0.0025	0.0009	0.0007	S2	0.0029	0.0019	0.0023	0.0024
S3	0.0435	0.0051	0.0022	0.0019	S3	0.0030	0.0031	0.0043	0.0061
	USA								
	M1	M2	M3	M4		M1	M2	M3	M4
S1	0.0032	0.0006	0.0007	0.0007					
S2	0.0043	0.0015	0.0019	0.0029					
S3	0.0054	0.0024	0.0033	0.0065					

five of the seven countries. However, for the United States and the UK the I(1) model provides better forecasts than the best I(2) model. As can be expected, the forecasting performance of the deterministic linear trend is very poor for all horizons.

The main conclusion from this exercise is that, on average, I(2) models, even the simplest ones that do not require any estimation, seem to be best for forecasting international growth in the countries considered in this example.

CONCLUSION

We have compared three time-series models in this paper. The three models forecast future growth by using a weighted average of the observed growths in the sample. Linear regression gives minimum weight to the last observed growth and maximum weight to the centre of the sample period. This implies that, for instance, if we use this method to forecast next year's gross national product (GNP) with a sample of 40 data, we are saying that the most informative item to forecast 1994 growth is the growth in 1974, whereas the last observed growth in 1993 receives a weight equal to the one in 1954. If we use an I(1) model, the growth is forecast by using a uniform weighting in all the years in the sample. In the GNP example the observed 1993 growth is as relevant as the one observed in 1960 or 1965 for forecasting 1994 growth. The logical requirement that the most relevant year to forecast GNP growth are the last observed growth is only accomplished by using an I(2) model. In particular, an ARIMA (0, 2, 1) model leads to an exponentially weighting of last observed growths.

Many econometric papers and some well-known books on time series (see, for instance, Brockwell and Davis, 1987, p. 25) use least squares regression on time as an alternative to differencing for removing a trend in a time series. However, the logical implications of both procedures are seldom analysed. It is important to stress that if a series follows an I(2) model but we detrend it by least squares regression on time, the residuals from this fit do not provide, in general, a sound basis for fitting an ARMA model, and the forecast performance of the procedure may be poor.

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APPENDIX 1

Using

$$\sum_{i=1}^m i^2 = \frac{(2m+1)m(m+1)}{6} \quad (\text{A1})$$

and

$$\sum_{t=1}^m t \sum_{j=t}^{-t+1} b_j = (b_1 + b_0) \sum_{i=1}^m i + (b_2 + b_{-1}) \sum_{i=2}^m i + \dots + (b_m + b_{1-m})m = \sum_{j=1}^m \left(\sum_{i=j}^m i \right) (b_j + b_{1-j})$$

we have

$$\hat{\beta} = \sum_{j=1}^m \omega_j (b_j + b_{1-j}) \quad (\text{A2})$$

where

$$\omega_j = \frac{3(j+m)(m-j+1)}{(2m+1)m(m+1)}$$

and the sum of all the weights ω_j adds up to one:

$$2 \sum_{j=1}^m \omega_j = \left(\sum_{i=1}^m i^2 \right)^{-1} \sum_{j=1}^m \sum_{i=j}^m i = 1$$

On the other hand, let $1' = (1, \dots, 1)$ be a vector of $2m$ ones. Then using equation (7)

$$1' \mathbf{V}^{-1} = (m, (2m-1), \frac{3}{2}(2m-2), \dots, \frac{i}{2}(2m-i+1), \dots, m)$$

and

$$1' \mathbf{V}^{-1} \mathbf{1} = \sum_{i=1}^{2m} \frac{i}{2} (2m-i+1) = \frac{m(2m+1)(m+1)}{3}$$

