A DECISION ANALYSIS MODEL FOR A SERIOUS MEDICAL PROBLEM*

DANIEL PEÑA SANchez DE RIVERA†

This paper presents a decision model for a serious medical problem: the diagnosis and treatment of undifferentiated liver disease with jaundice. The model formalizes the use of information before a treatment is chosen, taking account of prior information collected by the doctor from laboratory and clinical exploration. Then the model chooses the best treatment according to the patient's preference structure.

Since the best treatment in each case depends on the patient's preference for consequences, this aspect is central to the application of such models. Thus a main objective is to find a suitable criterion to measure the consequences in order that each patient's attitude can be taken into account. Our model was computerized and tested with fifty patients: the program duplicated in forty-four cases the decisions of expert doctors.

The model overcomes some of the difficulties observed in the manipulation of probabilities by clinicians. The results suggest that a Decision Analysis model may be a useful way to clarify the decision process of expert clinicians and to help in the education of new doctors. Finally, this kind of program can play a role in automating medical decision-making in such a way that the knowledge of the best experts can be made widely available.

(DECISION ANALYSIS—APPLICATIONS; UTILITY/PREFERENCE—APPLICATIONS)

1. Introduction

The application of Decision Analysis to clinical diagnosis and treatment is a fairly recent development. In 1967, Henscheke [6] published an application to cancer therapy. Other contributions can be found in [1], [4], [10], [11] and [13]. The reader is referred to these authors for a general overview of the problem and here we will only stress some of its main features.

First, diagnosis and treatment must be analyzed together. Medical diagnosis is probabilistic and does not, in general, determine the best treatment to follow. Second, the best treatment not only depends on the doctor's uncertainty but also on the preference structure of the patient. Third, the value of a medical test is linked to its usefulness in permitting the doctor to take better decisions and not merely to its capacity to reduce uncertainty; though these aims have many features in common, they are not identical. The value of a medical test must be evaluated in each particular case.

An important obstacle to the application of Decision Analysis in medical problems is the lack of an adequate criterion for measuring consequences. In business decision problems we are used to summarizing the consequences in terms of money. In such cases it is not difficult to find a preference structure consistent with the decision-maker's views. In medical decision problems it would appear that money cost is not a natural numeraire of consequences. We have tried in this paper to introduce the length of life as the numeraire for this kind of problem.

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In §2 we describe our medical problem and present the reasons that suggest the application of Decision Analysis. In §3 we sketch the main features of the model, discussing the assessment of probabilities in 3.1, the measurement of consequences in 3.2 and the evaluation of consequences in 3.3. §4 is dedicated to presenting some results of our experimentation with the model. Finally, in §5, several conclusions about the usefulness of this approach will be drawn.

2. The Medical Problem

The differential diagnosis of a jaundiced patient, whose disease does not become clear after routine (nonrisky) exploration and laboratory tests, is a very difficult task and one of the classic problems in Medicine. The difficulty arises because there are six possible diseases which mimic each other very closely. The diseases are:

1. Cholestatic hepatitis (HC),
2. Primary biliary cirrhosis (CBP),
3. Choledocho lithiasis (CL),
4. Carcinoma of bile ducts (CBD),
5. Carcinoma of the papilla of Vater (CPV),
6. Carcinoma of head of pancreas (CHP).

Differential diagnosis is important, first of all, because the treatment required for hepatitis and cirrhosis is medical, whereas the treatment for the other diseases is surgical. Secondly, the prognosis in the case of carcinoma depends on its rapid identification, so that any delay before surgical intervention increases the patient's risk. Thirdly, the differential diagnosis of these diseases cannot, generally speaking, be based merely on those sources of information with minimum mortality risk such as laboratory tests and radiology; it is necessary to apply tests that have a significant risk of mortality: liver biopsy and percutaneous transhepatic cholangiography. Finally, the results of treatment in patients with carcinoma, of the bile ducts or of the head of the pancreas, are poor and there is no general agreement among doctors as to the usefulness of surgery, especially in the case of older patients.

