MODELLING TIME SERIES OF CARBON DIOXIDE EMISSIONES IN ROMA CITY

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1. Introduction

The aim of the present work is to report and analyse time series of carbon dioxide emissions recorded in Roma city centre. In the first part of the paper the importance and the role of the carbon dioxide are explained; in the second part, using EVIEWS software, the series have been modelled taking into account the literature review, while at the end for generalizing the univariate AR models, the VAR model is used looking at interdependencies between multiple time series.

2. The Carbon dioxide

Carbon dioxide is a chemical compound, normally a colorless, odorless and neutral gas, and is composed of one carbon and two oxygen atoms, see Figure 1. It is often referred to by its formula **CO_2**. Carbon dioxide is present in the Earth's atmosphere at a concentration of approximately 0.000383 by volume (383 ppm) and is an important greenhouse gas due to its ability to absorb many infrared wavelengths of sunlight, and due to the length of time it stays in the atmosphere. It is also a major component of the carbon cycle. In its solid state, carbon dioxide is called dry ice. CO_2 has no liquid state at normal atmospheric pressure. Its density at standard temperature and pressure is around 1.98 kg/m³, about 1.53 times that of air. The carbon dioxide molecule (O=C=O) contains two double bonds and has a linear shape. It has no electrical dipole. As it is fully oxidized, it is not very reactive and is non-flammable and therefore is considered neutral.

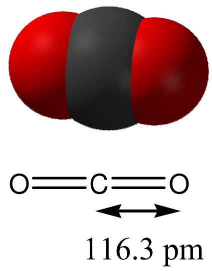


Figure 1: chemical structure of carbon dioxide

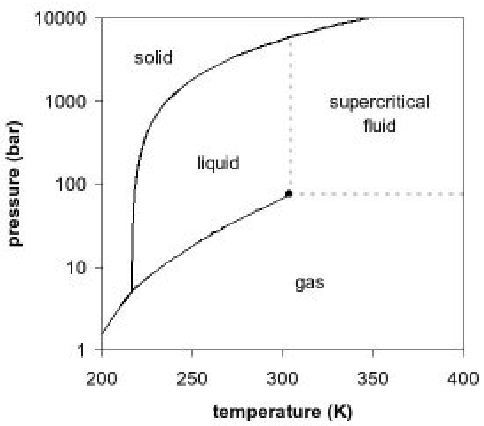


Figure 2: Carbon dioxide pressure-temperature phase diagram

Under normal atmospheric pressure (1 atm) at −78.5 °C, carbon dioxide changes directly from a solid phase to a gaseous phase through sublimation or gaseous to solid through deposition, see Figure 2. The solid form is typically called "dry ice". Liquid carbon dioxide forms only at pressures above 4.0-5.1 atm, depending on temperature. Specifically, the triple point is 416.7 kPa at -56.6 °C The critical point is 7821 kPa at 31.1 °C.[1]

2. Principle roles of CO_2

All aerobic organisms produce CO_2 when they oxidise carbohydrates, fatty acids and proteins in the mitochondria of cells; it is the prime energy source and the main metabolic pathway in heterotrophic organisms such as animals, and also a secondary energy source in prototroph organisms such as plants when not enough light is available for photosynthesis.

Biological role: Carbon dioxide is an end product in organisms that obtain energy from breaking down sugars, fats and amino acids with oxygen as part of their metabolism, in a process known as cellular respiration. This includes all plants, animals, many fungi and some bacteria. In higher animals, the carbon dioxide travels in the blood from the body's tissues to the lungs where it is exhaled. In plants using photosynthesis, carbon dioxide is absorbed from the atmosphere.

Role in photosynthesis: Plants remove carbon dioxide from the atmosphere by photosynthesis, also called carbon assimilation, which uses light energy to produce organic plant materials by combining carbon dioxide and water. Free oxygen is released as gas from the decomposition of water molecules, while the hydrogen is split into its protons and electrons and used to generate chemical energy via photophosphorylation. This energy is required for the fixation of carbon dioxide in the Calvin cycle to form sugars.

These sugars can then be used for growth within the plant through respiration. Carbon dioxide gas must be introduced into greenhouses to maintain plant growth, as even in vented greenhouses the concentration of carbon dioxide can fall during daylight hours to as low as 200 ppm, at which level photosynthesis is significantly reduced. Plants can potentially grow up to 50 percent faster in concentrations of 1000 ppm CO_2 when compared with ambient conditions.

Plants also emit CO_2 during respiration, so it is only during growth stages that plants are net absorbers. For example a growing forest will absorb many tonnes of CO_2 each year, however a mature forest will produce as much CO_2 from respiration and decomposition of dead specimens (e.g. fallen branches) as used in biosynthesis in growing plants. Regardless of this, mature forests are still valuable carbon sinks, helping maintain balance in the Earth's atmosphere.

