Measuring Intervention Effects on Multiple Time Series Subjected to Linear Restrictions: A Banking Example

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We consider the problem of estimating the effects of an intervention on a time series vector subjected to a linear constraint. Minimum variance linear and unbiased estimators are provided for two different formulations of the problem—(1) when a multivariate intervention analysis is carried out and an adjustment is needed to fulfill the restriction and (2) when a univariate intervention analysis was performed on the aggregate series obtained from the linear constraint, previous to the multivariate analysis, and the results of both analyses are required to be made compatible with each other. A banking example that motivated this work illustrates our solutions.

KEY WORDS: Accounting constraint; Linear estimators; Multivariate intervention; Restricted estimation; VARMA models..

Intervention analysis is nowadays a standard technique to study the effects of known events on a time series. It was proposed by Box and Tiao (1975) within the context of autoregressive integrated moving average (ARIMA) models. Since then, this technique has been extended to vector autoregressive moving average (VARMA) models by Abraham (1980) and to structural time series models by Harvey and Durbin (1986). Box and Tiao (1976) provided a test statistic to check if a given intervention has produced a statistically significant impact on a univariate time series. This test was generalized to multiple time series by Aczél (1992).

In this article we consider a multiple intervention analysis of a time series vector whose elements satisfy some exact linear restrictions. This situation was found in the study that motivated this work. A banking institution carried out a promotional campaign of its deposit and savings service and wanted to identify the possible effects on (a) new accounts, (b) stock variations, (c) cancellations, and (d) total amount. Because the increase in total amount from the previous to the current month is given by (1) + (2) − (3), a linear restriction is satisfied by the vector of time series every month. This fact should be taken into consideration, first to achieve accounting-consistent results and second to make use of all the information available to increase the statistical efficiency of the analysis. Similar situations may be faced when studying macroeconomic time series like gross domestic product (GDP) at the sectorial level (the sum of the sectors equals total GDP) or a consumer price index (CPI) at a disaggregate level (a weighted average of the disaggregated indexes equals the general CPI).

This article is organized as follows. Section 1 presents two optimal solutions to the problem, valid under different assumptions. These solutions are then illustrated by means of some theoretical examples. Section 2 is dedicated to an empirical application involving a set of data that refers to the aforementioned banking situation. Section 3 concludes with some final remarks.

1. OPTIMAL LINEAR ESTIMATION OF INTERVENTION EFFECTS

Let $\mathbf{Z}_t = (z_{1t}, \ldots, z_{kt})'$ be a vector of k time series variables observed at equispaced intervals of time $t = 1, \ldots, N$. We shall assume that $\{\mathbf{Z}_t\}$ admits the VARMA model

$$\Phi(B)(\mathbf{Z}_t - \boldsymbol{\mu}_t) = \Theta(B)\mathbf{a}_t, \tag{1}$$

where $\boldsymbol{\mu}_t = (\mu_{1t}, \ldots, \mu_{kt})'$ is a vector representing the level of the multiple time series and $\mathbf{a}_t = (a_{1t}, \ldots, a_{kt})'$ is a zero-mean white-noise process with positive definite variance–covariance matrix $\sum_a$. The matrix polynomials in the backshift operator B (such that $B\mathbf{Z}_t = \mathbf{Z}_{t-1}$ for every $\mathbf{Z}$ and t) are given by $\Phi(B) = I_k - \Phi_1 B - \cdots - \Phi_p B^p$ and $\Theta(B) = I_k - \Theta_1 B - \cdots - \Theta_q B^q$. The determinantal equation $|\Theta(x)| = 0$ has all its roots outside the unit circle. An alternative way of expressing (1) is as

$$\mathbf{Z}_t - \boldsymbol{\mu}_t = \Psi(B)\mathbf{a}_t, \tag{2}$$

where $\Psi(B)$ denotes an infinite polynomial satisfying $\Phi(B)\Psi(B) = \Theta(B)$. If $|\Phi(x)| = 0$ has all its roots outside the unit circle, $\{\mathbf{Z}_t\}$ is said to be stationary and (2) is well defined. Otherwise the process is nonstationary, and we shall assume that the generating process started at some

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finite time point in the past, with fixed initial conditions, for (2) to be well defined.

Let $X = \{\mathbf{Z}_1, \ldots, \mathbf{Z}_{T-1}\}$ be the dataset observed before the specific time point $t = T$ at which an inter- vention occurs. Then let $\mathbf{Z}^{(\mathrm{WI})} = (\mathbf{Z}_T^{(\mathrm{WI})'}, \ldots, \mathbf{Z}_N^{(\mathrm{WI})'})'$ be the vector of unobserved variables without intervention (WI) effects from time T onward. It is well known that $E(\mathbf{Z}_{T-1+h}^{(\mathrm{WI})}|X)$ provides the linear forecast of $\mathbf{Z}_{T-1+h}^{(\mathrm{WI})}$ with origin at $t = T-1, h$ steps ahead, with minimum MSE (mean squared error) and

$$\mathbf{Z}_{T-1+h}^{(\mathrm{WI})} - E(\mathbf{Z}_{T-1+h}^{(\mathrm{WI})}|X) = \sum_{j=0}^{h-1} \Psi_j \mathbf{a}_{T-1+h-j} \quad \text{for } h = 1, 2, \ldots, \tag{3}$$

where the Ψ_j's are the matrix coefficients appearing in $\Psi(B)$. This equation holds valid for both stationary and nonstationary processes, on the assumption of no change in the future behavior of $\{\mathbf{Z}_t\}$. When an intervention takes place at $t = T$, (3) becomes

$$\mathbf{Z}_{T-1+h} - E(\mathbf{Z}_{T-1+h}|X) = \boldsymbol{\eta}_{T-1+h} + \sum_{j=0}^{h-1} \Psi_j \mathbf{a}_{T-1+h-j} \quad \text{for } h = 1, 2, \ldots, \tag{4}$$

where $\boldsymbol{\eta}_{T-1+h}$ denotes the k vector of intervention effects at time $t = T-1+h$, for $h = 1, 2, \ldots$. Every element of this vector is assumed to follow its own dynamics and can be written as

$$(1 - \delta_{i,1}B - \cdots - \delta_{i,r_i}B^{r_i})(1-B)^{b_i}\eta_{it} = (\omega_{i,0} + \omega_{i,1}B + \cdots + \omega_{i,s_i}B^{s_i})P_t^{(T)} \tag{5}$$

for $i = 1, \ldots, k$, where $P_t^{(T)} = 1$ if $t = T$ and $P_t^{(T)} = 0$ otherwise. Besides, r_i, b_i, and s_i, as well as the δ's and ω's, are coefficients that depend only on the observed behavior of the ith variable. In particular, it is assumed that $(1 - \delta_{i,1}x - \cdots - \delta_{i,r_i}x^{r_i}) = 0$ has its roots outside the unit circle, for $i = 1, \ldots, k$.

Now let us note that $E(\mathbf{Z}_{T-1+h}|X) = E(\mathbf{Z}_{T-1+h}^{(\mathrm{WI})}|X)$ because X does not convey any information about the intervention. Thus, (3) and (4) can be written as

$$\begin{aligned} \mathbf{Z}^{(\mathrm{WI})} - E(\mathbf{Z}^{(\mathrm{WI})}|X) &= \Psi\mathbf{a} \\ \text{and} \quad \mathbf{Z} - E(\mathbf{Z}^{(\mathrm{WI})}|X) &= \boldsymbol{\eta} + \Psi\mathbf{a}, \end{aligned} \tag{6}$$

where $\mathbf{Z} = (\mathbf{Z}_T', \ldots, \mathbf{Z}_N')'$, and similar definitions hold for $\mathbf{Z}^{(\mathrm{WI})}, \boldsymbol{\eta}$, and $\mathbf{a}$. The $k(N-T+1) \times k(N-T+1)$ matrix Ψ is lower triangular with I_k on its main diagonal and Ψ_i on its ith subdiagonal, for $i = 1, \ldots, N-T$.