This problem has many features that suggest the application of Decision Analysis. The cost of uncertainty is high and the patient's preference structure should be central in obtaining the best strategy.

3. The Model

3.1. Basic Structure

The model is built on the following assumptions: (1) The patient has one and only one of the six possible diseases; (ii) The doctor can use two risky tests before applying treatment: liver biopsy (LB) and percutaneous transhepatic cholangiography (PTC); (iii) The treatments available are divided in two groups: medical and surgical.

The model structure is presented in Figure 1. The first decision the doctor has to make is whether or not to collect more information using one of the two risky tests. If a test is applied, one of the test results presented in Tables 1 and 2 will be obtained. Table 1 shows the mutually exclusive and exhaustive results for the liver biopsy test and Table 2 contains those of the PTC test. When the test results become known, the doctor will change his probability assessment over the diseases, taking account of the likelihood of the result obtained according to Bayes' rule.
TABLE 1

<table>
<thead>
<tr>
<th>Result</th>
<th>Possible Diseases given the result</th>
</tr>
</thead>
<tbody>
<tr>
<td>(h): Hepatitis</td>
<td>HC</td>
</tr>
<tr>
<td>(c): Cirrhosis</td>
<td>CBP</td>
</tr>
<tr>
<td>(e): Extrahepatic</td>
<td>CL, CBD, CPV, CHP</td>
</tr>
<tr>
<td>(d): Inconclusive</td>
<td>HC, CBP, CL, CBD, CPV, CHP</td>
</tr>
</tbody>
</table>

TABLE 2

<table>
<thead>
<tr>
<th>Result</th>
<th>Possible Diseases given the result</th>
</tr>
</thead>
<tbody>
<tr>
<td>(cbd): Carcinoma bile ducts</td>
<td>CBD</td>
</tr>
<tr>
<td>(ca): Carcinoma, head of pancreas or Papilla of Vater</td>
<td>CPV, CHP</td>
</tr>
<tr>
<td>(cl): Choledocholithiasis</td>
<td>CL</td>
</tr>
<tr>
<td>(ex): Extrahepatic</td>
<td>CL, CBD, CPV, CHP.</td>
</tr>
<tr>
<td>(ns): Not specific</td>
<td>HC, CBP, CL, CBD, CPV, CHP</td>
</tr>
<tr>
<td>(nv): Test provides no information</td>
<td>HC, CBP, CL, CBD, CPV, CHP</td>
</tr>
</tbody>
</table>

MT: Medical Treatment  
PTC: Percutaneous Transhepatic Cholangiography  
ST: Surgical Treatment  
LB: Liver Biopsy  
$r_{p}(k)$: Result of PTC.  
$r_{B}(j)$: Result of LB  
$D_i$: Diseases

Figure I.
The doctor, before choosing the treatment, can use both tests sequentially, one of them alone or no test at all. The final consequence for the patient will depend on the test used, the treatment applied and the real disease the patient has.

To analyse the problem we need: (i) the “a priori” probability for each disease, (ii) the likelihoods of the test results, given the disease and (iii) the preference structure over the consequences. With these inputs the model can be used to choose the strategy that maximizes expected utility. In the following sections we describe these inputs.

3.2. Probabilities

The model uses three kinds of probability assessment: the a priori probabilities, the likelihood of the test results and the conditional probabilities of the consequences given the treatment and disease.

The a priori probabilities summarize the information that the doctor has before starting the decision-making process outlined in Figure 1, that is, before applying either diagnostic tests with positive mortality risk or treatment. The doctor could rely on programs for sequential diagnosis, if these are available. Such programs compute posterior probabilities for each disease, given the medical history, physical symptoms and laboratory results for the given patient and using, as likelihoods, the relative frequency of these items of information in a large sample of patients who have suffered from the relevant diseases. The doctor could also make use of standard methods of assessing subjective probability distributions, as in [5],[7] and [10], to supplement or even substitute for the use of such programs.

