Detrending the Business Cycles:
Hodrick-Prescott and Baxter-King Filters

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Contents

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Abstract

The purpose of this paper is to see comparative analsis of two filters, Hodrick-Prescott and Baxter-King filters. The review of the literature on this topic show that the applicaion of the Hodrick-Prescott filter to the time series would produce spurious effect as well as poor approximation of the data near the end points is observed. Alternative filter which was proposed by Baxter- King (1995) is also revised. Although lost of data points are observed in the time series by the application of the filter, the problem of producing spuriousness in the time series is not as evident as it is in Hodrick-Prescott filter. Thus it would be more effective to apply Baxter-King filter to the time series.

1 Introduction

Measurement of the business cycles is very essential for the study of business cycles. In this context the main issue is separating the evolving trend component from the cyclical component. One of the most used, as well as most critisized detrending methods is Hodrick-Prescott filter, proposed by Hodrick and Prescott (1980). The purpose of this paper is to review the main characteristics and the critisizm of this filter found in the time series literature. In this context two main limitations are of interest: 1) poor approximation near the endpoints, and 2) spurious effect. One of the most considered alternative band-pass filter suggested by Baxter and King (1995) is used to carry the analysis.

The paper is built as follows: in the second section general detrending methods of business cycle is revised. Next the two filters : Hodrick-Prescott (H-P) and Baxter-King (B-K) filters are analyzed. In the third section the main problems of H-P filter are discussed and comparison with the B-K filter is given. Then summary and literature are presented. In the appendix section the MATLAB routines and the graphs are presented. For the empirical analysis the data is taken from the US macroeconomic time series. Three samples are used: gross national product (GNP), government consumption (GC) and government investment (GI).

2 Detrending the Business Cycles

Macroeconomic time series researchers face the immediate problem of decomposing the series into trend and cyclical components. Traditional way of extracting the linear trend from the time series has been a standard way of carrying this procedure.This traditional methods are essentially *ad hoc* which are designed with priory choice and without taking into consideration the statistical properties of the series it is applied to. This model usually is constructed in following way: the time series observed over the period $t = 1, \ldots \infty$ is decomposed into trend (growth, signal) and cyclical (noise) component. It is usually assumed that they are independent. As well as it is assumed that the seasonality has been removed from the time series under consideartion.

$$\begin{aligned} y_t &= g_t + c_t \\ E(g_t, c_t) &= 0 \end{aligned}$$

Applying the filter to the time series produces a new time series. As a simplest example we can see moving average filter

$$y_t^* = \sum_{h=-K}^{K} a_h y_{t-h} = a(L) y_t$$

where L is the lag operator and $h = 1, ..., K$, such that $a(L) = \sum_{h=-K}^{K} a_h L^h$. In the case if the MA process is symmetric , then $a_h = a_{-h}$. If the weights sum to 0 then it is shown that the symmetric MA has trend reduction properties. Thus one can write

$$a(L) = (1 - L)(1 - L^{-1})\psi(L)$$

where $\psi(L)$ is symmetric moving average with lags and leads $K - 1$.

The Cramer representation of the stationary time series is:

$$y_t = \int_{-\pi}^{\pi} \xi(\omega) d\omega$$

That is one can represent the time series as the integral of random periodic components, $\xi(\omega)$, which are mutually orthogonal, $E(\xi(\omega_1)\xi(\omega_2)) = 0$. Thus the filtered series can be expressed as:

$$y_t^* = \int_{-\pi}^{\pi} a\left(\omega\right) \xi(\omega) d\omega$$

where $a\left(\omega\right)$ is frequency responce function of the linear filter, and is equal: $a\left(\omega\right) = \sum_{h=-K}^{K} a_h e^{-i\omega h}$. The variance of the filtered series is thus:

$$var(y_t^*) = \int_{-\pi}^{\pi} \left|a\left(\omega\right)\right|^2 f_y(\omega) d\omega$$

where $|a(\omega)|^2$ is squared gain function of teh linear filter and $f_y(\omega) = var(\xi(\omega))$. The squared gain indicates the extent to which a moving average raises or loweres the contribution to variance in the filtered series from the level in the original series.

One of the issues of detrending the time series is designing filters to isolate specific frequencies from the data. A basic building block in filter designing is low pass filter, a filter which retains only slow-moving components of the data. An ideal low pass filter passes through the frequencies $-\underline{\omega} \leq \omega \leq \underline{\omega}$. High pass and band pass filters are constructed from the low pas filter. High pass filter passes components of the data with frequency equal or less than p while low pass filter passes the components of the data with frequency bigger than p. The ideal band pass filter is constructed using two low pas filters with cutoff frequencies $\underline{\omega}$ and $\overline{\omega}$, since it passes only frequencies in the range $\underline{\omega} \leq |\omega| \leq \overline{\omega}$.