The lowering of carbon dioxide in the atmosphere is largely due to absorption by plants, which convert it to sugars through photosynthesis.

Animal toxicity. Carbon dioxide content in fresh air varies and is between 0.03% (300 ppm - Parts Per Million; by volume) and 0.06% (600 ppm), depending on location. Exhaled breath is approximately 4.5% carbon dioxide. When inhaled in high concentrations (greater than 5% by volume, or 50000 ppm), it is immediately dangerous to the life and health of humans and other animals. The current threshold limit value (TLV) or maximum level that is considered safe for healthy adults for an 8-hour work day is 0.5% (5000 ppm). The maximum safe level for infants, children, the elderly and individuals with cardio-pulmonary health issues would be significantly less.

These figures are valid for carbon dioxide supplied in "pure" form. In indoor spaces occupied by humans the carbon dioxide concentration will also reach a level higher than in pure outdoor air. Concentrations higher than 1000 ppm will cause discomfort in more than 20% of occupants, and the discomfort will increase with increasing CO_2 concentration. The discomfort will be caused by various gases coming from human respiration and perspiration, and not by CO_2 itself. At 2000 ppm the majority of occupants will feel a significant degree of discomfort, and many will develop nausea and headache. The CO_2 concentration between 300 and 2500 ppm is used as an indicator of indoor air quality in spaces polluted by human occupation. *Human physiology:* According to a study by the USDA,[2] an average person's respiration generates approximately 450 liters (roughly 900 grams) of carbon dioxide per day.

3. Carbon dioxide in the Earth's atmosphere and pollution

Despite the low concentration, CO_2 is a very important component of the Earth's atmosphere because it absorbs infrared radiation at wavelengths of 4.26 µm (asymmetric stretching vibration mode) and 14.99 µm (bending vibration mode) and enhances the greenhouse effect to a great degree.[3] With a radioactive forcing of about 1.5 W/m^2, it is relatively twice as powerful as the next major forcing greenhouse gas, methane, and relatively ten times as powerful as the third, nitrous oxide. Carbon dioxide alone contributes up to 12% to the greenhouse effect.

At low concentrations, it induces fatigue in healthy people and chest pain in people with heart disease. At higher concentrations, it causes impaired vision and coordination, headaches, dizziness, confusion; nausea. It could cause flu-like symptoms that clear up after leaving home. It is fatal at very high concentrations. Acute effects are due to the formation of carboxyhemoglobin in the blood. At moderate concentrations it induces angina, impaired vision, and reduced brain function may result. As part of the current scientific opinion that excess amounts of carbon dioxide produced by humans in the atmosphere lead to global warming, various methods of limiting or removing the amount of carbon dioxide in the atmosphere have been suggested. Current debate on the subject mostly involves economic or political matters at a policy level.

4. The data collection

Carbon dioxide emissions is recorded daily, each hour, by many places (stations) in Roma city center, during 2002 year, (365 observations). The most common principles for CO sensors are infrared gas sensors (NDIR) and chemical gas sensors. In the following map (figure 3) the positioning of recording stations are shown. It is important to underline that they are placed in strategic points for tourist and citizen flows. .