The conditional probabilities for each test result, given the disease, which we will call the likelihoods of each test, written \( P_j(r|i) \), where \( j \) indicates the test, \( i \) the disease and \( r \) the test result, were collected from historical experience at the Hospital Francisco Franco in Madrid and from cases published in medical journals. The particular data used as input in our model are given in Tables 3 and 4.

### TABLE 3

**Likelihood Matrix for Liver Biopsy**

<table>
<thead>
<tr>
<th>Disease</th>
<th>Test Result</th>
<th>(h)</th>
<th>(c)</th>
<th>(e)</th>
<th>(d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HC</td>
<td>0.20</td>
<td>0.00</td>
<td>0.00</td>
<td>0.80</td>
<td></td>
</tr>
<tr>
<td>CBP</td>
<td>0.00</td>
<td>0.80</td>
<td>0.00</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td>CL</td>
<td>0.00</td>
<td>0.00</td>
<td>0.75</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>CBD</td>
<td>0.00</td>
<td>0.00</td>
<td>0.75</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>CPV</td>
<td>0.00</td>
<td>0.00</td>
<td>0.75</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>CHP</td>
<td>0.00</td>
<td>0.00</td>
<td>0.75</td>
<td>0.25</td>
<td></td>
</tr>
</tbody>
</table>

### TABLE 4

**Likelihood Matrix For Percutaneous Transhepatic Cholangiography**

<table>
<thead>
<tr>
<th>Disease</th>
<th>Test Result</th>
<th>(nv)</th>
<th>(ns)</th>
<th>(cbd)</th>
<th>(cl)</th>
<th>(ca)</th>
<th>(cx)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HC</td>
<td>0.260</td>
<td>0.740</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>CBP</td>
<td>0.260</td>
<td>0.740</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>CL</td>
<td>0.260</td>
<td>0.007</td>
<td>0.636</td>
<td>0.666</td>
<td>0.067</td>
<td>0.067</td>
<td>0.067</td>
</tr>
<tr>
<td>CBD</td>
<td>0.260</td>
<td>0.037</td>
<td>0.636</td>
<td>0.666</td>
<td>0.067</td>
<td>0.067</td>
<td>0.067</td>
</tr>
<tr>
<td>CPV</td>
<td>0.260</td>
<td>0.007</td>
<td>0.00</td>
<td>0.666</td>
<td>0.067</td>
<td>0.067</td>
<td>0.067</td>
</tr>
<tr>
<td>CHP</td>
<td>0.260</td>
<td>0.007</td>
<td>0.00</td>
<td>0.666</td>
<td>0.067</td>
<td>0.067</td>
<td>0.067</td>
</tr>
</tbody>
</table>

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The probabilities of each disease, given the information available at each stage of the process, are represented in a state vector $X$ with the same number of components as the number of diseases.

At the beginning of the decision tree in Figure 1, the state vector contains the a priori probabilities based on clinical exploration, laboratory tests and radiology. We define a likelihood vector of the result $r$ for the test $j$, $P_j(r \mid \cdot)$, as the vector whose components are the likelihoods of result $r$ for each disease. that is, as a column from Table 3 or Table 4.

