Econometrics
PhD in Business Administration and Quantitative Methods
Course 2004-2005
June 17th, 2005

Name:

Please, answer 4 of the following 6 questions. Time: 3 hours.

(1) For the regression model:

$$\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{\varepsilon}, \text{ with } \boldsymbol{\varepsilon} \sim \mathcal{N}(\boldsymbol{0}, \sigma^2\boldsymbol{I}),$$

(a) Obtain the mean and the variance of the ML estimator, $\hat{\sigma}^2 = \hat{\boldsymbol{\varepsilon}}'\hat{\boldsymbol{\varepsilon}}/n$, of σ^2.
(b) Prove that this estimator is consistent.

(2) The following statistics is used to test a set of J linear restrictions in the generalized regression model:

$$F[J, n-K] = \frac{(\boldsymbol{R}\widehat{\boldsymbol{\beta}} - \boldsymbol{r})'(\boldsymbol{R}(\boldsymbol{X}'\boldsymbol{\Omega}^{-1}\boldsymbol{X})^{-1}\boldsymbol{R}')^{-1}(\boldsymbol{R}\widehat{\boldsymbol{\beta}} - \boldsymbol{r})/J}{(\boldsymbol{y} - \boldsymbol{X}\widehat{\boldsymbol{\beta}})'\boldsymbol{\Omega}^{-1}(\boldsymbol{y} - \boldsymbol{X}\widehat{\boldsymbol{\beta}})/(n-K)},$$

where $\widehat{\beta}$ is the GLS estimator. Show that if $\boldsymbol{\Omega}$ is known and the innovations are normally distributed, this statistics is exactly distributed as a F with J and $n-K$ degrees of freedom. What assumptions about the regressors are needed to reach this conclusion?

(3) The constant elasticity of substitution production function may be written

$$\ln Y = \ln\gamma - \frac{\nu}{\rho}\ln(\delta K^{-\rho} + (1-\delta)L^{-\rho}) + \varepsilon.$$

A Taylor series approximation to this function around the point $\rho = 0$ is

$$\begin{aligned} \ln Y &= \ln\gamma + \nu\delta\ln K + \nu(1-\delta)\ln L - \tfrac{1}{2}\rho\nu\delta(1-\delta)(\ln K - \ln L)^2 + \varepsilon \\ &= \beta_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_5 + \varepsilon, \end{aligned}$$

where $x_2 = \ln K$, $x_3 = \ln L$, $x_4 = (\ln K - \ln L)^2$, and

$$\begin{array}{ll} \beta_1 = \ln\gamma & \gamma = \exp\beta_1 \\ \beta_2 = \nu\delta & \delta = \dfrac{\beta_2}{\beta_2+\beta_3} \\ \beta_3 = \nu(1-\delta) & \nu = \beta_2 + \beta_3 \\ \beta_4 = -\tfrac{1}{2}\rho\nu\delta(1-\delta) & \rho = \dfrac{-2\beta_4(\beta_2+\beta_3)}{\beta_2\beta_3}. \end{array}$$

We obtain the following ordinary least squares regression results:

$$\ln Y = 1.4677 - 0.1115\ln K + 1.1002\ln L + 0.1522(\ln K - \ln L)^2,$$

$$R^2 = 0.94677, \qquad \widehat{\boldsymbol{\varepsilon}}'\widehat{\boldsymbol{\varepsilon}} = 0.8018, \qquad n = 27,$$

$$\widehat{Var}(\widehat{\boldsymbol{\beta}}) = \begin{bmatrix} 0.16650 & & & \\ -0.10090 & 0.17320 & & \\ 0.08385 & -0.17890 & 0.18850 & \\ 0.03164 & -0.05192 & 0.05296 & 0.16220 \end{bmatrix}.$$

(a) Compute the implied estimates of the underlying parameters.
(b) Test the hypothesis that the coefficients on $\ln K$ and $\ln L$ sum to 1.
(c) Test the hypothesis that $\rho = 0$.

(4) Suppose that the regression model is:

$$y_i = \mu + \varepsilon_i,$$

where $\mathrm{E}[\varepsilon_i|x_i] = 0$, $\mathrm{Var}[\varepsilon_i|x_i] = \sigma^2 x_i^2$ and $x_i > 0$.
(a) Given a sample of observations on y_i and x_i, what is the GLS estimator of μ? What is its variance?
(b) What is the OLS estimator of μ, and what is the variance of the ordinary least squares estimator?

(5) For the instrumental variables estimator, $(\boldsymbol{Z}'\boldsymbol{X})^{-1}\boldsymbol{Z}'\boldsymbol{y}$, prove that

$$\widehat{\sigma}^2 = \frac{(\boldsymbol{y} - \boldsymbol{X}\widehat{\boldsymbol{\beta}}_{IV})'(\boldsymbol{y} - \boldsymbol{X}\widehat{\boldsymbol{\beta}}_{IV})}{n}$$

is a consistent estimator of σ^2.

(6) Assume that the model

$$y^* = \beta x^* + \varepsilon$$

conform to all of the assumptions of the classical normal regression model. If data on y^* and x^* were available, all the apparatus of the classical model would apply. We assume that

$$y = y^* + v \quad \text{with } v \sim \mathcal{N}(0, \sigma_v^2),$$
$$x = x^* + u \quad \text{with } u \sim \mathcal{N}(0, \sigma_u^2),$$

and

$$cov(x^*, u) = cov(y^*, v) = cov(u, v) = cov(u, \varepsilon) = cov(v, \varepsilon) = 0.$$

(a) Prove that when only x^* is measured with error, the squared correlation between y and x is less than that between y^* and x^*.
(b) Does the same hold true if y^* is also measured with error?