Approximate filter, $\alpha_K(\omega)$ is construced using the strategy of choosing the apprximating filter's weights a_h to minimize:

$$Q = \int_{-\pi}^{\pi} |\delta(\omega)|^2 \, d\omega$$

where $\delta(\omega) = \beta(\omega) - \alpha_K(\omega)$ is the discrepancy arising from approximation at frequency ω. $\beta(\omega)$ is specific filter which is to be approximated. The result of this maximization problem is general: the optimal approximating filter for a given maximum length K, is constructed by simly truncating the ideal filter's weights a_h at lag K.

Next, the charcteristics of two detrending methods, Hodrick-Prescott and Baxter-King filters are revised.

2.1 Hodrick-Prescott Filter

The Hodrick-Prescott (1980) filter is an ad hoc fixed, 2- sided MA filter which is constructed using penalty-function method. This filter optimally extracts the stochastic trend (unit root), moving smoothly over time. The filter is constructed as the solution to the problem of minimizing the variability in the cyclical component subject to a penalty for the variation in the second difference or the smoothness of the trend or *growth* component. The smoothness of cyclical componenet is calculated taking the sum of squares of its second difference.

$$\begin{aligned} y_t &= g_t + c_t \\ \pounds &= \min\left[\sum_{t=1}^{T}(y_t - g_t)^2 + \lambda \sum_{t=1}^{T}\left((g_{t+1} - g_t) - (g_{t-1} - g_{t-2})\right)^2\right] \end{aligned}$$

where y_t is the natural logarithm of the given series, g_t is the growth component , c_t are the deviations from the growth and λ is the smoothness parameter which panalyzes the variability in the growth function. The first term in the right-hand side is goodness of fit measure and the second term is sum of squares of the growth components second difference, i.e. smoothness of g_t.

Taking the derivatives of the minimization problem with respect to growth component the first order condition is:

$$\begin{aligned} \frac{\partial \pounds}{\partial g_t} &= 0 \\ H(L) &= \frac{\lambda L^{-2}(1-L)^4}{\lambda L^{-2}(1-L)^4+1} \end{aligned} \tag{2.1}$$

where the $H(L)$ is the representation of the trend elimination Hodrick-Prescott filter.

The Fourier transformation or the spectral representation (frequency response function) of the filter by King and Rebelo has the following form:

$$H(L) = \frac{4\lambda(1-\cos(\omega))^2}{4\lambda(1-\cos(\omega))^2+1}$$

where ω represents the frequency.

An alternative representation of the H-P filter is the Wiener- Kolmogorov derivation of the filter which provides an efficient and simple computational algorithm (Kaiser and Marvall, 1999). This representation is equivalent with Kalman and Danthine and Girardin filter. The estimator of the cyclical component is given as:

$$\widehat{c}_t = \xi(B,F)y_t = \left[k_{c(HP)} \frac{\nabla^2 \overline{\nabla}^2}{\theta_{H-P}(B)\theta_{H-P}(F)}\right] y_t$$

where $\xi(B,F)$ is symmetric, two-sided and convergent linear filter We are going to return to this represention in the next section.

The Hodrick-Prsecott filter shares some important properties with ideal high pass filter. An ideal high pass filter removes the low frequencies or the long cycle component, passing through the data with frequency lower than p. Therefore the Fourier transformation of ideal high pass filter is zero. We can see that the spectral H-P filter is zero at zero frequency, since $\cos(0) = 1$.It implies that the H-P filter generates stationary time series. [Figures $3, 4, 5$]

In the case of H-P filter the Fourier transformation is also treated as the gain function of the filter since the filter is symmetric by construction. Symmetricity in its turn eliminates the phase shift from the time series which is another desired property of H-P filter. Another important property is that the filter has a near unit gain at frequency equal to π ,since $\cos(\pi) = -1$.

The H-P filter as we saw in [Eq.1.1] depends on one variable : smoothness parameter, λ.The optimal value of this variable is calculated as:

$$\lambda = \frac{\sigma_g^2}{\sigma_c^2}$$

where σ_g^2 and σ_c^2 are the standard devations of the innovations in the growth and cyclical components respectively. Variations in λ alters the tradeoff between goodness of fit and smoothness degree which needs to be minimized. As penalty gets bigger , $\lambda \to \infty$ the smoothness of the growth component increases, thus making it linear trend . With quarterly data Hodrick and Prescott (1997) set the value of λ priori equal to 1600 in order to approximate it to the high pass filter.This choice is interpreted as defining the cyclical component as fluctuations with a period less than 8 years. For this value of λ the filter is close to an approximate high pass filter with cutoff frequency $\omega = \pi/16$. It means that if the quarterly data used then this choice makes the filter look like the ideal high pass filter which passes components of the data with periodicity $p = 32$. However with annual data $\lambda = 10$ (Baxter and King, 1995) and with monthly data $\lambda = 6400$ (Kaiser and Maravall, 2001) are chosen.

As it was indicated above H-P filter is also usually applied to the seasonally adjusted time series. In this framework it is essential that the seasonality does not contaminate the time series.