Then: (i) $P_j(r \mid \cdot)$ has as many components as there are diseases; (ii) $P_j(r \mid i) > 0 \ \forall i$;\[\\begin{array}{ccc}
\forall i \exists P_j(r) > 0, j = 1, 2.\end{array}\]

We define the transfer matrix of the test $j$ given the result $r$, $MT_j(r)$, as the diagonal matrix which has the components of $P_j(r \mid \cdot)$ in the main diagonal.

$$MT_j(r) = \begin{bmatrix}
P_j(r|1) & 0 & \ldots & 0 \\
0 & P_j(r|2) & 0 & \ldots \\
\vdots & \vdots & \ddots & \vdots \\
0 & \ldots & 0 & P_j(r|n)
\end{bmatrix}$$

The state vector after the test $j$ is carried out and the result $r$ obtained can be given by:

$$X = \frac{1}{X_0 \cdot P_j(r \mid \cdot)} \left[ X_0 \cdot MT_j(r) \right]$$  \(1\)

where $X_0$ is the state vector before the test and $X$ the state vector after the result $r$, the product $X_0 \cdot P_j(r \mid \cdot)$ is an inner product whereas $[X_0 \cdot MT_j(r)]$ is a matrix product.

Equation (1) merely represents Bayes’ rule in a convenient form for the computer.

### 3.3. The Consequences

The evaluation of consequences will depend on whether we take the point of view of the patient, of the doctor or of society. In our model, the patient’s point of view is chosen. We assume that the monetary cost, paid in the Spanish case by the Seguridad Social (Spanish Social Security Service), is not taken into account by the patient. Thus, only the risk and other nonmonetary consequences linked to each disease-treatment couple will be considered.

We have considered two methods for measuring the consequences. First, we have experimented with the approach suggested by Betaque [1], which summarizes all of the possible results in three groups: (i) patient’s condition improved, (ii) no change in patient’s condition and (iii) patient experiences a serious complication. According to Betaque’s method, if the result of a treatment is other than an improvement in the patient’s condition, the doctor continues to confront a decision problem. The process must be recycled in this case to determine a new test-treatment policy.

When we try to build a preference function following Betaque’s approach, several problems come to light. First, according to medical opinion, the meaning of the consequence depends very much on the patient’s age, a factor not included in that approach. Second, a change in the decision horizon, a factor not made explicit in Betaque’s approach, may change the result completely; doctors consulted insist that an
improvement in the patient's condition within fifteen days is not the same as improvement within a month or three months. Third, there is a risk of suboptimization, because the model does not consider the decisions that necessarily ensue when there is no change in the patient's condition or when a serious complication arises. Fourth, we should take into account the fact that knowledge with certainty of the patient's disease is potentially rewarding. In some cases, if there is no change in the patient's condition after treatment, the doctor can identify the patient's disease exactly while with other treatments the same consequences do not yield full information about the patient's situation. It appears that this fact ought to be included in the model but it is not clear how such an extension of Betaque's approach can be developed. Finally, the author's experience indicates that the description of the consequences in terms of Betaque's three categories may be difficult or confusing in some cases, hampering the transmission between physicians of experience that could lead to improvements in the model.

These difficulties lead us to take a new approach. There are four conditions that a meaningful system for the measurement of consequences must fulfill. First of all, the horizon for the consequences must be defined either by recovery or by death. Second, the patient's age must be explicitly taken into account. Third, it should be possible to establish a common numeraire for the consequences that allows us to define objective rules and to identify different categories of patients, taking into account their risk attitudes. Fourth, the procedure must be operational.

These considerations suggest that we specify the consequences in terms of:

- length of life,
- condition of life during the illness (absence or presence of pain, length of time in bed, etc.)

To obtain the length of life linked to a terminal point of the decision tree in Figure 1, we proceed as follows. We assume that any person of age \( x \) has a probability distribution of remaining length of life, \( pdl \) in what follows, that depends only on his sex and on the general state of health in the country in which he lives. To simplify, we take the year as time unit and work with discrete measures. Then, the \( pdl \) is easily calculated from a life table. Let \( q_x \) by the mortality rate for a person aged \( x \) according to the life table of his population, that is the probability of death within one year for an average person of age \( x \). Let \( P(x | x_0) \) be the conditional probability of death at age \( x \) for a person now of age \( x_0 \). Then:

\[
P(x | x_0) = q_x (1 - q_{x-1})(1 - q_{x-2}) \cdots (1 - q_{x_0}) \quad (x > x_0),
\]

\[
P(x_0 | x_0) = q_{x_0} \quad (x = x_0).
\]

Let us call this \( pdl \) the "general" \( pdl \) for people of age \( x_0 \) in this population.