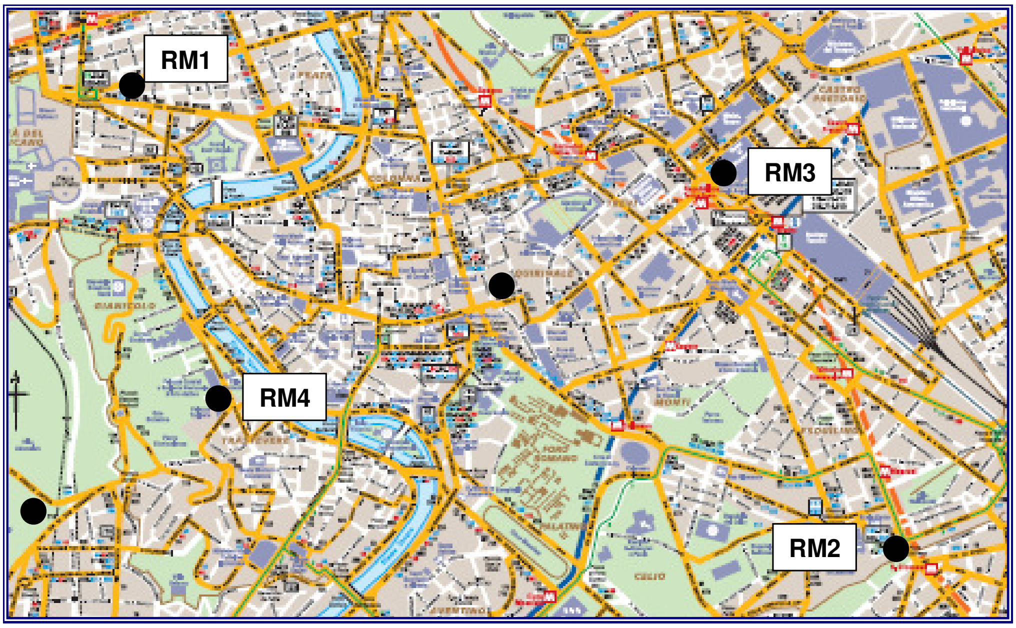
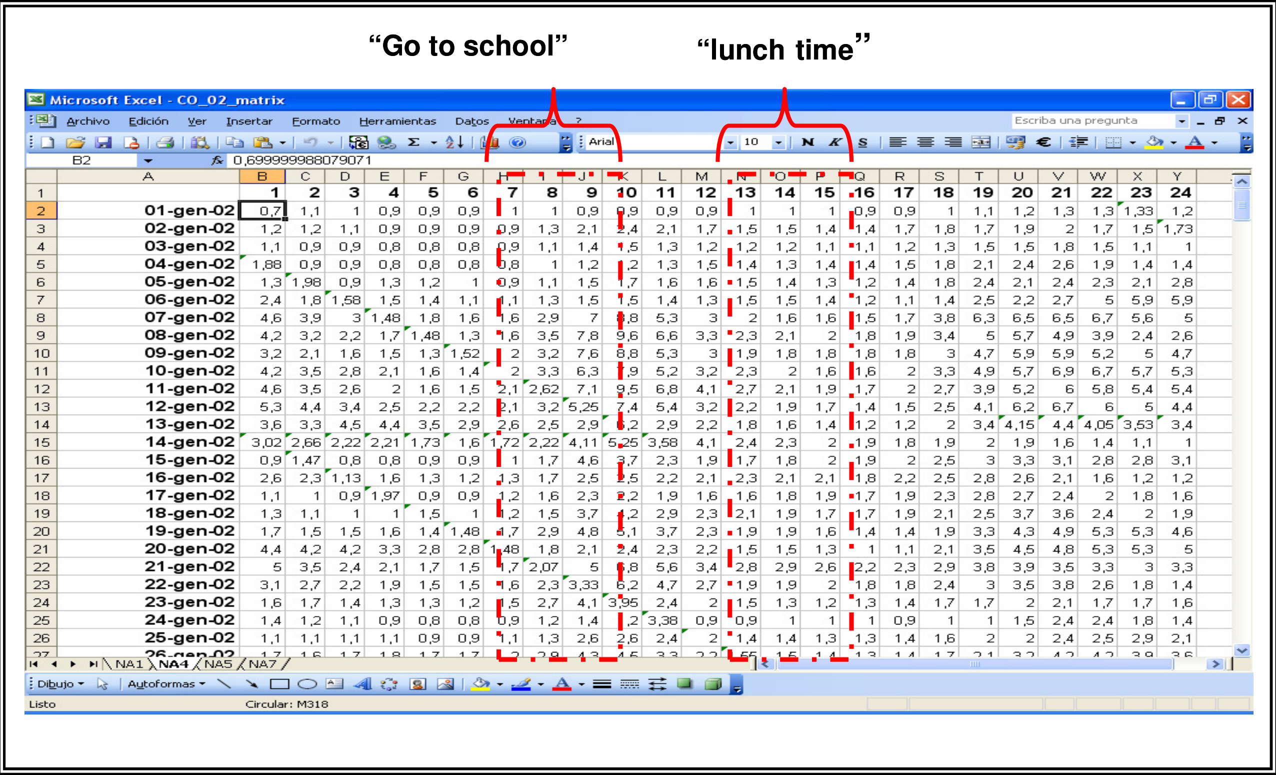


Figure 3: Recording stations positioning. (Source: our elaboration)

The place "RM3", for instance, is near the train station, "RM1" near the Vatican city, "RM4" is in the school district, while "RM2" is near one of the most important consular road: the Appia.

The data on which this paper is focused had been recorded in 2002. Since children are the most risk target, it has been decided to refer our study on the time in which children are more exposed to emissions and road traffic is higher, that is, the time "back to home from school" (lunch time, from 1 pm to 3 pm, figure 4). A study of more than 4,000 Dutch infants has concluded that young children who live close to busy roads are more at risk of developing respiratory diseases such as asthma For those children living close to busy roads, the study shows an average of 20% to 30% increased likelihood of asthma, wheezing, ear, nose and throat infections, colds and flu. (M.Jarret, 05).