Since the H-P filter in the equation (2.1) is infinite order MA , some modifications are needed in order to apply it to finite order data. One strategy of modification proposed by Baxter and King (1995) is truncating the weights at some fixed lag K. However an alternative method of applying the H-P fiter to finite sample used which relies on finding the optimal estimates of trend and cycle. This assumption is based on the original derivation of the H-P filter. Assume that the trend and cycle components are generated by the white noises which follow a particular probability model,

$$g_t = a_t^g, \ c_t = a_t^c$$

Knowing the magnitude of the standard deviations , $\sigma^2_{a_t^g}$ and $\sigma^2_{a_t^c}$ then it is possible to extract the estimates at each date of the finite sample. Considering that these estimates are the weighted average of the original series then the cyclical component at time t is:

$$c_t = \sum_h^T d_{ht} y_h$$

where h is lead/lag index. The weights of the filter sum to zero, $\sum_{h}^{T} d_{ht} = 0$. The modified filter behaves as the ideal filter towards to the middle of the series. But near the end points the gain functions differ sharply from each other. This problem is going to be mentioned below.

2.2 Baxter-King Filter

While high pass filter removes low frequencies from the data , band pass filter removes both low an high frequencies from the time series. Based on the Burns and Mitchell's (1946) definition of the business cycle Baxter and King (1995) develop method for measuring the business cycle which isolates the business cycle components bu applying the moving average to the macroeconomic data. The band pass filter designed by Baxter and King (1995) (B-K) passes through the components of time series with fluctuations between 6 (18 month) and 32 (96 month) quarters, removing higher and lower frequencies. They require 6 obectives which the filter should meet: 1) the fiter should extract a specified range of periodicities; 2) ideal band pass filter should not introduce phase shift; 3) filter should be optimal approxiamtion to the band passfilter; 4) approximation of the filter should result in a stationary time series thus to be able to eliminate the quadratic trend from the series; 5) the method should yield businss cycle component that are unrelated to the length of the sample period; 6) the method should be operational.

The method proposed by Baxter and King (1995) relies on the use of the symmetric finite odd-order $M = 2K + 1$ moving average such that:

$$\begin{aligned} y_t^* &= \sum_{h=-K}^{K} a_h y_{t-h} = \\ &= a_0 y_t + \sum_{h=1}^{K} a_h \left(y_{t-h} + y_{t+h} \right) \end{aligned} \tag{2.2}$$

which takes the form of a 24- quarter MA when applied to the quarterly data.

$$y_t^* = \sum_{h=-12}^{12} a_h y_{t-h} = a(L) y_t$$

The set of M weights a_h is obtained by truncating the ideal filter weights at M under the frequency response function constraint:

$$H_0 = \Delta t \sum_{h=-N/2}^{(N-1)/2} a_h$$

where N is the number of data points and Δt is the sampling periodicity. The frequency response function is built such that at $\omega = 0$ $H(0) = 0$ for band pass and high pass filters and $H(0) = 1$ for low pass filters.

The B-K filter coefficients are driven from the following maximization problem (Noullez and Iacobucci, 2005):

$$\min \int_{-(2\Delta t)^{-1}}^{(2\Delta t)^{-1}} \left| \left(\sum_{h=-K}^{K} a_h^{B-K} - a_h^{ideal} \right) e^{-i2\pi n\omega \Delta t2} \right|^2 d\omega$$

Solving the maximization problem subject to the (2.2) shows that B-K coefficients are equal to the ideal filter coefficients shifted by some constant:

$$a_h^{B-K} = a_h^{ideal} + \frac{H(0) - \Delta t \sum_{h=-K}^{K} a_h^{ideal}}{M\Delta t}$$

Since the filter depends on M and not on N the performance of the filter does not change as the number of the observations increase.

The B-K flter has some desirable properties. Because of the symmetrycity property the filter does not present phase shift. Next being of constant finite length the filter is stationary. Another desirable property of the filter we can see from the following implications made by Baxter and King (1995). Considering the lag operator $a(L) = \sum_{h=-K}^{K} a_h L^h$ and applying furher simplifications they show that their filter can be factorized as:

$$a(L) = -(1 - L)(1 - L^{-1})\psi_K(L)$$

where $\psi_K(L)$ is symmetric moving average with $K - 1$ lags and leads. Tus the filter is able to render stationarity to the time series which contain up to two unit roots. Further the filter is incencitive to deterministic trends, so that it is not used in the edges of the series.

Additionally, adding the filter to the time series results in the lost of K observations both in the beginning and in the end of the series. But choosing low values for K results in poor approximation of the flter to the ideal high pass filter. Thus in order to apply the truncation to the filter lag , K choice should depend on the length of the data and necessity of how well the filter should be approximated. The authors propose putting $K \geq 12$, without considering the number of observations, sampling frequencs or the band to be extracted, since a value of $K = 12$ for the passband (6, 32) quarters is found to be basically equivalent to higher values as 16 or 20. Equatng K=12 would thus mean loosing 24 observaions from the series

3 Problems Associated with Hodrick- Prescott Filter

Above some characteristics of H-P filter were revised. However there are some drawbacks associated with H-P filter. The main problems with H-P which were highlighted by researchers are the poor approximation of the filter near the endpoints and the spurious effect it can produce when applied to the series. In this part we are going to see these problems in more details.