Suppose now that a person age \( x_0 \) begins suffering from the illness \( D \). If this disease has mortality risk, then this person will experience an unfavorable change in his \( pdl \) that will depend on the disease as well as the treatment. Let \( p_{x, D, T} \) be the incremental probability due to the disease \( D \) for person aged \( x \), when the treatment \( T \) is applied. We have made the following assumptions about the doctor's behavior in order to evaluate these probabilities: (i) If the illness does not require surgical treatment but this is applied erroneously, the doctor discovers the mistake and then goes to medical treatment. The effect of the surgical procedure in this case is to increase the mortality rate of the illness due to time lost in correct diagnosis. (ii) If the patient does not recover within a given time when the medical treatment is incorrectly applied, the
doctor will undertake surgery. The resulting delay in applying the correct treatment increases the risk of mortality.

With these assumptions, we can evaluate the probability of dying \( P_{x, D, T} \) for each disease-treatment couple. We have used historical data from published cases when the correct treatment is applied. When the wrong treatment is chosen first, we have used subjective evaluation from expert clinicians, given the lack of published cases. In some diseases, choledocholithiasis or hepatitis for example, \( P_{x, D, T} \) is zero after some \( x \) because of the fact that, if the patient does not die, he will recover completely. In other diseases, cirrhosis or carcinoma for example, there is a risk of mortality that lasts for a longer time. Let \( p_T \) be the incremental mortality rate for treatments. We assume that \( p_T = 0 \) for medical procedures. When surgery is applied, there is always a risk of sudden death. We have taken this risk as independent of the illness. To measure this probability we have obtained subjective evaluations from five doctors and take the average value of 0.05 in our model. This value was calculated assuming that the anesthetic used does not produce hepatic damage.

The probability distributions of death that are needed in the model have to summarize all of the causes of death. We make the basic hypothesis that every patient aged \( x \) faces the “general” mortality rate \( q_x \) associated with this age. When this person contracts illness \( D \) and receives treatment \( T \), he faces an additional incremental mortality rate \( p_{x, D, T} \). The events “general death”, “disease death” and “treatment death” are taken as mutually exclusive. This hypothesis is reasonable if none of the possible diseases has a strong influence on the rate of “general” mortality. Thus we in fact assume that the probability of death within one year for a person of age \( x \) for any cause other than disease and treatment is the “general” mortality risk for this age, \( q_x \).

Let us call \( t_x \) the total mortality rate for a person age \( x \) with disease \( D \) and treatment \( T \). Then:

\[
t_x = q_x + p_{x, D, T} + p_T \quad (x = x_0),
\]

\[
t_x = q_x + p_{x, D, T} \quad (x > x_0),
\]

where \( x_0 \) is the age of the patient at the time of the analysis. We have taken the age of 45 as our standard age, since it is in approximately this age group that the differential diagnosis of jaundiced patients is most difficult.

We write \( P_{D, T}(x \mid x_0) \) for the conditional pdl for the patient with disease \( D \) and treatment \( T \):

\[
P_{D, T}(x \mid x_0) = t_x (1 - t_{x-1}) \ldots (1 - t_{x_0}) \quad (x > x_0)
\]

\[
P_{D, T}(x_0 \mid x_0) = t_{x_0} \quad (x = x_0),
\]

where \( x_0 \) is the standard age.

Table 5 shows the mean of the twelve basic distributions for each of the six diseases and two treatments. Obviously, the expected value of these distributions is the life expectancy for each disease-treatment couple for our standard patient.

Under a linear preference structure for the consequences, the differences between the correct and the incorrect treatment is a global measure of the opportunity cost for each disease. It is clear from Table 5 that, under the linear preference structure, the doctor will try mainly to distinguish the HC, that requires medical treatment, from CL and CPV, that require surgery.

