Initial Enlargment: General Results

Bernardo D'Auria
email: bernardo.dauria@uc3m.es
web: www.est.uc3m.es/bdauria

Additional Material

ICMAT / UC3M

Initial Enlargment: General Results

Initial Enlargment: General Results ([1] 5.9.3)

Given a random variable L, we consider the filtration $\mathbb{F}^{(L)}$ made by

$$\mathcal{F}_t^{(L)} = \mathcal{F}_t \vee \sigma(L)$$

or more precisely $\mathcal{F}_t^{(L)} = \cap_{\epsilon>0}\{\mathcal{F}_{t+\epsilon} \vee \sigma(L)\}$.

Definition

There exists a family of *regular conditional distributions* $\lambda_t(\omega, dx)$ that satisfies

$\lambda_t(\cdot, A)$ is a version of $\mathbb{E}[\mathbb{1}\{L \in A\}|\mathcal{F}_t]$

$\forall\omega$, $\lambda_t(\omega, \cdot)$ is a probability distribution on $\mathbb{R}$.

Jacod's Criterion

Proposition (Jacod's Criterion. [1] 5.9.3.1)

Suppose that, for each $t < T$, $\lambda_t(\omega, dx) \ll \nu(dx)$ where ν is the law of L. Then, every $\mathbb{F}$-semi-martingale $(X_t, t < T)$ is also an $\mathbb{F}^{(L)}$-semi-martingale.

Moreover, if $\lambda_t(\omega, dx) = p_t(\omega, x)\,\nu(dx)$ and if X is an $\mathbb{F}$-martingale, its decomposition in the filtration $\mathbb{F}^{(L)}$ is

$$X_t = \tilde{X}_t + \int_0^t \frac{d\langle p(L), X\rangle}{p_s(L)} \ ,$$

where $p(L) = (p_t(L), t \geq 0)$.

Jacod's Criterion - general settings i

For a bounded Borel function f, let $(\lambda_t(f), t \geq 0)$ be the continuous version of the martingale $(\mathbb{E}[f(L)|\mathcal{F}_t], t \geq 0)$. There exists a predictable kernel $\lambda_t(dx)$ such that

$$\lambda_t(f) = \int f(x)\,\lambda_t(dx)$$

and by the predictable representation property applied to the martingale $(\mathbb{E}[f(L)|\mathcal{F}_t], t \geq 0)$, there exists a predictable process $\hat{\lambda}_t(dx)$ such that

$$\lambda_t(f) = \mathbb{E}[f(L)] + \int_0^t \hat{\lambda}_s(f)\,dB_s\ .$$

Jacod's Criterion - general settings ii

Proposition ([1] 5.9.3.2)

Assume that there exists a predictable kernel $\hat{\lambda}_t(dx)$ such that

$$dt \text{ a.s.}, \hat{\lambda}_t(f) = \int f(x)\,\hat{\lambda}_t(dx)\ .$$

In addition $dt \times d\mathbb{P}$ a.s. the measure $\hat{\lambda}_t(dx)$ is absolutely continuous w.r.t. $\lambda_t(dx)$:

$$\hat{\lambda}_t(dx) = \rho(t,x)\,\lambda_t(dx)\ .$$

Then, if X is an $\mathbb{F}$-martingale, there exists a $\mathbb{F}^{(L)}$-martingale $\tilde{X}$, such that

$$X_t = \tilde{X}_t + \int_0^t \rho(s,L)\,d\langle X,B\rangle\ .$$

Examples ([1] 5.9.3.3)

Example

Enlargement with B_1.

Example

Enlargement with $M_1 = \sup_{s \leq 1} B_s$.

We use the formula:

$$\mathbb{E}[f(M^B)|\mathcal{F}_t] = F(1-t, B_t, M_t^B)$$

where $M_t^B = \sup_{s \leq t} B_s$ and

$$F(s,a,b) = \sqrt{\frac{2}{\pi s}}\left(f(b)\int_0^{b-a} e^{-\frac{u^2}{2s}}\,du + \int_b^{\infty} f(u)\,e^{-\frac{(u-a)^2}{2s}}\,du\right)$$

Bibliography

Bibliography

- M. Jeanblanc, M. Yor and M. Chesney (2009). *Mathematical methods for financial markets*. Springer, London.
- D. Revuz and M. Yor (1999). *Continuous Martingales and Brownian Motion.* Springer Verlag, Berlin. 3rd ed.
- K. Itö (1976). *Extension of stochastic integrals.* Proc. Intern. Symp. on Stochastic Differential Equations, 95–109.