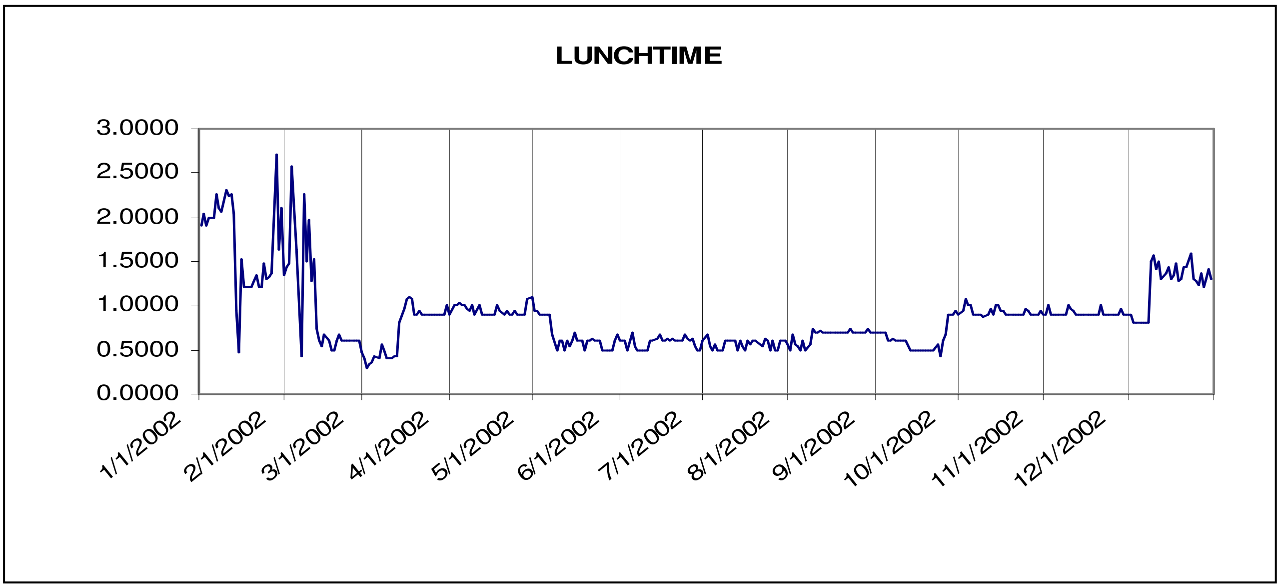
The U.S. National Ambient Air Quality Standards for outdoor air are 9 ppm (40,000 micrograms per meter cubed) for 8 hours, and 35 ppm for 1 hour. (Source: NAAQS, 2004)