Another problems associated with the filter is the presence of leakage and compression when applied to the time series. As it was indicated above for quarterly data setting $\lambda = 1600$ the filter can well approximate the ideal high-pass filter with cut-off frequency $\omega = \pi/16$. Applying both the ideal high pass and H-P filter to the data one can observe a rounded peak in the H-P filter.(Pedersen, 2001) which is due to the leakage and the compression of the H-P filter. Leakage refers to the phenomenon that the filter passes through the frequencies which it was designed to supress whereas compression is the tendency that the filter has less than unit frequency response for frequencies above the cut-of frequency, ω.

3.1 End Point Estimation

As it was already mentioned one of the significant critics of the H-P filter is related to its poor approximation near the end points. Above the application of the H-P filter to the finite samples by Baxter and King was already reviewed. It is shown that in the beginning of the sample period the filter d_{ht} has quite different properties that the ideal high pass filter. Also phase shift is observed in the beginning of the sample. But towards the middle of the series the H-P filter behaves very close to the ideal band-pass filter.

In order to evaluate the effect of the filter they choose AR(1) process:

$$y_t = \rho y_{t-1} + a_t$$

where the variance of the innovations is $Var(a_t) = 1$ and $\rho = 0.95$. After applying the filter to the series the variance of the observaations differ though in the real data this pattern is not observed. Their investigation shows that H-P filter does not generate as many useful estimates of the cyclical component as there are data points. That is the filter weights start to settle down after 12 observations.

Another analysis of the revision implied by H-P filtering was carried out by Kaiser and Maravall (1999). Two features of the revision analyzed are: 1) magnitude and 2) duration of the revision, i.e. the value of K at which the filter can be safely truncated. In order to carry the analysis WK version of the filter is used.Also it should be indicated that the analysis are carried by applying the

H-P filter to the X11-SA series. Assuming that the process is following the ARIMA process

$$\phi(B)\nabla^d y_t = \theta(B)a_t$$

with $0 < d < 4$, $\phi(B)$ stationary and $\theta(B)$ invertible.

After the simpifications the estimator of the estimator of the cycle becomes:

$$\widehat{c}_t = \xi(B,F)a_t = \left[k_{c(HP)}\frac{\nabla^4\theta(B)}{\theta_{H-P}(B)\phi(B)\theta_{H-P}(B)}\right]a_{t+2}$$

where $k_c = Var(c_t)/Var(a_t)$. Analysis of the equation show that the spectrum of the estimator is determined partially by the structure of the H-P filter and and partially by the dynamic structure of the series. Thus using the spectral expression of the estimator the revision of the estimator is found to be:

$$r_{t|t} = \sum_{h=-K}^{K} \xi_h a_{t+h}$$

Setting the variance of the innovations equal to 1 the variance of the revision is

$$Var(r_{t|t}) = \sum_{h=-K}^{K} (\xi_h)^2$$

Applying the analysis to three models, white noise, random walk, and IMA(2, 2), the magnitude of the revision and duration of the revision, that is the number of the periods needed for the estumatior to converge are exhibited. It was found out that the magnitude of the revision was approximately 34% of the one period-ahead foerecast , implying that magnitude was not negligible. As to the revision period, for all three methods it lasted more than to years (Regina and Maravall, 2001). Since the H-P filter is often applied to the seasonally adjusted X11 series as it was aplready noted above, where filter X11 also produces revisions the revisions associated with the analysis would reflect the combined effect of the two filtter. In order to see this effect the authors consider the white noise case. That is, since $y_t = a_t$ the revision analysis would illustrate "pure filter" effect. This shows that after 13 quarters 95% of the revision variance disappers. Thus it takes 13 observations for the filter to settle down. Also final analysis show that highly moving trends and seasonals are subject to bigger, longer-lasting revisions.

As we can see from the analysis it takes the filter 12 or 13 observations to settle down. Thus it iwould be straightforward to drop these amount of observations both in the beginning and in the end of the series. But in this case H-P filter is not preferred to B-K filter since the same or more number observations are lost by applying both filters to the series.

3.2 Spurious Effect

Another most critisized feature of the H-P filter is the spurious effect it can produce when applied to the series.