We suppose that a univariate time series $\{y_t\}$ is also observed and is related to $\{\mathbf{Z}_t\}$ by

$$y_t = \mathbf{c}'\mathbf{Z}_t \quad \text{for } t = 1, \ldots, N, \tag{7}$$

with $\mathbf{c} = (c_1, \ldots, c_k)' \neq \mathbf{0}$ a known constant vector that defines the form of an accounting constraint among the multiple time series under study. The set of restrictions (7) applying for $t = T, \ldots, N$ can be expressed as

$$\mathbf{Y} = C\mathbf{Z} \quad \text{with } C = I_{N-T+1} \otimes \mathbf{c}', \tag{8}$$

where $\mathbf{Y} = (y_T, \ldots, y_N)', \otimes$ denotes Kronecker product, and C is a full-rank matrix.

The problem lies in estimating the intervention effects $\boldsymbol{\eta}$, where

$$\mathbf{Z} = \mathbf{Z}^{(\mathrm{WI})} + \boldsymbol{\eta}. \tag{9}$$

These effects will be represented by the linear form

$$\boldsymbol{\eta} = L\boldsymbol{\beta}, \tag{10}$$

where L is a $k(N-T+1) \times l$ matrix of coefficients that depends on the polynomial in the left side of (5) and $\boldsymbol{\beta}$ is a fixed vector of dimension $l \leq k + s_1 + \cdots + s_k$ containing the ω's. Of course, the estimation procedure must take into consideration the restrictions (8). In fact, (8) precludes the application of a multivariate intervention analysis on the augmented time series vector $\{(\mathbf{Z}_t', y_t)'\}$, which, at first sight, could be deemed appropriate, but the estimation would not be possible because of the accounting relationship $(\mathbf{c}', -1)(\mathbf{Z}_t', y_t)' = 0$.

1.1 Primary Solution

We suggest carrying out a preliminary estimation of the ω's, the δ's, and the VARMA model parameters of $\{\mathbf{Z}_t\}$, disregarding the restrictions (8). This joint estimation provides the matrices Ψ and Σ_a, as well as the values of the δ's, all of which are henceforth assumed known. The preliminary estimation of the ω's produces an estimated vector $\mathbf{b} = (\hat{\omega}_{1,0}, \ldots, \hat{\omega}_{1,s_1}, \ldots, \hat{\omega}_{k,0}, \ldots, \hat{\omega}_{k,s_k})'$ and its corresponding (positive definite) estimated variance–covariance matrix $\widehat{\mathrm{var}}(\mathbf{b}) = \Sigma_u$. The $\mathbf{b}$ estimator is unbiased, although inefficient, because it does not take into account the restriction. It is related to $\boldsymbol{\beta}$ by means of

$$\mathbf{b} = \boldsymbol{\beta} + \mathbf{u}, \tag{11}$$

where $\mathbf{u}$ is a random-error term uncorrelated with $\mathbf{Z}$ such that $E(\mathbf{u}) = \mathbf{0}$ and $\mathrm{var}(\mathbf{u}) = \Sigma_u$. Even though our main interest is to estimate $\boldsymbol{\beta}$, we realize that (9) and (10) imply

$$\mathbf{Z} = \mathbf{Z}^{(\mathrm{WI})} + L\boldsymbol{\beta} \tag{12}$$

so that $\mathbf{Z}^{(\mathrm{WI})}$ and $\boldsymbol{\beta}$ are linked together and must be estimated simultaneously. Similarly, (6) allows us to write

$$E(\mathbf{Z}^{(\mathrm{WI})}|X) = \mathbf{Z}^{(\mathrm{WI})} + \mathbf{e}, \tag{13}$$

with $\mathbf{e} = -\Psi\mathbf{a}$ such that $\mathrm{var}(\mathbf{e}|X) = \Psi(I_{N-T+1} \otimes \Sigma_a)\Psi' = \Sigma_e$ and $E(\mathbf{e}\mathbf{u}'|X) = 0$.

We now put Equations (13) and (11) together to form the system

$$\begin{pmatrix} E(\mathbf{Z}^{(\mathrm{WI})}|X) \\ \mathbf{b} \end{pmatrix} = \begin{pmatrix} \mathbf{Z}^{(\mathrm{WI})} \\ \boldsymbol{\beta} \end{pmatrix} + \begin{pmatrix} \mathbf{e} \\ \mathbf{u} \end{pmatrix}, \tag{14}$$

and (8) can also be expressed as

$$\mathbf{Y} = (C \;\; CL)\begin{pmatrix} \mathbf{Z}^{(\mathrm{WI})} \\ \boldsymbol{\beta} \end{pmatrix}. \tag{15}$$

This situation is related to the restricted-parameter estimation problem of linear regression (see, for instance, Seber 1977), although here we are not just estimating fixed parameters but also forecasting a random vector. A general solution to this problem is provided in the Appendix for completeness. Thus, (14)–(15) are of the form (A.1) in the Appendix, with $\Sigma_Y = 0$ and $H = (C\ CL)$, an $(N-T+1)\times[k(N-T+1)+l]$ matrix with rank $N-T+1$, so that expressions (A.2)–(A.3) can be used and yield the following results:

$$\begin{aligned}\hat{\mathbf{Z}}^{(\mathrm{WI})} &= E(\mathbf{Z}^{(\mathrm{WI})}|X) + \Sigma_e C'\Lambda^{-1} \\ &\quad \times [\mathbf{Y} - CE(\mathbf{Z}^{(\mathrm{WI})}|X) - CL\mathbf{b}] \\ &= A_z(\mathbf{Y} - CL\mathbf{b}) + (I_{k(N-T+1)} - A_z C) \\ &\quad \times E(\mathbf{Z}^{(\mathrm{WI})}|X) \end{aligned} \tag{16}$$

and

$$\begin{aligned}\hat{\beta} &= \mathbf{b} + \Sigma_u L'C'\Lambda^{-1}[\mathbf{Y} - CE(\mathbf{Z}^{(\mathrm{WI})}|X) - CL\mathbf{b}] \\ &= A_\beta[\mathbf{Y} - CE(\mathbf{Z}^{(\mathrm{WI})}|X)] + (I_l - A_\beta CL)\mathbf{b}, \end{aligned} \tag{17}$$

where

$$\begin{aligned}\Lambda &= C\Sigma_e C' + CL\Sigma_u L'C', \quad A_z = \Sigma_e C'\Lambda^{-1}, \\ \text{and } A_\beta &= \Sigma_u L'C'\Lambda^{-1}. \end{aligned} \tag{18}$$

Furthermore, the MSE matrix $\Sigma_z = E[(\hat{\mathbf{Z}}^{(\mathrm{WI})} - \mathbf{Z}^{(\mathrm{WI})})(\hat{\mathbf{Z}}^{(\mathrm{WI})} - \mathbf{Z}^{(\mathrm{WI})})']$, the variance–covariance matrix $\Sigma_\beta = \mathrm{var}(\hat{\beta})$, and the covariance matrix $\Sigma_{z\beta} = E[(\hat{\mathbf{Z}}^{(\mathrm{WI})} - \mathbf{Z}^{(\mathrm{WI})})(\hat{\beta} - \beta)']$ is given by

$$\begin{aligned}\Sigma_z &= (I_{K(N-T+1)} - A_z C)\Sigma_e, \\ \Sigma_\beta &= (I_l - A_\beta CL)\Sigma_u, \\ \text{and } \Sigma_{z\beta} &= -A_z CL\Sigma_u. \end{aligned} \tag{19}$$

In summary, to use this solution in practice we should apply the following two-step procedure: (1) Carry out a preliminary multivariate intervention analysis on $\{\mathbf{Z}_t\}$; that is, estimate first a VARMA model with data from $t = 1$ up to $t = T-1$ and obtain $E(\mathbf{Z}^{(\mathrm{WI})}|X)$, then include the intervention effects in the model and reestimate it with all the available data to obtain $\Psi, \Sigma_a, \Sigma_e, L, \mathbf{b}$, and Σ_u. (2) Take into account the restrictions among the series by revising the estimates using (16)–(19).