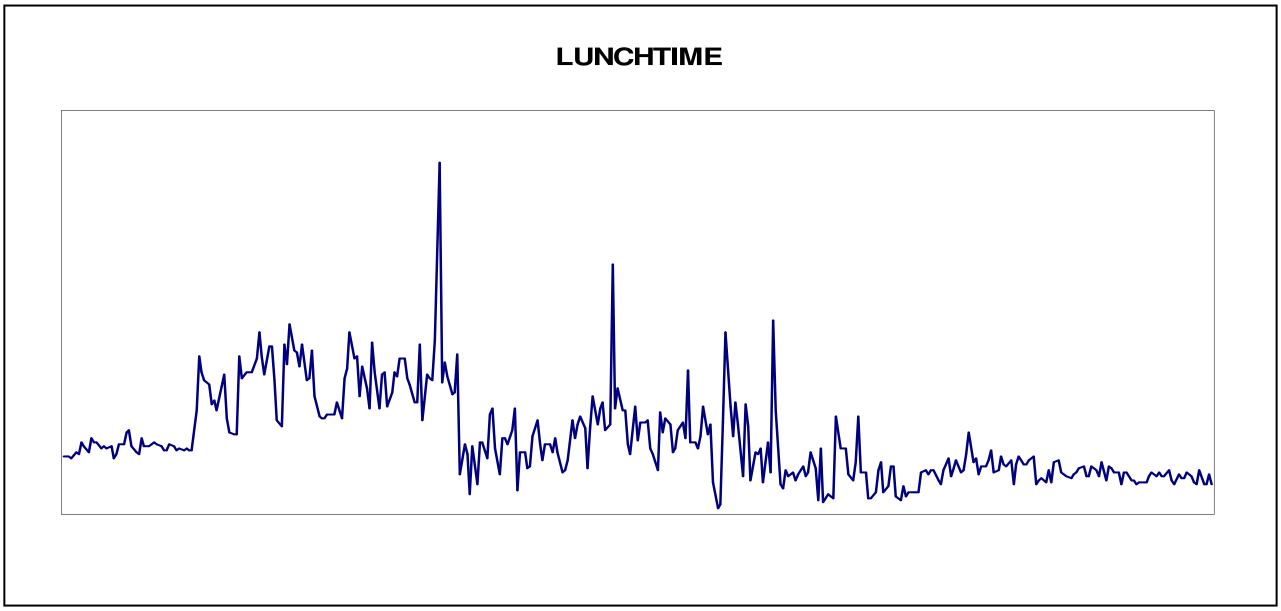


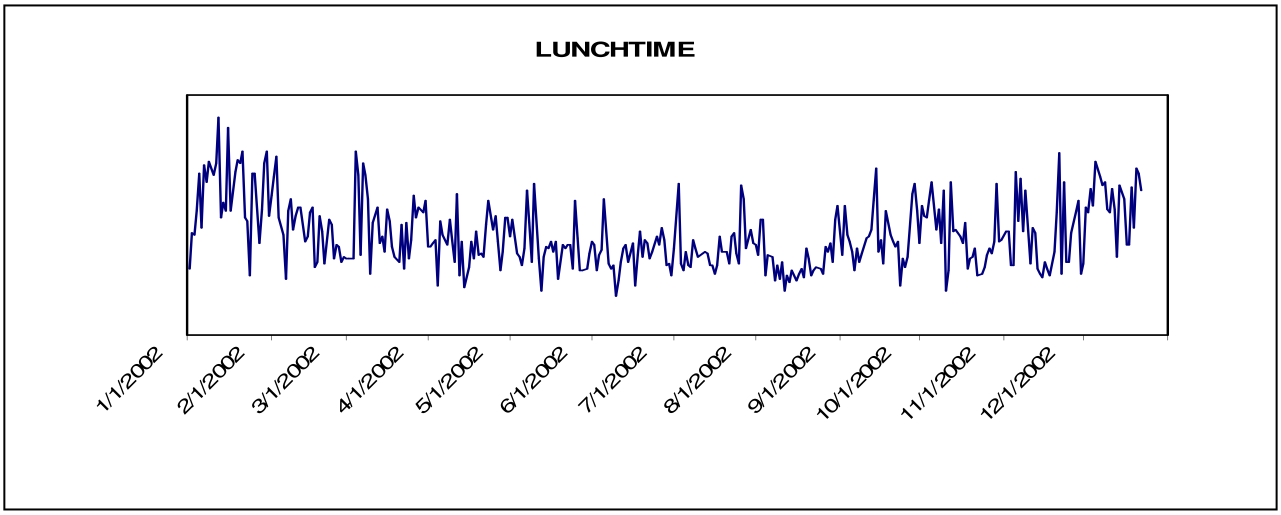
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y
1		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
2	01-gen-02	0,7	1,1	1	0,9	0,9	0,9	1	1	0,9	0,9	0,9	0,9	1	1	1	0,9	0,9	1	1,1	1,2	1,3	1,3	1,33	1,2
3	02-gen-02	1,2	1,2	1,1	0,9	0,9	0,9	0,9	1,3	2,1	2,4	2,1	1,7	1,5	1,5	1,4	1,4	1,7	1,8	1,7	1,9	2	1,7	1,5	1,73
4	03-gen-02	1,1	0,9	0,9	0,8	0,8	0,8	0,9	1,1	1,4	1,5	1,3	1,2	1,2	1,2	1,1	1,1	1,2	1,3	1,5	1,5	1,8	1,5	1,1	1
5	04-gen-02	1,88	0,9	0,9	0,8	0,8	0,8	0,8	1	1,2	1,2	1,3	1,5	1,4	1,3	1,4	1,4	1,5	1,8	2,1	2,4	2,6	1,9	1,4	1,4
6	05-gen-02	1,3	1,98	0,9	1,3	1,2	1	0,9	1,1	1,5	1,7	1,6	1,6	1,5	1,4	1,3	1,2	1,4	1,8	2,4	2,1	2,4	2,3	2,1	2,8
7	06-gen-02	2,4	1,8	1,58	1,5	1,4	1,1	1,1	1,3	1,5	1,5	1,4	1,3	1,5	1,5	1,4	1,2	1,1	1,4	2,5	2,2	2,7	5	5,9	5,9
8	07-gen-02	4,6	3,9	3	1,48	1,8	1,6	1,6	2,9	7	8,8	5,3	3	2	1,6	1,6	1,5	1,7	3,8	6,3	6,5	6,5	6,7	5,6	5
9	08-gen-02	4,2	3,2	2,2	1,7	1,48	1,3	1,6	3,5	7,8	9,6	6,6	3,3	2,3	2,1	2	1,8	1,9	3,4	5	5,7	4,9	3,9	2,4	2,6
10	09-gen-02	3,2	2,1	1,6	1,5	1,3	1,52	2	3,2	7,6	8,8	5,3	3	1,9	1,8	1,8	1,8	1,8	3	4,7	5,9	5,9	5,2	5	4,7
11	10-gen-02	4,2	3,5	2,8	2,1	1,6	1,4	2	3,3	6,3	7,9	5,2	3,2	2,3	2	1,6	1,6	2	3,3	4,9	5,7	6,9	6,7	5,7	5,3
12	11-gen-02	4,6	3,5	2,6	2	1,6	1,5	2,1	2,62	7,1	9,5	6,8	4,1	2,7	2,1	1,9	1,7	2	2,7	3,9	5,2	6	5,8	5,4	5,4
13	12-gen-02	5,3	4,4	3,4	2,5	2,2	2,2	2,1	3,2	5,25	7,4	5,4	3,2	2,2	1,9	1,7	1,4	1,5	2,5	4,1	6,2	6,7	6	5	4,4
14	13-gen-02	3,6	3,3	4,5	4,4	3,5	2,9	2,6	2,5	2,9	6,2	2,9	2,2	1,8	1,6	1,4	1,2	1,2	2	3,4	4,15	4,4	4,05	3,53	3,4