Less smooth comparing with B-K filter. By construction the spectrum of the H-P filter is zero at zero frequency, as well as at seasonal frequencies, $\omega = \pi/2$ and $\omega = \pi$. This fixed structure results in spurious effect (Kaiser and Maravall, 2001). That is two peak structure of the spectrum results in the possibbility of obtaining spurious autocorrelation structure of the series.. Also the presence of three zeros will induce peaks in the spectrum which would result in a spurious periodic cycle

In order to detect the spurious crosscorrelation in the H-P filtered series Kaiser and Maravall (1999) carry simulations for white noise and random walk series. For the first series no spurious crosscorrelation was detected , although for random walk a moderate spurious effect was detected. The spurious autocorrelations were analyzed using calibration technique, i.e. validating the econmic model by comparing the previous ACF with the one implied by the observed economic variable. To carry the analysis the following AR(2) process was considered:

$$(1 - 1.293B + .49B^2)c_t = a_t$$

Detrending the SA series with H-P filter the ACF for observed series was obtained. The anlysis showed that the ACF of the cycle contained in the series was considerably different that the one obtained by filtering, thus showing significant distortions implied by the H-P filtering.

For the anaysis of the spurious periodic cycles the series spectrum was considered, in order to see which variation was actually passed. For this purpose white noise and random walk series were used. For the white noise input it was found that the peaks of the AR process which was fit to the filtered series capture the peaks of the spectrum of the filtered white noise series. It was concluded that H-P filter is likely to produce spurious periodic cycles when applied to white noise series.

In Figures 8, 9, 10 we can see the H-P filtered series compared with B-K filtered series. Though there is a close correspondence between the filtered series, the B-K filtered series looks more smooth. As well as in the B-Kfiltered series we can not observe the two-peakedness as we do in H-P filtered series. Thus the presence of the spurious results are less probable in the series by application B-K filter to the time series.

4 Summary

The purpose of this paper was to see the review the critics of the Hodrick-Prescott filter introduced in the literature and compare it to the filter propsed by Baxter and King (1995). Despite the fact that the filter has several desirable features which we saw above there are some problems associated with the application of H-P filter to the time series which limits the usefullness of the filter for the analysis of the data. These problems included mainly the end point estimation and spurious effect. That is in the beginning and towards to the end of the data the behavior of the filter is not reliable, thus some points from the data are to be dropped off. Anther problem associated with the filter is the spurious effect it could produce when applied to the series.

Another filter which was represented by Baxter and King (1995) was reviewed as well. Form the discussion above it could is concluded that though B-K filter also contain the problem of lost of the data in the beginning and in the end of the series. But since the filter does not produce as much spurious effect as H-P filter does using B-K filter in business cycles would be more appropriate.

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5 Appendix

5.1 Matlab Routines

5.1.1 Baxter King Filter

```
%Program name:  BPF
   function yf=bpf(y,up,dn,K);
   %bpf.m
   %Program to compute bp filter
   %Inputs are:
   %y:  data(rows=observations,columns=series)
   %up:period corresponding to highest frequency (e.g.,6)
   %dn:  period corresponding to lowest frequency (e.g.,32)
   %K: number of terms in approximating moving average
   %[calls filtk.m (filter with symmetric weights) as subroutine],
The
   %frequencies are chosen according to Burns and Mitchell.  With quarterly
   %data x=[32 6].  Since I am using monthly data x vector is [96 18].
   up=96; %
   dn=18;
   K=12;
   x=[up dn];
   disp(”)
   disp('bpf(y,up,dn,K):band pass filtering of series y with symmetric
MA(2K+1)')
   disp(”)
   disp(' for additional information see:')
   disp(”)
   disp(' M.Baxter and R.G. King')
   disp(”)
   disp(' Measuring Business Cycles:')
   disp(' Approximate Band-Bass Filters')
   disp(' for Macroeconomic Time Series')
   disp(”)
   disp('Filter extracts components between period of:')
   disp(' up dn')
   disp(x)
   %pause(2)
   if(up>dn)
    disp('Periods reversed:  switcing indices up & dn')
    disp(”)
    dn=x(1);
    up=x(2);
   end
   if (up<2)
```

```
 up=2;
 disp('Higher periodicity >max:  Setting up=2')
 disp('')
end
%convert to column vector
[r c]=size(y);
if(r<c)
 y=y'
 disp('There are more columns than rows:  Transposing data matrix')
 disp('')
end
%Implied Frequencies
omubar=2*pi/up;
omlbar=2*pi/dn;
% An approximate low pass filter , with a cutoff frequency of "ombar",
%has a frequency response function
%alpha(om)=a0+2*a1cos(om)+...+2*aKcos(K om)
%and the ak's are given by:
%a0=ombar/(pi) ak=sin(k ombar)/(k pi)
%where ombar is the cutoff frequency.
% A band pass filter is the difference between two
%low pass filter,bp(L)=bu(L)-bl(L) with bu(L) being filter with
high cutoff
%point and bl(L) being filter with low cutoff
%point.  Thus the weights are differences of weights for two low
pass
%filters.
% Construct filter weights for bandpass filter (a(0)...a(K)).
akvec=zeros(1,1:K+1);
akvec(1)=(omubar-omlbar)/(pi); %weight at k=0
for k=1:K;
 akvec(k+1)=(sin(k*omubar)-sin(k*omlbar))/(k*pi); %weight at k=1,2,...K
end
%Impose constraint on frequency response at om=0
%If high pass filter this amounts to requiring that weights sum
to zero
%If low pass filter this amounts to requiring that weights sum to
one
if (dn>1000)
 disp('dn>1000:assuming low pass filter')
 phi=1;
else
 phi=0;
end
%Sum of weights without constraint
theta=akvec(1)+2*sum(akvec(2:K+1));
```

```
%amount to add to each nonzero lag/lead to get sum=phi
theta=phi-(theta/(2*K+1));
%adjustment of weights
akvec=akvec+theta;
%filter the time series
yf=filtk(y,akvec);
if(r<c)
 yf=yf;
end


%Program name :  FILTK.M
function yf=filtk(y,a);
%Filter data with a filter with symmetric filter with weights data
is
%organized (rows=obs,columns=series)
%a=[a0,a1,...,aK];
K=max(size(a))-1; %max lag;
T=max(size(y)); %number of observations;
%Set vector of weights
avec=zeros(1,2*K+1);
avec(K+1)=a(1);
for i=1:K;
 avec(K+1-i)=a(i+1);
 avec(K+1+i)=a(i+1);
end
yf=zeros(y);
for t=K+1:1:T-K
 yf(t,:)=avec*y(t-K:t+K,:);
end
```