Remarks: (1) The precision of $\hat{\beta}$ is at least as high as that of b because $\Sigma_u - \Sigma_\beta$ is a positive semidefinite matrix. (2) The restriction $\mathbf{Y} = C(\hat{\mathbf{Z}}^{(\mathrm{WI})} + L\hat{\beta})$ is fulfilled by our solution, as required by (15). It does not follow, however, that $\mathbf{Z} = \hat{\mathbf{Z}}^{(\mathrm{WI})} + L\hat{\beta}$, even though (12) assumes that it happens with the true values. Therefore, $\hat{\mathbf{Z}}^{(\mathrm{WI})}$ should not be considered as a proper series adjusted for intervention effects. (3) In case we were indeed interested in imposing as restriction (a) $\mathbf{Z} = \mathbf{Z}^{(\mathrm{WI})} + L\beta$ instead of just the accounting constraint (b) $\mathbf{Y} = C(\mathbf{Z}^{(\mathrm{WI})} + L\beta)$, we must realize that (a) implies (b) for any matrix C. Therefore (a) is more restrictive than (b), which holds only for a particular C matrix. Of course, the corresponding solution for (a) can be obtained by the result in the Appendix, in the same way we obtained (16)–(19), and it is given by (16) and (17) when $C = I$ and $\mathbf{Y} = \mathbf{Z}$.

1.2 An Alternative Solution

We now consider a different formulation of the problem that takes into account the results of a univariate intervention analysis performed on the series $\{y_t\}$ of (7). We may need this solution when, for instance, the univariate time series $\{y_t\}$ has been analyzed first, obtaining the $(N-T+1)$ vector $\hat{\eta}_y$ of estimated intervention effects. Afterwards, a complementary multivariate intervention analysis of $\{\mathbf{Z}_t\}$ is carried out, and due to the link between $\{y_t\}$ and $\{\mathbf{Z}_t\}$, we want the results to be compatible with those obtained from the univariate analysis. To do so, we add the additional (unbinding) restriction

$$\hat{\eta}_y = CL\beta + \varepsilon, \tag{20}$$

where ε is a zero-mean random vector uncorrelated with $\mathbf{Z}^{(\mathrm{WI})}$ and $(\mathbf{e}', \mathbf{u}')$ of (14). The (positive definite) variance–covariance matrix $\mathrm{var}(\varepsilon) = \Sigma_\varepsilon$ is known and given by $\Sigma_\varepsilon = \widehat{\mathrm{var}}(\hat{\eta}_y)$. We consider now the system (14) together with the new set of restrictions

$$\begin{pmatrix} \mathbf{Y} \\ \hat{\eta}_y \end{pmatrix} = \begin{pmatrix} C & CL \\ 0 & CL \end{pmatrix} \begin{pmatrix} \mathbf{Z}^{(\mathrm{WI})} \\ \beta \end{pmatrix} + \begin{pmatrix} \mathbf{0} \\ \varepsilon \end{pmatrix} \tag{21}$$

in place of the previous (15). Then we realize that (14) and (21) are such that the two equations pertaining to β—that is, (11) and (20)—do not involve the known datasets in (13) and (15). Therefore, we can first estimate β by applying the result in the Appendix to (11) and (20); that is,

$$\begin{aligned}\hat{\beta}^A &= \mathbf{b} + \Sigma_u L'C'(CL\Sigma_u L'C' + \Sigma_\varepsilon)^+(\hat{\eta}_y - CL\mathbf{b}) \\ &= A_\beta^A \hat{\eta}_y + (I_l - A_\beta^A CL)\mathbf{b}, \end{aligned} \tag{22}$$

with

$$\begin{aligned}\Sigma_\beta^A &= (I_l - A_\beta^A CL)\Sigma_u \\ A_\beta^A &= \Sigma_u L'C'(CL\Sigma_u L'C' + \Sigma_\varepsilon)^+, \end{aligned} \tag{23}$$

where the superscript A stands for alternative solution and $+$ for Moore–Penrose (generalized) inverse.

Once $\hat{\beta}^A$ and Σ_β^A have been obtained, we can write $\beta = \hat{\beta}^A + \varepsilon_\beta$, where ε_β is a zero-mean random vector uncorrelated with e and $\mathbf{Z}^{(\mathrm{WI})}$ and with $\mathrm{var}(\varepsilon_\beta) = \Sigma_\beta^A$. Hence the first row of (21) becomes

$$\mathbf{Y} - CL\hat{\beta}^A = C\mathbf{Z}^{(\mathrm{WI})} + CL\varepsilon_\beta, \tag{24}$$

and using (13) we obtain (by the result in the Appendix)

$$\begin{aligned}\hat{\mathbf{Z}}^{(\mathrm{WI})A} &= E(\mathbf{Z}^{(\mathrm{WI})}|X) + \Sigma_e C'(C\Sigma_e C' + CL\Sigma_\beta^A L'C')^{-1} \\ &\quad \times [\mathbf{Y} - CL\hat{\beta}^A - CE(\mathbf{Z}^{(\mathrm{WI})}|X)] \\ &= A_z^A(\mathbf{Y} - CL\hat{\beta}^A) \\ &\quad + (I_{k(N-T+1)} - A_z^A C)E(\mathbf{Z}^{(\mathrm{WI})}|X), \end{aligned} \tag{25}$$

with

$$\begin{aligned}\Sigma_z^A &= (I_{k(N-T+1)} - A_z^A C)\Sigma_e \\ A_z^A &= \Sigma_e C'(C\Sigma_e C' + CL\Sigma_\beta^A L'C')^{-1}. \end{aligned} \tag{26}$$

Thus, to apply this solution in practice, we suggest a two-step procedure as with the primary solution. In the first step we now require an additional univariate intervention analysis of $\{y_t\}$ basically to obtain $\hat{\boldsymbol{\eta}}_y$ and Σ_ε.

Remarks: (1) None of the restrictions imposed by $\hat{\boldsymbol{\eta}}_y$ and $\mathbf{Y}$ are binding; therefore, they are not fulfilled exactly by $\hat{\boldsymbol{\beta}}^{\mathrm{A}}$ and $\hat{\mathbf{Z}}^{(\mathrm{WI})\mathrm{A}}$. (2) A variant of this alternative solution might be of interest in some cases in which the accounting constraint must be met exactly. In such a case all we have to do is to assume that once $\hat{\boldsymbol{\beta}}^{\mathrm{A}}$ has been obtained it has a fixed value so that $\boldsymbol{\varepsilon}_\beta = \mathbf{0}$ in (24). As a result, Expressions (25)–(26) remain valid with $\Sigma_\beta^{\mathrm{A}} = 0$. (3) We should emphasize that the univariate intervention analysis was assumed to be carried out with no awareness of the behavior of $\{\mathbf{Z}_t\}$. Therefore, there may be some cases in which (20) would not make sense in the light of the information provided by $\{\mathbf{Z}_t\}$—for instance, when an intervention effect is distinguishable on a particular series element of $\{\mathbf{Z}_t\}$ that has a small weight on the aggregate series so that such an effect is not appreciable on $\{y_t\}$. In that case we suggest either revising the univariate analysis, paying attention specifically to the intervention in consideration, or else discarding the univariate results and employing our primary solution.