15	14-gen-02	3,02	2,66	2,22	2,21	1,73	1,6	1,72	2,22	4,11	5,25	3,58	4,1	2,4	2,3	2	1,9	1,8	1,9	2	1,9	1,6	1,4	1,1	1
16	15-gen-02	0,9	1,47	0,8	0,8	0,9	0,9	1	1,7	4,6	3,7	2,3	1,9	1,7	1,8	2	1,9	2	2,5	3	3,3	3,1	2,8	2,8	3,1
17	16-gen-02	2,6	2,3	1,13	1,6	1,3	1,2	1,3	1,7	2,5	2,5	2,2	2,1	2,3	2,1	2,1	1,8	2,2	2,5	2,8	2,6	2,1	1,6	1,2	1,2
18	17-gen-02	1,1	1	0,9	1,97	0,9	0,9	1,2	1,6	2,3	2,2	1,9	1,6	1,6	1,8	1,9	1,7	1,9	2,3	2,8	2,7	2,4	2	1,8	1,6
19	18-gen-02	1,3	1,1	1	1	1,5	1	1,2	1,5	3,7	4,2	2,9	2,3	2,1	1,9	1,7	1,7	1,9	2,1	2,5	3,7	3,6	2,4	2	1,9
20	19-gen-02	1,7	1,5	1,5	1,6	1,4	1,48	1,7	2,9	4,8	5,1	3,7	2,3	1,9	1,9	1,6	1,4	1,4	1,9	3,3	4,3	4,9	5,3	5,3	4,6
21	20-gen-02	4,4	4,2	4,2	3,3	2,8	2,8	1,48	1,8	2,1	2,4	2,3	2,2	1,5	1,5	1,3	1	1,1	2,1	3,5	4,5	4,8	5,3	5,3	5
22	21-gen-02	5	3,5	2,4	2,1	1,7	1,5	1,7	2,07	5	6,8	5,6	3,4	2,8	2,9	2,6	2,2	2,3	2,9	3,8	3,9	3,5	3,3	3	3,3
23	22-gen-02	3,1	2,7	2,2	1,9	1,5	1,5	1,6	2,3	3,33	6,2	4,7	2,7	1,9	1,9	2	1,8	1,8	2,4	3	3,5	3,8	2,6	1,8	1,4
24	23-gen-02	1,6	1,7	1,4	1,3	1,3	1,2	1,5	2,7	4,1	3,95	2,4	2	1,5	1,3	1,2	1,3	1,4	1,7	1,7	2	2,1	1,7	1,7	1,6
25	24-gen-02	1,4	1,2	1,1	0,9	0,8	0,8	0,9	1,2	1,4	[illegible]	3,38	0,9	0,9	1	1	1	0,9	1	1	1,5	2,4	2,4	1,8	1,4
26	25-gen-02	1,1	1,1	1,1	1,1	0,9	0,9	1,1	1,3	2,6	2,6	2,4	2	1,4	1,4	1,3	1,3	1,4	1,6	2	2	2,4	2,5	2,9	2,1

Figure 4: the database

By taking the mean of the three observations in the period "lunch time" four times series have been obtained, as shown in the following picture 5. (RM1, RM2, RM3, RM4). They show different trends, with some outliers and no stationarity.







