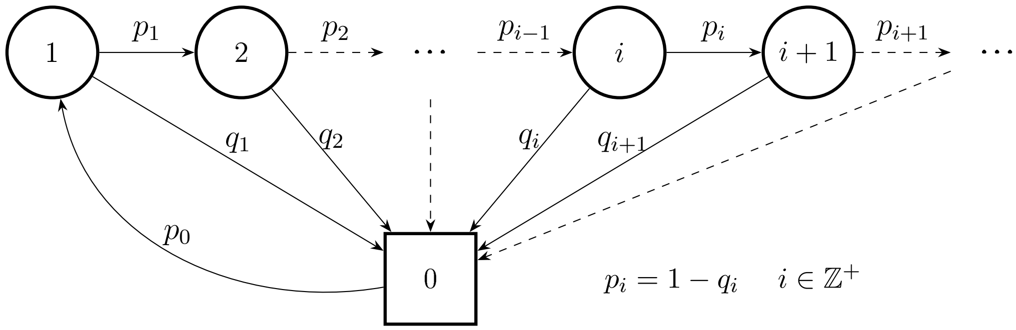

$$P = \begin{bmatrix} q_0 & p_0 & 0 & 0 & \cdots \\ q_1 & 0 & p_1 & 0 & \cdots \\ q_2 & 0 & 0 & p_2 & \cdots \\ q_3 & 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \end{bmatrix} \Longrightarrow \pi P = \left(\sum_{i \in S} \pi_i p_{ij},\ j \in \mathbb{Z}^+ \right)$$

$$\begin{aligned}
&\overset{\pi=\pi P}{\Longrightarrow} \begin{cases} \pi_0 &= \displaystyle\sum_{i=0}^{\infty} q_i \pi_i \\ \pi_i &= p_{i-1}\pi_{i-1} \quad i \geq 1 \ \Rightarrow \pi_i = p_{i-1}p_{i-2}\cdots p_0\pi_0 \end{cases} \\
&\Longrightarrow \pi_0 = \pi_0 \sum_{i=0}^{\infty} q_i p_{i-1} p_{i-2} \cdots p_0 \\
&\overset{\text{M.P.}}{\Longrightarrow} \pi_0 = \pi_0 \sum_{i=0}^{\infty} \mathbb{P}_0(X_1 = 1, X_2 = 2, \ldots, X_i = i, X_{i+1} = 0) \\
&\overset{\sigma-\text{sum}}{\Longrightarrow} \pi_0 = \pi_0 \mathbb{P}_0 \left(\bigcup_{i=0}^{\infty} \{X_1 = 1, X_2 = 2, \ldots, X_i = i, X_{i+1} = 0\} \right) \\
&\Longrightarrow \pi_0 = \pi_0 \mathbb{P}_0 \left(\bigcup_{i=0}^{\infty} \{T_0 = i+1\} \right) = \pi_0 \left(1 - \mathbb{P}_0(T_0 = \infty)\right) \\
&\qquad = \pi_0 \left(1 - \mathbb{P}_0 \left(X_1 = 1, X_2 = 2, \ldots\right)\right) \\
&\qquad \overset{\text{M.P.}}{=} \pi_0(1-p),
\end{aligned}$$

donde,

(*i*) $T_0 = \text{ínf}\{n > 0 : X_n = 0\}$, con $\text{ínf}\{\emptyset\} = \infty$.

(*ii*) $p = \prod_{i=0}^{\infty} p_i$.

(*iii*) M.P. representa la propiedad de Markov.

(*iv*) $\sigma-$sum indica la propiedad de sigma aditividad de la probabilidad $\mathbb{P}_0$

Basta tomar $(p_i)_{i\geq 0}$ de forma que $p > 0$ para no tener medida estable no-nula.
Otra vía para llegar al mismo resultado es notar que la serie $\sum_{i=0}^{\infty} q_i p_{i-1} p_{i-2} \cdots p_0$ es telescópica, esto es, viene dada en la forma $\sum_{i=0}^{\infty} a_{i+1} - a_i$ con $a_i = p_{i-1} p_{i-2} \cdots p_0$, con $a_1 = 1$. Luego

$$\sum_{i=0}^{\infty} q_i p_{i-1} p_{i-2} \cdots p_0 = a_0 - a_\infty = 1 - p.$$