1.3 Some Illustrative Theoretical Examples

To shed some light on the estimates produced by the previous solutions, let us consider a very simple example of a bivariate series driven by a white-noise process; that is,

$$\begin{pmatrix} z_{1t} \\ z_{2t} \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} + \begin{pmatrix} a_{1t} \\ a_{2t} \end{pmatrix} \quad \text{with } \Sigma_a = \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix}.$$

Suppose that both series are subjected to an intervention producing a pulse effect at time T and that the aggregate series is obtained as $y_t = z_{1t} + z_{2t}$. Here we have $\Psi = I_2$; $E(\mathbf{Z}^{(\mathrm{WI})}|X) = (\mu_1, \mu_2)'$ for $t = T, T+1, \ldots, N$; $\boldsymbol{\eta} = L\boldsymbol{\beta}$ with $L = (I_2, O_2, \ldots, O_2)'$, and $\boldsymbol{\beta} = (\omega_{10}, \omega_{20})'$, while $\mathbf{Y} = C\mathbf{Z}$ with $C = I_{N-T+1} \otimes \mathbf{c}'$ and $\mathbf{c}' = (1, 1)$. A preliminary intervention analysis on $\{\mathbf{Z}_t\}$ produces $\mathbf{b} = (\tilde{\omega}_{10}, \tilde{\omega}_{20})$ so that Expression (11) holds, and let us call

$$\Sigma_u = \begin{pmatrix} \sigma_{(1)}^2 & \sigma_{(12)} \\ \sigma_{(12)} & \sigma_{(2)}^2 \end{pmatrix}.$$

Then, by calling $\sigma_0^2 = \sigma_1^2 + 2\sigma_{12} + \sigma_2^2, \sigma_{(0)}^2 = \sigma_{(1)}^2 + 2\sigma_{(12)} + \sigma_{(2)}^2, \alpha_i = (\sigma_i^2 + \sigma_{12})/(\sigma_0^2 + \sigma_{(0)}^2), \alpha_{(i)} = (\sigma_{(i)}^2 + \sigma_{(12)})/(\sigma_0^2 + \sigma_{(0)}^2)$, and $\delta_i = (\sigma_i^2 + \sigma_{12})/\sigma_0^2$ for $i = 1, 2$, we can write

$$A_z = \begin{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} & O_{2\times(N-T)} \\ \mathbf{O}_{2(N-T)} & I_{N-T} \otimes \begin{pmatrix} \delta_1 \\ \delta_2 \end{pmatrix} \end{pmatrix}$$

$$A_\beta = \begin{pmatrix} \alpha_{(1)} \\ \alpha_{(2)} \end{pmatrix} (1, 0, \ldots, 0),$$

and using (16) we obtain

$$\hat{\mathbf{Z}}_T^{(\mathrm{WI})} = \begin{pmatrix} \mu_1 + \alpha_1(y_T - \mu_1 - \mu_2 - \tilde{\omega}_{10} - \tilde{\omega}_{20}) \\ \mu_2 + \alpha_2(y_T - \mu_1 - \mu_2 - \tilde{\omega}_{10} - \tilde{\omega}_{20}) \end{pmatrix}$$

and

$$\hat{\mathbf{Z}}_t^{(\mathrm{WI})} = (\ \mu_1 + \delta_1(y_T - \mu_1 - \mu_2)\mu_2 + \delta_2(y_T - \mu_1 - \mu_2)\ ) \quad \text{for } t = T+1, T+2, \ldots, N.$$

Similarly, the intervention parameters are estimated as

$$\hat{\boldsymbol{\beta}} = \begin{pmatrix} \tilde{\omega}_{10} + \alpha_{(1)}(y_T - \mu_1 - \mu_2 - \tilde{\omega}_{10} - \tilde{\omega}_{20}) \\ \tilde{\omega}_{20} + \alpha_{(2)}(y_T - \mu_1 - \mu_2 - \tilde{\omega}_{10} - \tilde{\omega}_{20}) \end{pmatrix} = \begin{pmatrix} \hat{\omega}_{10} \\ \hat{\omega}_{20} \end{pmatrix}$$

with variance–covariance matrix

$$\Sigma_\beta = \begin{pmatrix} \sigma_{(1)}^2 + \alpha_{(1)}^2(\sigma_0^2 + \sigma_{(0)}^2) & - \\ \sigma_{(12)} - \alpha_{(1)}\alpha_{(2)}(\sigma_0^2 + \sigma_{(0)}^2) & \sigma_{(2)}^2 - \alpha_{(2)}^2(\sigma_0^2 + \sigma_{(0)}^2) \end{pmatrix}.$$

A few comments about these results are now in order. On the one hand, because $\alpha_1 + \alpha_{(1)} + \alpha_2 + \alpha_{(2)} = 1$, we see that $\hat{z}_{1T}^{(\mathrm{WI})} - \hat{\omega}_{10} + \hat{z}_{2T}^{(\mathrm{WI})} - \hat{\omega}_{20} = y_T$ and similarly as $\delta_1 + \delta_2 = 1$, we get $\hat{z}_{1T}^{(\mathrm{WI})} + \hat{z}_{2T}^{(\mathrm{WI})} = y_t$ for $t = T+1, \ldots, N$. Thus, the restriction imposed by the aggregated series is strictly fulfilled. On the other hand, both $\hat{\omega}_{10}$ and $\hat{\omega}_{20}$ take into account the preliminary estimates and adjust them by adding a weighted discrepancy between $y_T - \mu_1 - \mu_2 = [z_{1T} - E(z_{1T}^{(\mathrm{WI})}|X)] + [z_{2T} - E(z_{2T}^{(\mathrm{WI})}|X)]$ and $\tilde{\omega}_{10} + \tilde{\omega}_{20}$. The involved weights, $\alpha_{(1)}$ and $\alpha_{(2)}$ are ratios that increase (decrease) with respect to $\sigma_0^2 + \sigma_{(0)}^2$. Furthermore, the variances of $\hat{\omega}_{10}$ and $\hat{\omega}_{20}$ are clearly smaller than those of $\tilde{\omega}_{10}$ and $\tilde{\omega}_{20}$.

Let us suppose now that a univariate intervention analysis has been carried out on $\{y_t\}$, as well as a bivariate intervention analysis on $\{\mathbf{Z}_t\}$. Then the alternative solution is called for with $\hat{\boldsymbol{\eta}}_y = (\omega, 0, \ldots, 0)'$ and $\Sigma_\varepsilon = \operatorname{diag}(\sigma_\omega^2, 0, \ldots, 0)$ given. In this case we observe that

$$(CL\Sigma_u L'C' + \Sigma_\varepsilon)^+ = (\sigma_{(0)}^2 + \sigma_\omega^2)^{-1} \begin{pmatrix} 1 & - \\ \mathbf{O}_{N-T} & O_{(N-T)\times(N-T)} \end{pmatrix}.$$