We measure the effect of each test by its associated probability of death: 0.002 for
biopsy and 0.006 for cholangiography. In this way the cost of the test for a particular patient, measured by expected reductions of length of life, depends on the patient’s age. Therefore the cost of the test is not constant for all patients. We believe this method of introducing the cost of information in our model to be quite realistic.

The pdf for the triplet: test $M$, treatment $T$, disease $D$, for a patient aged $x$ is:

$$p_{M,D,T}(x_0|x_0) = p_M + (1 - p_M)p_{D,T}(x_0|x_0) \text{ for } x = x_0,$$

$$p_{M,D,T}(x|x_0) = (1 - p_M)p_{D,T}(x|x_0) \text{ for } x > x_0,$$

where $p_M$ is the mortality of the test.

We attempted to quantify the “conditions of life” factors in our model by means of a two-stage procedure, relying on the responses of doctors familiar with the typical discomfort factors associated with these diseases and treatments, who attempted to simulate patient responses. First, we tried to establish the trade-off between the number of days in bed with different degrees of discomfort (high, medium and low), following Raiffa [10]. Second, we attempted to establish the trade-off between length-of-life probabilities and number of days in bed under standard discomfort conditions. We found that the preference orderings for this second choice problem were very close to lexicographic. Therefore, we decided to ignore “conditions of life” factors in the main analysis and treat them by means of sensitivity analysis once an optimal strategy was found.

3.4. Preferences

The preference structure for our problem is defined over the remaining length of life of the patient. Two kinds of preference structure were considered: (i) linear preferences and (ii) decreasing risk aversion. The patient with linear preferences would, of course, select that treatment which maximizes his life expectancy. Risk aversion in this context would imply that the patient will prefer a treatment $T_1$ with expected life $E_1$ to another treatment $T_2$ with expected life $E_2$, even if $E_1 < E_2$, if the probability distribution of death in the short run with $T_1$ is sufficiently lower than with $T_2$. Decreasing risk aversion implies that this relative preference goes down when the expected life with both treatments is increased to the same degree. Linear preferences are especially interesting to consider, because they define a “neutral” doctor’s attitude and, from the point of view of resource allocation for medical planning, might rule the behavior of society as a whole. Risk aversion seems to be a quite common individual preference structure. This attitude might explain why many people are not willing to undergo surgery even when their life expectancy would increase. In the author’s experience,
decreasing risk aversion for length of life is a very common attitude among people with risk aversion.

To build a preference curve with decreasing risk aversion, we have adapted the lottery technique [10] to our problem in order to obtain the input points for a program of the Schlaifer type [12], which constructs a decreasing-risk-aversion curve for a person, once three points are fixed. In order to show how our model works, we have used as an example a curve built with three points obtained from a doctor showing risk aversion. Our approach was as follows. First, we assumed that the preference index for the “general” pdl is always unity. With this fact in mind, we asked this person the following question: “You suffer from a disease that will certainly cause your death in exactly N years, if you do not receive medical treatment. This treatment is not painful, and does not last long, but you will face a 1 – p probability of sudden death due to the treatment. If you survive the treatment, then you will be restored to full health with certainty. Will you accept this treatment?”

The question, which is sketched in Figure 2, was chosen so that only the values of p and N, come into the evaluations. We modified the numbers p and N until obtaining the indifferent point. Then, the utility or preference for N years of life was set: \( u(N) = p \). With this procedure the input points for Schlaifer’s program [12] were: (0, 0 years), (0.25, 2 years), (0.5, 6 years), (0.75, 15 years), (1, 40 years). The corresponding curve was used throughout the analysis whenever we assumed decreasing risk aversion.

\[ P(x \mid x_0): \text{probability distribution of life of a healthy person aged } x_0, \ u(0) = 0, \ u[P(x \mid x_0)] = 1. \]

\[ \text{Figure 2.} \]

Table 6 displays the results of applying this curve to the same probability distributions that are reflected in the life expectancies of Table 5. The final utility is re-expressed in terms of remaining years of life. We call this quantity the life certainty equivalent, LCE.