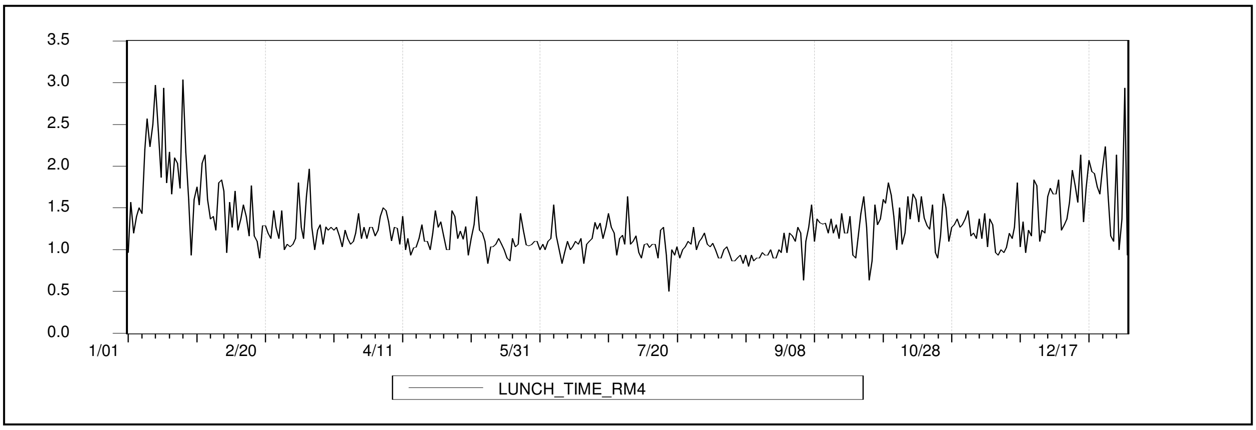
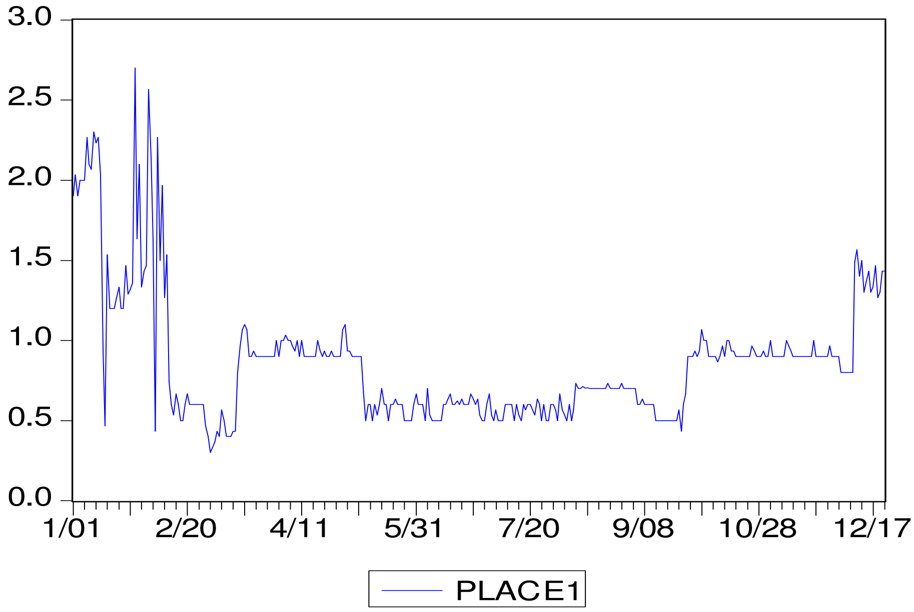


Figure 5: the original times series of carbon dioxide emissions:RM1, RM5, RM4, RM7,respectively. (Source: our elaboration)

5. Modelling time series

In this part of the paper we choose a model for each of the four series (Place_1, Place_2, Place_3, Place_4). In the last part, the VAR model **(Vector autoregression)** to capture the evolution and the interdependencies between multiple time series is chosen.