Hence, if we call $\delta_{(i)} = (\sigma_{(i)}^2 + \sigma_{(12)})/(\sigma_{(0)}^2 + \sigma_\omega^2)$ for $i = 1, 2$, we obtain

$$A_\beta^{\mathrm{A}} = \begin{pmatrix} \delta_{(1)} \\ \delta_{(2)} \end{pmatrix} (1, 0, \ldots, 0);$$

thus

$$\hat{\boldsymbol{\beta}}^{\mathrm{A}} = \begin{pmatrix} \tilde{\omega}_{10} + \delta_{(1)}(\omega - \tilde{\omega}_{10} - \tilde{\omega}_{20}) \\ \tilde{\omega}_{20} + \delta_{(2)}(\omega - \tilde{\omega}_{10} - \tilde{\omega}_{20}) \end{pmatrix} = \begin{pmatrix} \hat{\omega}_{10}^{\mathrm{A}} \\ \hat{\omega}_{20}^{\mathrm{A}} \end{pmatrix}$$

and

$$\Sigma_\beta^{\mathrm{A}} = \begin{pmatrix} \sigma_{(1)}^2 - \delta_{(1)}^2(\sigma_{(0)}^2 + \sigma_\omega^2) & - \\ \sigma_{(12)} - \delta_{(1)}\delta_{(2)}(\sigma_{(0)}^2 + \sigma_\omega^2) & \sigma_{(2)}^2 - \delta_{(2)}^2(\sigma_{(0)}^2 + \sigma_\omega^2) \end{pmatrix}.$$

Then, to obtain $\hat{\mathbf{Z}}^{(\mathrm{WI})\mathrm{A}}$, we first calculate

$$(C\Sigma_e C' + CL\Sigma_\beta^A L'C')^{-1} = \begin{pmatrix} (\sigma_{(0)}^2 + \sigma_\omega^2)/[\sigma_0^2\sigma_{(0)}^2 + \sigma_\omega^2(\sigma_0^2 + \sigma_{(0)}^2)] & - \\ \mathbf{O}_{N-T}' & \sigma_0^{-2} I_{N-T} \end{pmatrix}$$

and define $\delta_i^* = \delta_i \sigma_0^2(\sigma_{(0)}^2 + \sigma_\omega^2)/[\sigma_0^2\sigma_{(0)}^2 + \sigma_\omega^2(\sigma_0^2 + \sigma_{(0)}^2)]$ for $i = 1, 2$. Therefore we get

$$A_z^{\mathrm{A}} = \begin{pmatrix} \begin{pmatrix} \delta_1^* \\ \delta_2^* \end{pmatrix} & O_{2\times(N-T)} \\ \mathbf{O}_{2(N-T)} & I_{N-T} \otimes \begin{pmatrix} \delta_1 \\ \delta_2 \end{pmatrix} \end{pmatrix}$$

so that

$$\hat{\mathbf{Z}}_T^{(\mathrm{WI})\mathrm{A}} = \begin{pmatrix} \mu_1 + \delta_1^*(y_T - \mu_1 - \mu_2 - \hat{\omega}_{10}^{\mathrm{A}} - \hat{\omega}_{20}^{\mathrm{A}}) \\ \mu_2 + \delta_2^*(y_T - \mu_1 - \mu_2 - \hat{\omega}_{10}^{\mathrm{A}} - \hat{\omega}_{20}^{\mathrm{A}}) \end{pmatrix}$$

and

$$\hat{\mathbf{Z}}_t^{(\mathrm{WI})\mathrm{A}} = \begin{pmatrix} \mu_1 + \delta_1(y_T - \mu_1 - \mu_2) \\ \mu_2 + \delta_2(y_T - \mu_1 - \mu_2) \end{pmatrix}$$

$$\text{for } t = T+1, T+2, \ldots, N.$$

We can see here that $\hat{z}_{1T}^{(\mathrm{W})\mathrm{A}} + \hat{z}_{2T}^{(\mathrm{WI})\mathrm{A}} \neq y_T$ unless $\sigma_\omega^2 = 0$, in which case $\delta_i^* = \delta_i$ for $i = 1, 2$. As it happens with the primary solution, to obtain $\hat{\omega}_{10}^{\mathrm{A}}$ we start with $\tilde{\omega}_{10}$ and add a weighted discrepancy, in this case between ω and $(\tilde{\omega}_{10} + \tilde{\omega}_{20})$. Besides we notice that $(\sigma_\omega^2 + \sigma_{(0)}^2)\delta_{(i)} = (\sigma_0^2 + \sigma_{(0)}^2)\alpha_{(i)}$ for $i = 1, 2$ so that the weights involved in $\hat{\boldsymbol{\beta}}^{\mathrm{A}}$ are larger (smaller) than those used to get $\hat{\boldsymbol{\beta}}$ when σ_0^2 is larger (smaller) than σ_ω^2. A similar reasoning is applicable to the reduction (increase) in variance shown by Σ_β and $\Sigma_\beta^{\mathrm{A}}$. In particular, when $\sigma_\omega^2 = 0$ (see Remark 2 of the alternative solution), $\hat{\boldsymbol{\beta}}^{\mathrm{A}}$ turns out to be more efficient than $\hat{\boldsymbol{\beta}}$.

2. A BANKING APPLICATION

The data considered in this application were obtained from a banking institution (Bank X) and are presented here in disguised form for confidentiality reasons. They are monthly observations on new accounts $\{\mathrm{NA}_t\}$, stock variations $\{\mathrm{SV}_t\}$, cancelled accounts $\{\mathrm{CA}_t\}$, and total amount $\{\mathrm{TA}_t\}$ from January 1990 to April 1996. The intervention under study is a promotional campaign launched by Bank X in May 1994 and relaunched in March 1995. In Figure 1(a) we show $\{\mathrm{NA}_t\}, \{\mathrm{SV}_t\}$, and $\{\mathrm{CA}_t\}$ together, whereas Figure 1(b) shows the behavior of $\{\mathrm{TA}_t\}$, which is obtained as $\mathrm{TA}_t = \mathrm{TA}_{t-1} + \mathrm{NA}_t + \mathrm{SV}_t - \mathrm{CA}_t$. Thus, if we let $y_t = (1 - B)\mathrm{TA}_t, \mathbf{Z}_t = (\mathrm{NA}_t, \mathrm{SV}_t, \mathrm{CA}_t)'$, and $\mathbf{c} = (1, 1, -1)'$, we are in the situation considered in Section 1.

2.1 Application of the Primary Solution

As a first step, we built a VARMA model for $\{\mathbf{Z}_t\}$ with data from $t = 1$ up to $t = T - 1$, with $T = 53$ corresponding to May 1994. The resulting model was $(I_3 - \Phi_1 B - \Phi_2 B^2)(I_3 - \Phi_{12}B^{12})\mathbf{Z}_t = \boldsymbol{\nu} + \mathbf{a}_t$ and was estimated by maximum likelihood with the following results (note that $\boldsymbol{\nu} = \Phi(1)\boldsymbol{\mu}$):

$$\hat{\Phi}_1 = \begin{pmatrix} 0 & 0 & 0 \\ .139(.063) & 0 & -.744(.300) \\ 0 & 0 & .308(.138) \end{pmatrix},$$