As a test of Betaque’s procedure, we have used his method to obtain the preferences from the same person whose answers led to Table 6. The new scoring of the disease-treatment couple does not show the same ranking, indicating an inconsistency between the two methods. However, when we extend the horizon of analysis for Betaque’s approach and assume that the doctor learns from the response of the patient to treatment and shifts to the correct treatment after some period of time, the ranking of the disease-treatment couples is the same as that of Table 6. In our opinion, this...
suggests that there is a risk of suboptimization when the doctor does not consider the whole chain of decisions that he will have to make when the first treatment chosen does not produce recovery of the patient.

4. Experiments with the Model

The study of the strategies recommended by the model when the patient has a non-zero a priori probability for more than three of the six possible diseases, reveals a large number of admissible strategies. Furthermore it is difficult to state simple decision rules to represent the doctor's optimal strategy for each possible a priori state vector. The only feature that can be established with generality is that it is always better to begin with biopsy than with cholangiography, if some test is to be made. This is due to the lower mortality risk of biopsy and, although the cholangiography is, generally speaking, more informative, producing a greater reduction of entropy on average, its greater mortality risk is not compensated by the reduction in uncertainty.

Although the principal use for this model is as a computerized decision-making tool, we have tried to obtain simple decision rules for the more common situations. We have studied the case of the differential diagnosis of hepatitis (HC) and choledocholithiasis (CL) in detail, because this case is very frequent and also involves a high uncertainty cost, as we have seen in Tables 3 and 4.

Table 7 displays the best strategy for this problem according to the values of $P(\text{CL})$, the a priori probability of choledocholithiasis. One interesting result is that, out of fourteen decision strategies that the doctors consulted considered as available, only five are ever optimal. The analysis assumes a linear preference structure for the consequences.

As a test of the strategies indicated by the model, we have compared its results with what is generally considered good medical practice. We consider fifty hypothetical patients. Some of these cases were based on real patients, but in the other cases unusual features were introduced to cover the maximum range of possibilities. These cases were presented to three clinicians with a wide experience in this field. The agreement between the model and the doctors was very good in 44 out of 50 cases. In these 44 cases, the strategy chosen by the doctor and that of the model were identical or did not differ by more than three months of life expectancy, a difference on average of only 1% in life expectancy.

To measure the reliability of the answers, a consistency test was introduced, based on the following principle: the doctor's strategy at any point of the analysis should depend solely on the state vector at this point. Then, we introduced patients in whom the state vector, after some test result, was identical to the a priori state vector for
<table>
<thead>
<tr>
<th>$P(\text{CL}) = p$</th>
<th>Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \leq p \leq 0.05$</td>
<td>Medical Treatment (MT)</td>
</tr>
<tr>
<td></td>
<td>(h)→MT</td>
</tr>
<tr>
<td></td>
<td>(e)→ST</td>
</tr>
<tr>
<td></td>
<td>(d)→MT</td>
</tr>
<tr>
<td>$0.05 \leq p \leq 0.625$</td>
<td>Biopsy:</td>
</tr>
<tr>
<td></td>
<td>(b)→MT</td>
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<tr>
<td></td>
<td>(e)→ST</td>
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<tr>
<td></td>
<td>(d)→Cholangiography: (nv)→MT</td>
</tr>
<tr>
<td></td>
<td>(n)→MT</td>
</tr>
<tr>
<td></td>
<td>any other case ST</td>
</tr>
<tr>
<td>$0.625 \leq p \leq 0.74$</td>
<td>Biopsy:</td>
</tr>
<tr>
<td></td>
<td>(b)→MT</td>
</tr>
<tr>
<td></td>
<td>(e)→ST</td>
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<td></td>
<td>(d)→Cholangiography: (nv)→ST</td>
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<tr>
<td></td>
<td>(n)→MT</td>
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<tr>
<td></td>
<td>any other case ST</td>
</tr>
<tr>
<td>$0.74 \leq p \leq 0.9$</td>
<td>Biopsy:</td>
</tr>
<tr>
<td></td>
<td>(b)→MT</td>
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<td></td>
<td>(e)→ST</td>
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<td>(d)→Cholangiography: (nv)→ST</td>
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<td></td>
<td>(n)→MT</td>
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<tr>
<td></td>
<td>any other case ST</td>
</tr>
<tr>
<td>$0.9 \leq p \leq 1$</td>
<td>Surgical Treatment (ST)</td>
</tr>
</tbody>
</table>