5.1.2 Hodrick- Prescott Filter

```
%Program Name:  HPF
% If x is a column vector of length LENGTH
% xtr=HP_mat\x; delivers the HP-trend and
% xhp=x-xtr; delivers the HP-filtered series
% This program computes HP_mat, given the length of
% some given column vector x
% I will use HP_LAMBDA = 6400, unless you assign a different value
beforehand.
HP_LAMBDA = 6400;
disp('If x is a column vector of length LENGTH,');
disp('xtr=HP_mat\x; delivers the HP-trend and');
disp('xhp=x-xtr; delivers the HP-filtered series');
disp('This program computes HP_mat, given the length of');
```

```
disp('some given column vector x');
LENGTH = max(size(x));
if ~exist('HP_LAMBDA'),
 HP_LAMBDA = 1600;
end;
% The following piece is due to Gerard A. Pfann
 HP_mat = [1+HP_LAMBDA, -2*HP_LAMBDA, HP_LAMBDA, zeros(1,LENGTH-3);
 -2*HP_LAMBDA,1+5*HP_LAMBDA,-4*HP_LAMBDA,HP_LAMBDA, zeros(1,LENGTH-4);
 zeros(LENGTH-4,LENGTH);
 zeros(1,LENGTH-4),HP_LAMBDA,-4*HP_LAMBDA,1+5*HP_LAMBDA,-2*HP_LAMBDA;

 zeros(1,LENGTH-3), HP_LAMBDA, -2*HP_LAMBDA, 1+HP_LAMBDA ];
 for iiiii=3:LENGTH-2;
 HP_mat(iiiii,iiiii-2)=HP_LAMBDA;
 HP_mat(iiiii,iiiii-1)=-4*HP_LAMBDA;
 HP_mat(iiiii,iiiii)=1+6*HP_LAMBDA;
 HP_mat(iiiii,iiiii+1)=-4*HP_LAMBDA;
 HP_mat(iiiii,iiiii+2)=HP_LAMBDA;
 end;
xtr=HP_mat\x;
xhp=x-xtr;
```