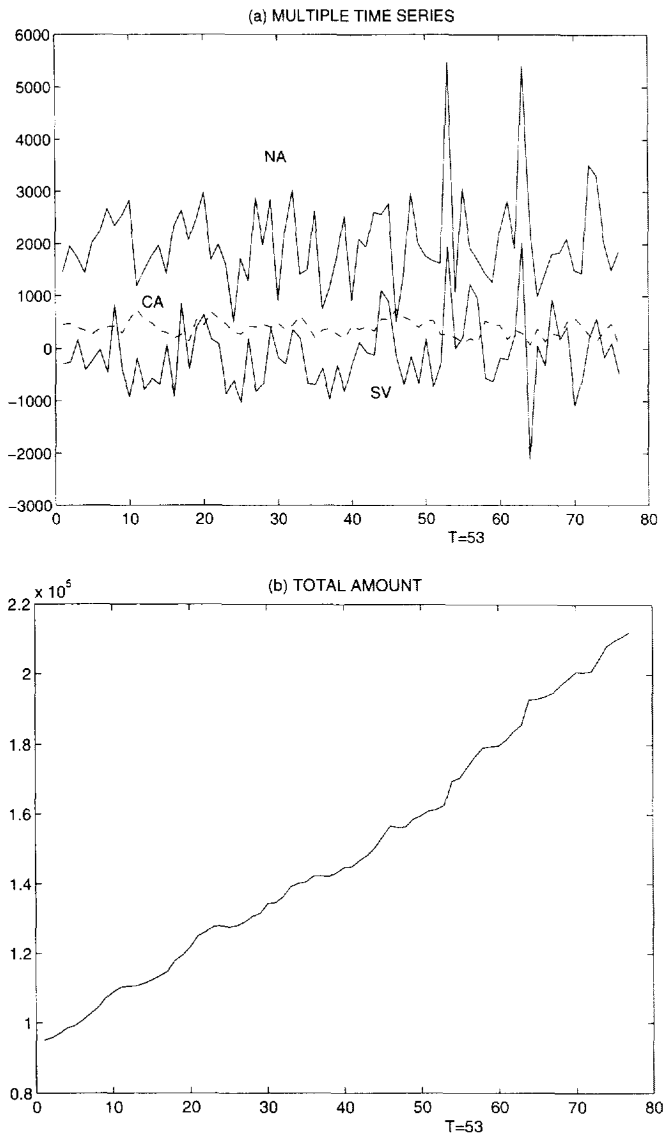

Figure 1. Observed Data: (a) New Accounts, Stock Variations, and Cancelled Time Series Data, (b) Total Amount Time Series Data.

$$\hat{\Phi}_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & .063(.038) & 0 \end{pmatrix}$$

$$\hat{\Phi}_{12} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & .469(.110) & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\hat{\boldsymbol{\nu}} = \begin{pmatrix} 1{,}930.9(113.7) \\ 0 \\ 296.2(61.8) \end{pmatrix},$$

and

$$\hat{\Sigma}_a = \begin{pmatrix} 492{,}497 & — & — \\ 56{,}387 & 175{,}361 & — \\ -12{,}447 & 21{,}462 & 13{,}519 \end{pmatrix}$$

with standard errors in parentheses. To judge the adequacy of the fitted model we present both the univariate Ljung–Box statistics calculated from 24 lags for each individual series of residuals NA : $Q = 26.1$; SV : $Q = 27.0$; and CA : $Q = 15.2$, and the Hosking (1980) multivariate portmanteau statistics with values 102.1 and 180.3 for the first 12 and 24 lags, respectively, that were considered reasonably appropriate. The plot of the multivariate residuals did not show any signs of inadequacy.

This model provided us the following explanation for the individual series: $\{\mathrm{NA}_t\}$ fluctuates around a constant level regardless of the past behavior of $\{\mathbf{Z}_t\}$; $\{\mathrm{SV}_t\}$ is affected by the values of both NA and CA in the previous month and by its own value one year before so that a seasonal fluctuation is incorporated; $\{\mathrm{CA}_t\}$ depends on its own value one month ago and on the value of SV recorded 2 and 14 months before the current month, with a variability much lower than those of the other two series. These results indicate the presence of inertia in the inflows and outflows of money into the system. When new accounts are opened, NA increases its value and a change will be reflected on SV next month. Thus NA is a leading indicator for SV. Moreover, if cancellations occur at time t (due for instance, to a promotional campaign of a competitor bank), they will be reflected in less inflows from previous accounts.

Second, we tested for significant intervention effects on $\{\mathbf{Z}_t\}$ with the aid of the Q^* statistic proposed by Aczél (1992), which yielded a value $Q^*_{\mathrm{calc}} = 231.2$. When comparing this figure against a chi-squared distribution with 72 df, we concluded that a significant impact was felt by $\{\mathbf{Z}_t\}$, even though we know that such a distribution must be considered as an approximation to a true F distribution; see Lütkepohl (1991, sec. 4.6) in this respect. The specific dynamic forms of the interventions for each individual series were postulated from the empirical evidence provided by the forecast errors with origin at the time point $t = 52$ as compared with their two standard error limits (see Fig. 2). Besides, subject-matter considerations also gave support to the following forms of type (5):

$$\begin{aligned}
\mathrm{NA} &: \eta_{1t} = (\omega_{1.0} + \omega_{1.10}B^{10})P_t^{(53)} \\
\mathrm{SV} &: \eta_{2t} = (\omega_{2.0} + \omega_{2.10}B^{10} + \omega_{2.11}B^{11})P_t^{(53)} \\
\mathrm{CA} &: \eta_{3t} = 0.
\end{aligned}$$

Therefore, to use the linear form (10) we employed a 72×5 matrix L with 1's in entries (1,1), (2,2), (31,1), (32,2), and (35,2) and with 0's everywhere else. The corresponding parameter vector became $\beta = (\omega_{1.0}, \omega_{1.10}, \omega_{2.0}, \omega_{2.10}, \omega_{2.11})'$.

A VARMA model with interventions was then estimated and yielded the following results:

$$\hat{\Phi}_1 = \begin{pmatrix} 0 & 0 & 0 \\ .168(.049) & 0 & -.819(.232) \\ 0 & 0 & .282(.107) \end{pmatrix},$$

$$\hat{\Phi}_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & .122(.033) & 0 \end{pmatrix}.$$

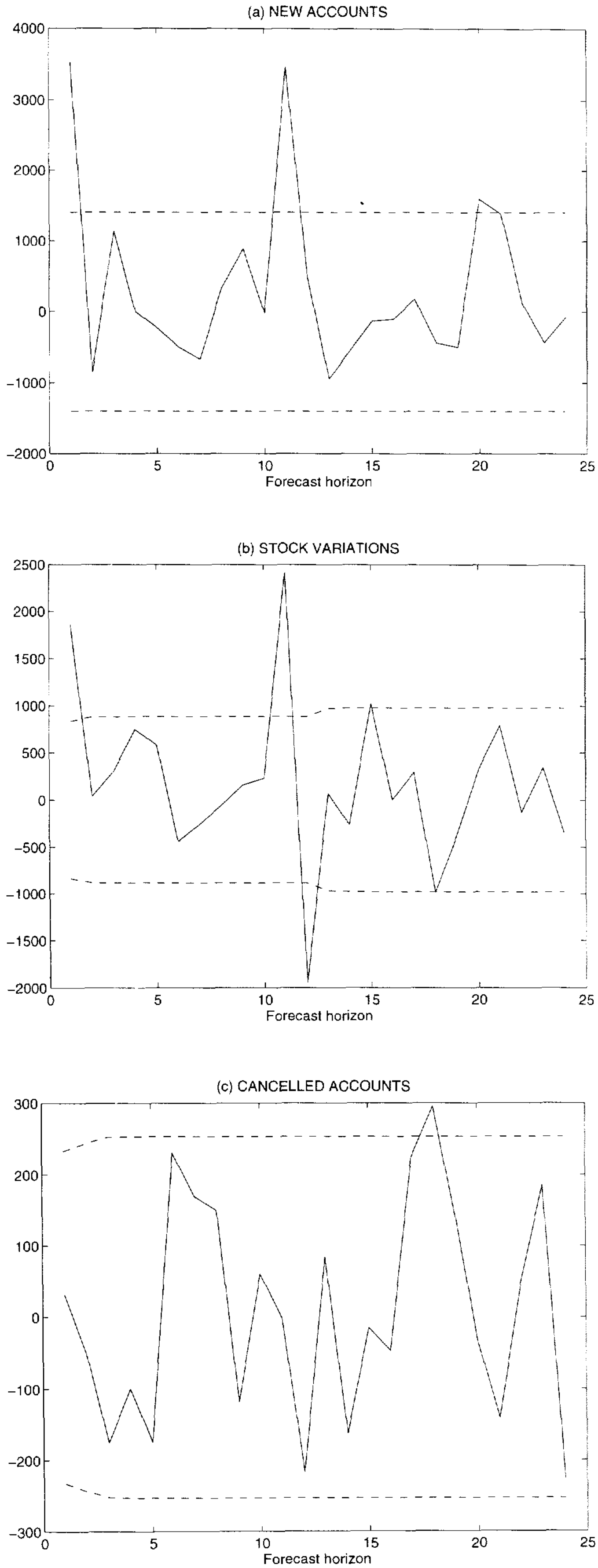

Figure 2. Prediction Errors for the Multiple Time Series: Prediction Errors and Two Standard Error Limits for each Element of the Multiple Time Series Vector.

