Econometrics
PhD in Business Administration and Quantitative Methods
Course 2005-2006
June 29th, 2006

Name:

Please, answer 4 of the following 6 questions. Time: 3 hours.

(1) Consider the following regression model:

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i,$$

where the x_i are non-stochastic and the $\{\varepsilon_i\}$ are i.i.d. with the following density function:

$$f_\varepsilon(x) = \begin{cases} \lambda x^{-(\lambda+1)} & \text{if } x \geq 1, \\ 0 & \text{otherwise} \end{cases}.$$

Assuming that λ is unknown ($\lambda > 2$), obtain the maximum likelihood estimators for λ, β_0 and β_1.

(2) The constant elasticity of substitution production function may be written

$$\ln Y = \ln\gamma - \frac{\nu}{\rho}\ln(\delta K^{-\rho} + (1-\delta)L^{-\rho}) + \varepsilon.$$

A Taylor series approximation to this function around the point $\rho = 0$ is

$$\begin{aligned} \ln Y &= \ln\gamma + \nu\delta\ln K + \nu(1-\delta)\ln L - \tfrac{1}{2}\rho\nu\delta(1-\delta)(\ln K - \ln L)^2 + \varepsilon \\ &= \beta_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_5 + \varepsilon, \end{aligned}$$

where $x_2 = \ln K$, $x_3 = \ln L$, $x_4 = (\ln K - \ln L)^2$, and

$$\begin{aligned} \beta_1 &= \ln\gamma & \gamma &= \exp\beta_1 \\ \beta_2 &= \nu\delta & \delta &= \frac{\beta_2}{\beta_2+\beta_3} \\ \beta_3 &= \nu(1-\delta) & \nu &= \beta_2+\beta_3 \\ \beta_4 &= -\tfrac{1}{2}\rho\nu\delta(1-\delta) & \rho &= \frac{-2\beta_4(\beta_2+\beta_3)}{\beta_2\beta_3}. \end{aligned}$$

We obtain the following ordinary least squares regression results:

$$\ln Y = 1.4677 - 0.1115\ln K + 1.1002\ln L + 0.1522(\ln K - \ln L)^2,$$

$$R^2 = 0.94677, \qquad \widehat{\boldsymbol{\varepsilon}}'\widehat{\boldsymbol{\varepsilon}} = 0.8018, \qquad n = 27,$$

$$\widehat{Var}(\widehat{\boldsymbol{\beta}}) = \begin{bmatrix} 0.16650 & & & \\ -0.10090 & 0.17320 & & \\ 0.08385 & -0.17890 & 0.18850 & \\ 0.03164 & -0.05192 & 0.05296 & 0.16220 \end{bmatrix}.$$

(a) Compute the implied estimates of the underlying parameters.
(b) Test the hypothesis that $\rho = 0$.

(3) Consider the following regression model:

$$y_i = \beta_0 + \beta_1 i^\gamma + \varepsilon_i,$$

where the $i = 1, 2, \ldots, n$ and the $\{\varepsilon_i\}$ are such that $\mathrm{E}[\varepsilon_i] = 0$, $\mathrm{E}[\varepsilon_i \varepsilon_j] = \sigma^2 I(i = j)$ and $\sigma > 0$. Prove that the OLS estimator of (β_0, β_1) converges in quadratic mean if and only if $-1/2 < \gamma < 0$ or $\gamma > 0$.

(4) Consider the following regression:

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + u_i,$$

where $\mathrm{E}[u_i] = \mathrm{cov}(x_{2i}, u_i) = 0$, $\mathrm{E}[u_i^2] = \sigma_i^2$ and $\mathrm{cov}(x_{2i}, u_i) \neq 0$.

(a) Derive the probability limit of the OLS estimator of the parameters β_0, β_1 and β_2 denote by $\widehat{\beta}_0$, $\widehat{\beta}_1$ and $\widehat{\beta}_2$, respectively.
(b) Is $\widehat{\beta}_2$ a consistent estimator of β_2?

(5) Consider the simple regression $y_i = \beta x_i$. Show that the estimator $\widetilde{\beta} = \bar{y}/\bar{x}$ is unbiased and that its variance is larger than that of the OLS estimator.

(6) The following model is specified:

$$\begin{array}{lllllll} y_1 & = & & \gamma_1 y_2 & +\beta_{11} x_1 & & & +\varepsilon_1, \\ y_2 & = & \gamma_2 y_1 & & & +\beta_{22} x_2 & +\beta_{32} x_3 & +\varepsilon_2. \end{array}$$

All variables are measured as deviations from their means. The sample of 25 observations produces the following matrix of sum of squares and cross–products:

$$\begin{bmatrix} 20 & 6 & 4 & 3 & 5 \\ 6 & 10 & 3 & 6 & 7 \\ 4 & 3 & 5 & 2 & 3 \\ 3 & 6 & 2 & 10 & 8 \\ 5 & 7 & 3 & 8 & 15 \end{bmatrix}.$$

(a) Estimate the two equations by OLS.
(b) Estimate the parameters of the two equations by 2SLS. Also, estimate the asymptotic covariance matrix of the 2SLS estimates.
(c) Estimate the reduced–form coefficient matrix by OLS and indirectly by using your structural estimates from part (b).