Testing relevant hypotheses in functional data analysis

Holger Dette

Ruhr-Universität Bochum

Abstract

Functional data analysis is typically conducted within the L^2-Hilbert space framework. There is by now a fully developed statistical toolbox allowing for the principled application of the functional data machinery to real-world problems, often based on dimension reduction techniques such as functional principal component analysis. At the same time, there have recently been a number of publications that sidestep dimension reduction steps and focus on a fully functional L^2-methodology. This paper goes one step further and develops data analysis methodology for functional time series in the space of all continuous functions. The work is motivated by the fact that objects with rather different shapes may still have a small L^2-distance and are therefore identified as similar when using an L^2-metric. However, in applications it is often desirable to use metrics reflecting the visualization of the curves in the statistical analysis. The methodological contributions are focused on developing two-sample and change-point tests as well as confidence bands, as these procedures appear do be conducive to the proposed setting. Particular interest is put on relevant differences; that is, on not trying to test for exact equality, but rather for pre-specified deviations under the null hypothesis.

The procedures are justified through large-sample theory. To ensure practicability, non-standard bootstrap procedures are developed and investigated addressing particular features that arise in the problem of testing relevant hypotheses. The finite sample properties are explored through a simulation study and an application to annual temperature profiles.