$$\hat{\Phi}_{12} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & .443(.087) & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\hat{\nu} = \begin{pmatrix} 1.946.2(86.7) \\ 0 \\ 308.1(48.2) \end{pmatrix},$$

$$\hat{\Sigma}_a = \begin{pmatrix} 467.665 & — & — \\ 77.477 & 167.883 & — \\ -14.725 & 2.721 & 14.410 \end{pmatrix},$$

$$\mathbf{b} = \begin{pmatrix} 3.630.8(631.2) \\ 3.402.3(448.9) \\ 2.175.6(262.0) \\ 1.525.9(650.8) \\ -1.765.7(344.8) \end{pmatrix},$$

and

$$\Sigma_u = \begin{pmatrix} 398.390 & — & — & — & — \\ 14.370 & 421.070 & — & — & — \\ 30.580 & 32.260 & 68.650 & — & — \\ 1.690 & 1.780 & 3.800 & 423.570 & — \\ 1.770 & 59.950 & 3.980 & 220 & 118.870 \end{pmatrix}.$$

Again we judged the adequacy of this model by looking both at the Ljung–Box statistics for each individual series of residuals with 24 lags—NA : $Q = 23.3$. SV : $Q = 18.3$, and CA : $Q = 18.1$—and at the multivariate Hosking (1980) portmanteau statistics for the first 12 and 24 lags with values 106.1 and 178.3, respectively, which were also considered appropriate. The plot of the multivariate residuals did not show any signs of inadequacy. From the latter model we calculated the polynomial appearing in (2) by solving the equation $\Phi(B)\Psi(B) = I$. Then we were able to obtain the matrix Σ_e described after Expression (13), which is required to work out the proposed solutions. The values of $\mathbf{b}$ and Σ_u can be interpreted as saying that the promotional campaign of May 1994 increased the amount of both NA and SV in $3.630.8 \pm 631.2$ and $2,175.6 \pm 262.0$ units, respectively. Similarly, the March 1995 campaign produced increases amounting to $3.420.3 \pm 448.9$ on NA and $1.525.9 \pm 262.0$ on SV. An overshooting effect occurred in SV, however, because it diminished its value by $1.765.7 \pm 344.8$ units during April 1995. Thus, the bank's customers took advantage of the promotion in March and made deposits in both existing or new accounts, but part of that money was taken out from the accounts in April. In fact we may say that the second campaign served basically to attract new customers because the deposits of the old customers stayed in the bank only for one month.

The second step of our suggested procedure was then applied to adjust the estimates of the intervention parameters according to our primary solution (16)–(19). Thus, we obtained

$$\hat{\beta} = \begin{pmatrix} 3.592.6(509.2) \\ 3.657.5(560.0) \\ 2.186.0(246.2) \\ 1.747.6(559.8) \\ -1.656.7(321.2) \end{pmatrix},$$

and

$$\Sigma_{\beta} = \begin{pmatrix} 259.350 & — & — & — & — \\ -370 & 313.580 & — & — & — \\ -1.550 & 20.730 & 60.600 & — & — \\ 590 & -104.460 & -4.660 & 313.370 & — \\ -600 & 43.100 & 2.280 & -9.300 & 103.160 \end{pmatrix},$$

which, when compared with $\mathbf{b}$ and Σ_u, allow us to appreciate the effect of considering explicitly the accounting constraint. In this application we observe that the magnitude of the effects remained basically the same, but a substantial gain in efficiency was obtained. Similarly Figure 3

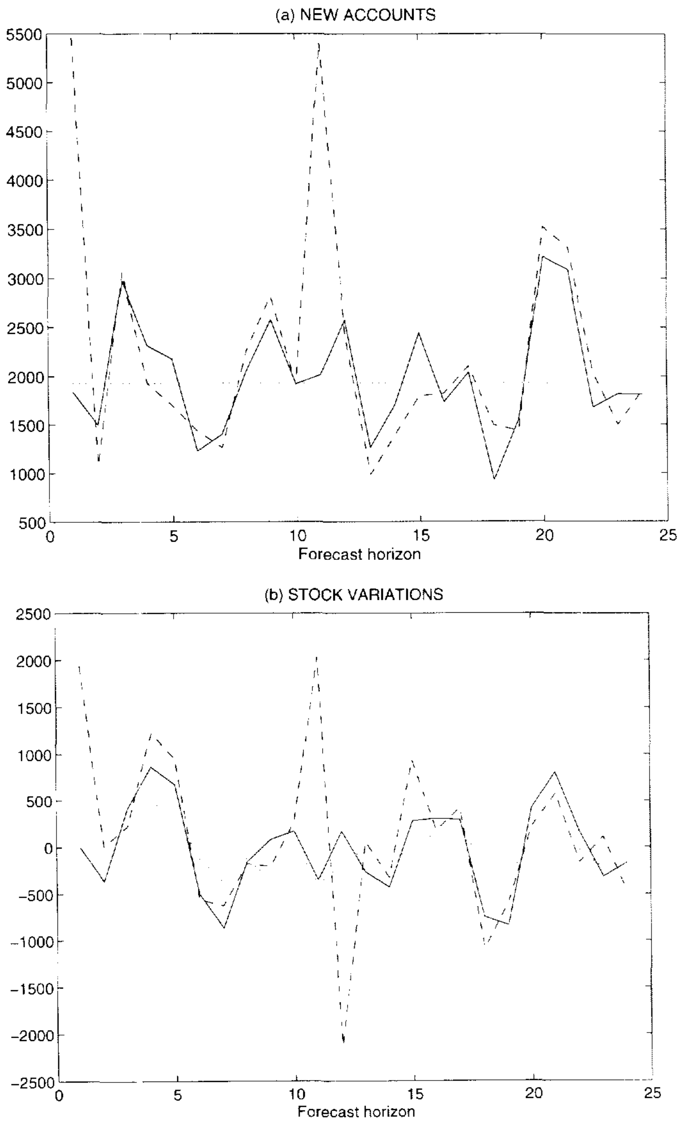

Figure 3. Observed, Predicted, and Estimated Data: Elementwise Comparison of the Observed Data (- - -) and Estimated Data Free of Intervention Effects (———).

allows us a visual comparison of the predicted and estimated series without intervention effects, $E(\mathbf{Z}^{(\mathrm{WI})}|X)$ and $\hat{\mathbf{Z}}^{(\mathrm{WI})}$, against the observed series $\mathbf{Z}$. It is worth mentioning that an indirect estimation of the intervention effects on $\{y_t\}$ may be accomplished by calculating $\hat{\boldsymbol{\eta}}_y = CL\hat{\boldsymbol{\beta}}$ with $\widehat{\mathrm{var}}(\hat{\boldsymbol{\eta}}_y) = CL\sigma_\beta L'C'$. In our example this estimation produced a 24-dimensional vector $\hat{\boldsymbol{\eta}}_y$ with nonzero values in entries 1, 11, and 12. Thus, the corresponding values $5{,}778.9 \pm 562.9, 5{,}405.1 \pm 646.6$, and $-1{,}656.7 \pm 321.2$ are interpretable as the effects of the promotional campaign on the flow series $\{\mathrm{TA}_t - \mathrm{TA}_{t-1}\}$ for the months May 1994 and March and April 1995.