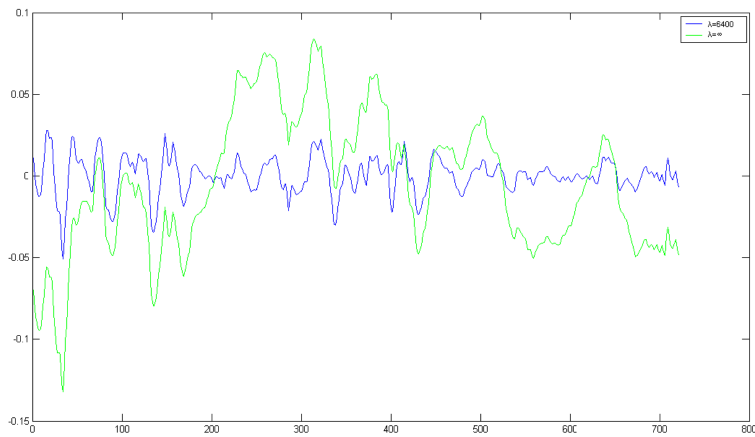

Figure 1: Smoothness of λ

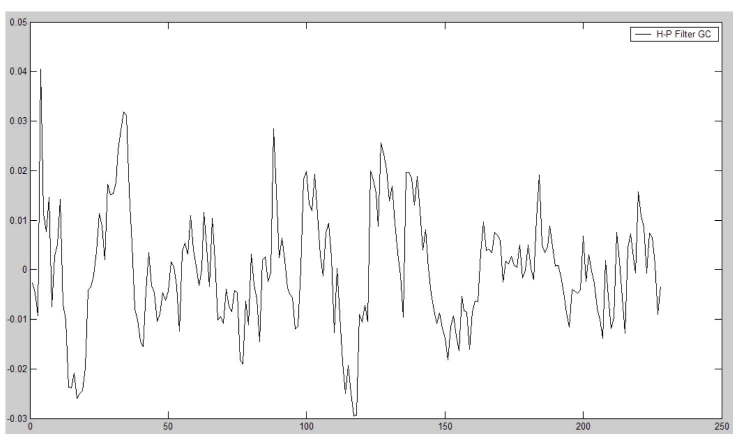

Figure 2: H-P Filter Government Consumption

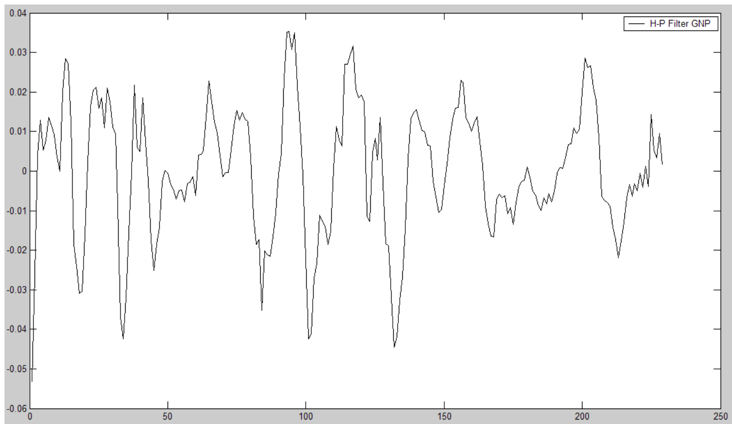

Figure 3: H-P Filter, Gross National Product

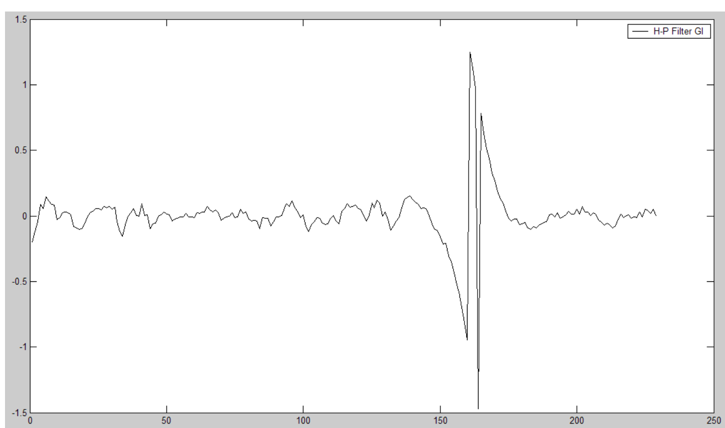

Figure 4: H-P Filter, Government Investment

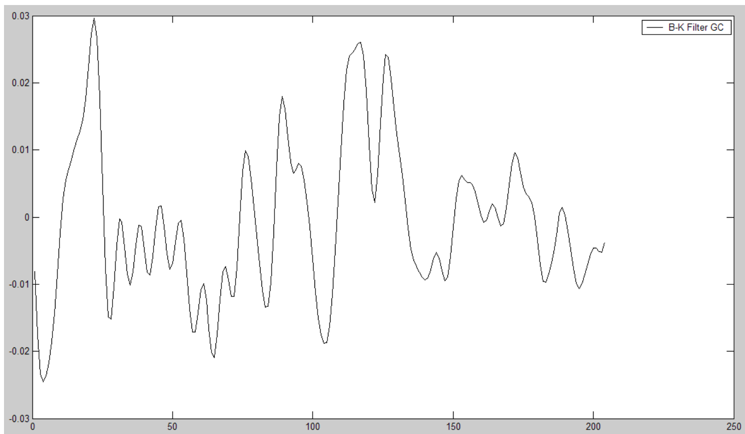

Figure 5: B-K Filter, Government Consumption

