Problem Set # 2

(1) The following data were generated by a Weibull distribution: [1]

1.3043	0.4895	1.2742	1.4019	0.3255	0.2996	0.2642
1.0878	1.9461	0.4761	3.6454	0.1534	1.2357	0.9638
0.3345	1.1227	2.0296	1.2797	0.9608	2.0070	

(a) Obtain the maximum likelihood estimates of α and β, and give the asymptotic covariance matrix for the estimates.
(b) Carry out the Wald test of the hypothesis that $\beta = 1$.
(c) Obtain the maximum likelihood estimates of α under the hypothesis that $\beta = 1$.
(d) Using the results of parts (a) and (b), carry out a likelihood ratio test of the hypothesis that $\beta = 1$.
(e) Carry out a Lagrange multiplier test of the hypothesis that $\beta = 1$.

(2) For the classical regression model $\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{\varepsilon}$ with no constant term and K regressors, what is

$$plim \frac{R^2/K}{(1-R^2)/(n-K)},$$

assuming that the true value of $\boldsymbol{\beta}$ is zero?

(3) Prove that, under the hypothesis that $\boldsymbol{R\beta} = \boldsymbol{r}$, the estimator

$$s_R^2 = \frac{(\boldsymbol{y} - \boldsymbol{X}\widehat{\boldsymbol{\beta}}_R)'(\boldsymbol{y} - \boldsymbol{X}\widehat{\boldsymbol{\beta}}_R)}{n-K+J},$$

where J is the number of restrictions, is unbiased for σ^2.

(4) What is the covariance matrix of the GLS estimator and the difference between it and the OLS estimator?

$$\mathrm{Cov}(\widehat{\boldsymbol{\beta}}_{GLS}, \widehat{\boldsymbol{\beta}}_{GLS} - \widehat{\beta}_{OLS}),$$

where $\widehat{\boldsymbol{\beta}}_{GLS} = (\mathbf{X}'\boldsymbol{\Omega}^{-1}\mathbf{X})^{-1}\mathbf{X}'\boldsymbol{\Omega}^{-1}\boldsymbol{y}$, and $\widehat{\boldsymbol{\beta}}_{OLS} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\boldsymbol{y}$.

(5) Suppose that y has the following density function:

$$f(y|\boldsymbol{x}) = \frac{e^{-y/\boldsymbol{\beta}'\boldsymbol{x}}}{\boldsymbol{\beta}'\boldsymbol{x}}, \ y > 0.$$

Then $\mathrm{E}[y|\boldsymbol{x}] = \boldsymbol{\beta}'\boldsymbol{x}$ and $\mathrm{Var}[y|\boldsymbol{x}] = (\boldsymbol{\beta}'\boldsymbol{x})^2$. For this model, prove that GLS and MLE are the same.

[1] The density function of a Weibull distribution is $f(x) = \alpha\beta x^{\beta-1}e^{-\alpha x^{\beta}}$ for $x > 0$ and $\alpha, \beta > 0$.

(6) Assume that the model

$$y^* = \beta x^* + \varepsilon$$

conform to all of the assumptions of the classical normal regression model. If data on y^* and x^* were available, all the apparatus of the classical model would apply. [2] We assume that

$$y = y^* + v \quad \text{with } v \sim \mathcal{N}(0, \sigma_v^2),$$

and

$$x = x^* + u \quad \text{with } u \sim \mathcal{N}(0, \sigma_u^2).$$

(a) Prove that when only x^* is measured with error, the squared correlation between y and x is less than that between y^* and x^*.

(b) Does the same hold true if y^* is also measured with error?

[2] Notice that here $*$ denotes unobserved variables.