2.2 Application of the Alternative Solution

We now start by performing a univariate intervention analysis on $\{y_t\}$. An ARIMA model with intervention effects was constructed as indicated by Box and Tiao (1975), producing as a result

$$\begin{aligned}
&(1 + \underset{(.126)}{.258}\, B^2)(1 - \underset{(.103)}{.686}\, B^{12})y_t \\
&\quad = \underset{(251.7)}{1{,}476.9} + (\underset{(632.9)}{5{,}802.7} + \underset{(632.5)}{6{,}161.4}\, B^{10} - \underset{(626.7)}{1{,}023.1}\, B^{11}) \\
&\qquad \times P_t^{(53)} + a_t
\end{aligned}$$

with $\hat{\sigma}_y = 770.9$ and Ljung–Box statistic $Q(24) = 19.5$, which did not show any evidence of inadequacy. All we need from this model is the vector $\hat{\boldsymbol{\eta}}_y$ containing the estimated intervention effects and its corresponding variance–covariance matrix. In this case $\hat{\boldsymbol{\eta}}_y$ is of dimension 24 and has values 5,802.7, 6,161.4, and $-1{,}023.1$ in entries 1, 11, and 12, respectively, and 0's in its remaining entries. Similarly the matrix $\Sigma_\varepsilon = \widehat{\mathrm{var}}(\hat{\boldsymbol{\eta}}_y)$ became a diagonal 24-dimensional matrix with values 400,683, 400,051, and 392,954 in entries 1, 11, and 12, and 0's everywhere else.

By applying expressions (22)–(23), we obtained

$$\hat{\boldsymbol{\beta}}^{\mathrm{A}} = \begin{pmatrix} 3{,}647.5(447.3) \\ 3{,}906.8(518.1) \\ 2{,}216.6(238.7) \\ 1{,}946.4(527.8) \\ -1{,}533.8(297.2) \end{pmatrix}$$

and

$$\Sigma_\beta^{\mathrm{A}} = \begin{pmatrix} 200{,}070 & — & — & — & — \\ -12{,}810 & 268{,}470 & — & — & — \\ -15{,}720 & 14{,}600 & 56{,}020 & — & — \\ -6{,}320 & -142{,}610 & -9{,}080 & 278{,}600 & — \\ -2{,}070 & 25{,}360 & 700 & -20{,}370 & 88{,}330 \end{pmatrix},$$

and a comparison with the previous values of $\hat{\boldsymbol{\beta}}, \Sigma_\beta, \mathbf{b}$, and Σ_u can now be established. In this application we observe that the elements in the diagonal of $\Sigma_\beta^{\mathrm{A}}$ are only slightly smaller than those of Σ_β but much smaller than those of Σ_u, indicating that the alternative solution is preferable in terms of precision. Furthermore, the estimated values $\hat{\boldsymbol{\beta}}^{\mathrm{A}}$ differ from $\hat{\boldsymbol{\beta}}$ and even more from $\mathbf{b}$. Special mention should be made of the last element of $\hat{\boldsymbol{\beta}}^{\mathrm{A}}$—that is, $\hat{\omega}_{2,11}^{\mathrm{A}} = -1{,}533.8$—which is affected by the (nonsignificant) estimate of that effect derived from the univariate model. Thus, we should be cautious when applying this alternative solution for the potential presence of bias, even though the precision gets increased. We do not include graphs of the elements of $\hat{\mathbf{Z}}^{(\mathrm{WI})\mathrm{A}}$, because they show essentially the same pattern as those in Figure 3.

Finally, it is important to keep in mind that the solutions presented should not be considered as competitors but as complementary of each other because they emerge from different and excluding assumptions about the use of the dataset $\{y_t\}$. Moreover, in our case we were fortunate in that the univariate intervention analysis produced a dynamic form for the intervention effects compatible with those forms for $\{\mathbf{Z}_t\}$. Had that not been the case, we would rather discard the univariate results and rely solely on the multivariate analysis. In closing this section we should mention that all the computations for this application were carried out with the aid of the SCA statistical system, release V.1 (Scientific Computing Associates, P.O. Box 625, Dekalb, IL 60115) and the MATLAB, version 4.2c (The Math Works, Inc., 24 Prime Park Way, Natick, MA 01760).

3. CONCLUDING REMARKS

Multiple time series intervention analysis will be more useful in practice when, besides the time series data, we are able to take into account additional information. In this article we have shown how to incorporate efficiently the information provided by a linear restriction (an accounting constraint) when estimating intervention effects.

We have provided two solutions that apply under different assumptions. Each solution is optimal in the sense of providing linear and unbiased estimators with minimum MSE. We advocate the use of the primary solution whenever possible, to avoid inconsistencies in the results obtained from independent univariate and multivariate intervention analyses. Nevertheless, the alternative solution stands on its own merit as an optimal solution when it can be justified. From a purely theoretical perspective, both solutions are obtained as special cases of a fairly general result provided in the Appendix that had already appeared in the literature. Therefore, we do not claim originality in that respect. In fact, our aim is essentially to provide a new tool that will enable a time series analyst to incorporate into his/her analysis the extra piece of information conveyed by an accounting constraint, through a statistically sound procedure.

The accompanying illustrative example deals with a real banking problem that motivated this work. We present this application in detail so that an interested analyst can apply our suggested solutions using it as a guide. Such an application serves also to compare numerically the results produced by the two solutions and understand their appropriateness under different circumstances.

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APPENDIX: LINEAR ESTIMATION OF A RESTRICTED RANDOM VECTOR

We consider the problem of estimating the random vector $\mathbf{Z}$ based on the following two relationships:

$$\mathbf{W} = \mathbf{Z} + \mathbf{a}; \quad \mathbf{Y} = H\mathbf{Z} + \mathbf{v}, \tag{A.1}$$

where $\mathbf{W}$ and $\mathbf{Y}$ are some known data vectors, $\mathbf{a}$ and $\mathbf{v}$ are zero-mean random-error vectors uncorrelated with $\mathbf{Z}$ and such that $E(\mathbf{aa}') = \Sigma_W, E(\mathbf{vv}') = \Sigma_Y$, and $E(\mathbf{av}') = 0$. Furthermore, Σ_W and Σ_Y are known nonsingular matrices and H is a known constant matrix. Then the minimum MSE linear and unbiased estimator of $\mathbf{Z}$ is given by

$$\begin{aligned}\hat{\mathbf{Z}} &= \mathbf{W} + \Sigma_W H'(H\Sigma_W H' + \Sigma_Y)^{+}(\mathbf{Y} - H\mathbf{W}) \\ &= A\mathbf{Y} + (I - AH)\mathbf{W}\end{aligned} \tag{A.2}$$

with MSE matrix

$$\begin{aligned}E[(\hat{\mathbf{Z}} - \mathbf{Z})(\hat{\mathbf{Z}} - \mathbf{Z})'] &= \Sigma_W \\ &\quad - \Sigma_W H'(H\Sigma_W H' + \Sigma_Y)^{+} H\Sigma_W \\ &= (I - AH)\Sigma_W,\end{aligned} \tag{A.3}$$

where $A = \Sigma_W H'(H\Sigma_W H' + \Sigma_Y)^{+}$. A proof of this result was given by Catlin (1989).

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