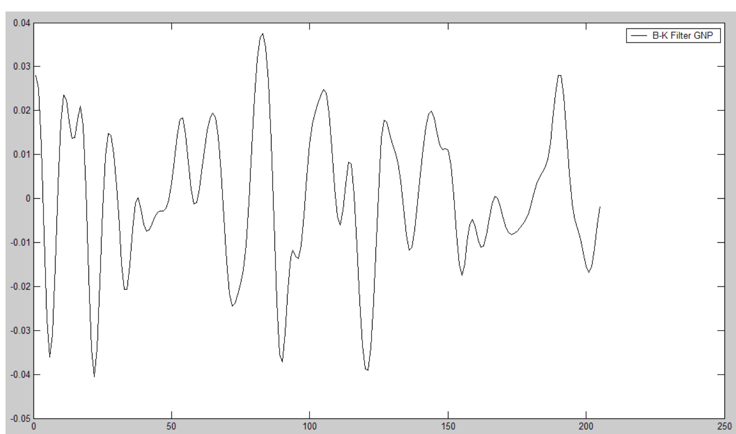

Figure 6: B-K Filter, Gross National Product

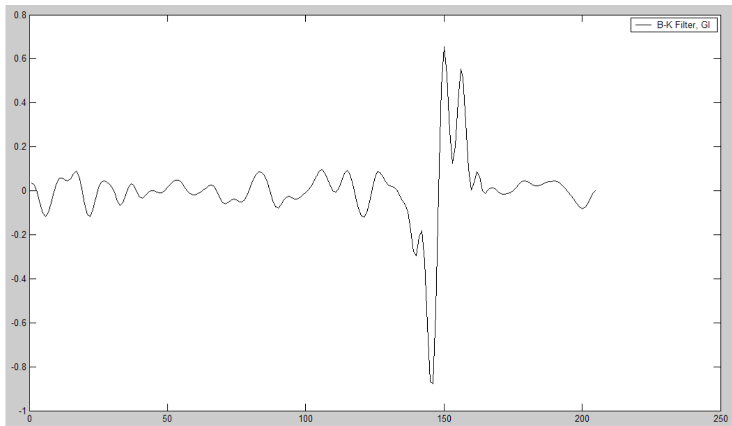

Figure 7: B-K Filter, Government Investment

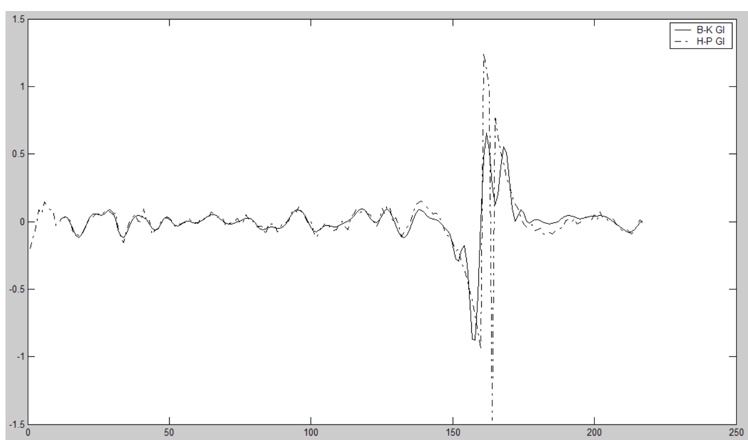

Figure 8: H-P Filter vs B-K Filter

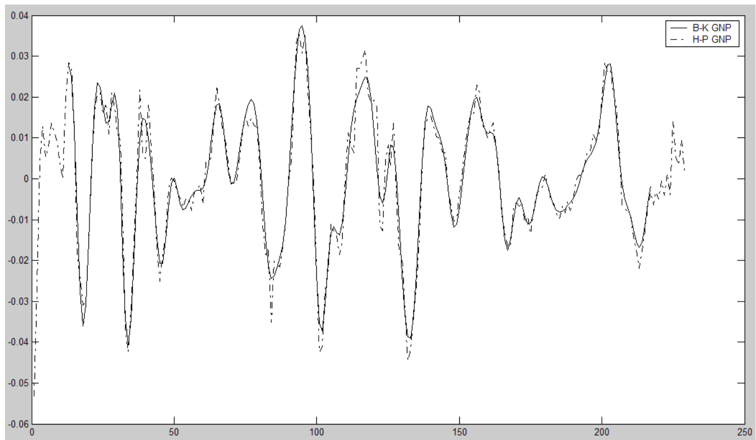

Figure 9: H-P Filter vs B-K Filter

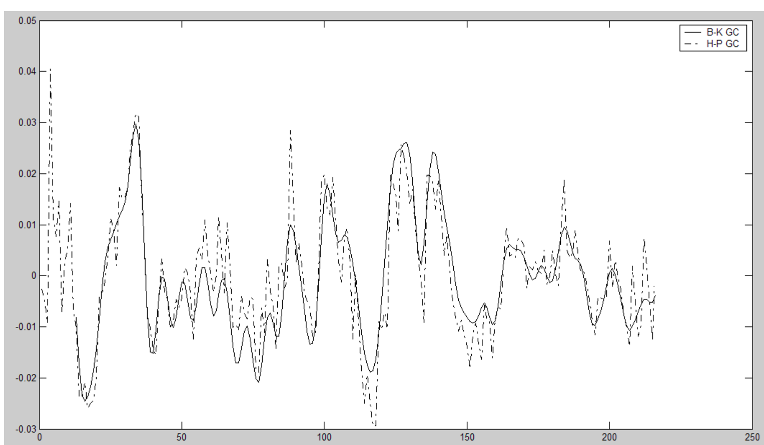

Figure 10: H-P Filter vs B-K Filter