MT: Medical Treatment; ST: Surgical Treatment; (h), (e), (d) results of biopsy (see Table 1); (nv), (n) results of cholangiography (Table 2).

another patient. In three cases out of the six where agreement between the model and the doctors was not reached, the physicians were not consistent in the above sense. Discussing this point with the doctors, we came to the conclusion that the modification of a priori opinion, given the test result, is less strong than expected from Bayes’ rule. This “inertia effect” or “conservatism effect” has been found elsewhere [9].

The disagreement in the other three cases may be due to “diagnosis illusion”. By this, we mean that doctors choose a strategy that leads rapidly to an exact knowledge of the disease even though this is not optimal on the grounds of the patient's expected length of life. This might be due to the maximization of doctors expected utility instead of patients. Future research on this problem might reveal whether this effect is widespread in medical practice or not.

The strategies given by the model with decreasing risk aversion were identical in 39 out of 50 cases with those generated under linear preferences. In the eleven cases in which the decision strategy was not the same, we found the same pattern: the patient with decreasing risk aversion is not willing to take a test with low mortality or to go through a surgical treatment even though his life expectancy would increase with it. This kind of behavior agrees with the behavior expected.

### 5. Conclusions

The main conclusion of this paper is that the length of life as described in this article is a useful criterion for summarizing the relevant effects of a medical policy.

Furthermore, the total probability distribution of length of life offers an objective criterion to evaluate the different treatments. The approach presented in section 3.2 shows that this pdt depends on the patient’s age and sex and thus the best treatment does also. For example, in our case, the surgical treatment is optimal for carcinoma of the head of the pancreas if the patient has linear preferences and is a male under 54 or...
female under 59, but it is not necessarily optimal in other cases. Furthermore, the \textit{pdf}\ depends on the general health situation of the population, i.e. on the life expectancy for healthy people. Thus the optimal medical strategy for a given community need not be optimal for another population with a very different “general” probability distribution of life. Finally, the length-of-life criterion allows us to establish objective decision rules based on linear preference structures and to link this kind of analysis with the general problem of health resources allocation.

This paper also confirms the useful role of Decision Analysis in the study of serious medical problems. First, Decision Analysis, as in other areas, offers a systematic methodology for the structuring and analysis of complex problems in which differences in expert opinions can be understood and to some extent resolved through the explicit introduction of subjective probabilities and preferences. Second, this approach may well be effective in training new doctors. Schwartz [13] has said, “The reward and evaluation system in medical schools is based largely on one’s store of information rather than on one’s decisions-making capability”. This method permits the storage of information in a computer and puts the emphasis on the physician’s decision process. Third, this methodology provides a guide to hospitals on the kinds of information to collect systematically and its relative importance as far as decisions are concerned. Fourth, this approach can be easily linked to automatic-diagnosis programs that have been developed over the last two decades.

Further research on the trade-off between length and conditions of life might well expand the field of application for this methodology.\footnote{The author is indebted to Arthur B. Treadway for helpful comments on an earlier draft of this paper.}

References