Therefore the estimate is given by

$$\hat{\beta} = \sum_{i=1}^{2m} \frac{3i(2m-i+1)}{m(2m+1)(m+1)} b_{i-m}$$

However,

$$\begin{aligned}\hat{\beta} &= \sum_{i=1}^{2m} \frac{3i(2m-i+1)}{m(2m+1)(m+1)} b_{i-m} + \sum_{j=1}^m \frac{3(m+j)(m-j+1)}{2m(2m+1)(m+1)} b_j = \sum_{j=1}^m \frac{3(m-j+1)(m+j)}{2m(2m+1)(m+1)} b_{1-j} \\ &\quad + \sum_{j=1}^m \frac{3(m+j)(m-j+1)}{2m(2m+1)(m+1)} b_j \\ &= \sum_{j=1}^m \omega_j (b_j + b_{1-j})\end{aligned}$$

in agreement with equation (A.2).

APPENDIX 2

Year	France	Germany	Italy	Japan	Spain	UK	USA
1960	1 843 797	856 480	299 823	63 530	9 997 409	201 042	1 864 700
1961	1 945 321	891 267	322 014	68 758	11 268 160	207 630	1 854 445
1962	2 075 117	933 451	341 991	74 883	12 388 150	209 774	1 952 558
1963	2 186 077	962 240	363 885	84 091	13 577 180	217 831	2 096 600
1964	2 328 572	1 026 340	374 066	93 910	14 255 234	229 627	2 213 900
1965	2 439 832	1 081 450	386 290	99 376	15 146 669	235 432	2 336 100
1966	2 567 050	1 111 960	409 409	109 947	16 244 187	239 850	2 473 400
1967	2 687 399	1 108 750	438 795	122 132	16 949 235	245 261	2 536 400
1968	2 801 853	1 169 990	467 514	137 866	18 067 357	255 247	2 641 400
1969	2 997 718	1 257 090	496 023	155 068	19 676 720	260 529	2 715 800
1970	3 169 517	1 321 400	522 366	171 674	20 512 121	266 433	2 714 400
1971	3 320 319	1 361 160	530 750	178 993	21 465 753	271 734	2 791 800
1972	3 455 696	1 419 120	545 077	193 712	23 215 030	281 202	2 934 400
1973	3 642 994	1 488 190	583 826	208 484	25 023 181	301 667	3 086 600
1974	3 741 918	1 492 080	615 520	207 197	26 429 127	296 503	3 068 400
1975	3 729 687	1 471 220	599 197	213 123	26 572 397	294 236	3 043 500
1976	3 893 818	1 549 800	638 619	222 098	27 450 289	302 201	3 193 800
1977	4 031 470	1 593 910	660 174	232 566	28 229 612	309 155	3 336 400
1978	4 167 773	1 641 640	684 480	243 891	28 642 478	320 439	3 490 000
1979	4 300 036	1 709 170	725 461	257 390	28 654 512	329 601	3 579 200
1980	4 359 708	1 727 510	756 197	266 741	29 027 187	322 432	3 563 800
1981	4 411 784	1 730 520	760 366	276 287	28 975 987	318 195	3 632 900
1982	4 513 213	1 714 140	761 991	285 022	29 429 760	323 576	3 551 800
1983	4 548 895	1 740 900	769 370	292 721	30 082 958	335 406	3 675 000
1984	4 616 372	1 789 350	790 036	305 208	30 524 354	343 232	3 900 700
1985	4 700 143	1 823 180	810 580	320 419	31 321 697	356 096	4 016 649
1986	4 813 350	1 863 770	834 262	328 839	32 323 992	370 860	4 119 600
1987	4 917 827	1 890 280	860 422	342 340	34 147 515	388 750	4 243 300
1988	5 126 894	1 960 510	895 397	363 592	35 910 027	405 512	4 410 600
1989	5 320 876	2 027 330	921 714	380 735	37 611 409	414 053	4 525 300
1990	5 440 386	2 130 500	942 271	400 627	38 973 365	416 206	4 555 000
1991	5 501 546	2 209 640	955 817	418 347	39 839 726	406 842	4 496 100

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