Place 1 (ROMA_1)



To check stationarity of the series place_1 (or roma_1) Dickey-Fuller test was used:

Dickey-Fuller unit root test

ADF Test Statistic	-3.383272	1% Critical Value*	-3.4509
		5% Critical Value	-2.8700
		10% Critical Value	-2.5712

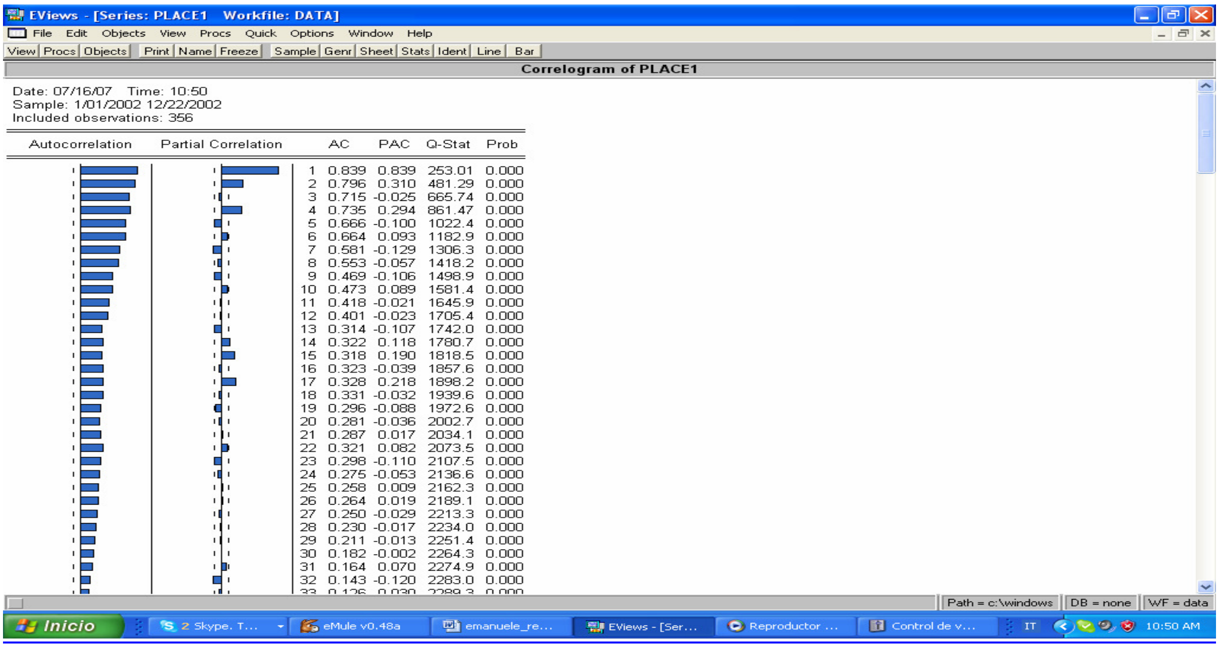
*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(PLACE1)
Method: Least Squares
Date: 07/13/07 Time: 18:51
Sample(adjusted): 1/06/2002 12/22/2002
Included observations: 351 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
PLACE1(-1)	-0.093492	0.027634	-3.383272	0.0008
D(PLACE1(-1))	-0.344311	0.056070	-6.140768	0.0000
D(PLACE1(-2))	-0.123533	0.055705	-2.217604	0.0272
D(PLACE1(-3))	-0.308220	0.055182	-5.585529	0.0000
D(PLACE1(-4))	0.057928	0.053222	1.088426	0.2772
C	0.076450	0.025598	2.986551	0.0030

R-squared	0.295459	Mean dependent var	-0.001614
Adjusted R-squared	0.285249	S.D. dependent var	0.221804
S.E. of regression	0.187519	Akaike info criterion	-0.492922
Sum squared resid	12.13143	Schwarz criterion	-0.426925
Log likelihood	92.50775	F-statistic	28.93617
Durbin-Watson stat	1.987829	Prob(F-statistic)	0.000000

We can see that the series place_1 (roma_1) is not stationary. The correlogram of residuals is:



Date: 07/16/07 Time: 10:50
Sample: 1/01/2002 12/22/2002
Included observations: 356

	AC	PAC	Q-Stat	Prob
1	0.839	0.839	253.01	0.000
2	0.796	0.310	481.29	0.000
3	0.715	-0.025	665.74	0.000
4	0.735	0.294	861.47	0.000
5	0.666	-0.100	1022.4	0.000
6	0.664	0.093	1182.9	0.000
7	0.581	-0.129	1306.3	0.000
8	0.553	-0.057	1418.2	0.000
9	0.469	-0.106	1498.9	0.000
10	0.473	0.089	1581.4	0.000
11	0.418	-0.021	1645.9	0.000
12	0.401	-0.023	1705.4	0.000
13	0.314	-0.107	1742.0	0.000
14	0.322	0.118	1780.7	0.000
15	0.318	0.190	1818.5	0.000
16	0.323	-0.039	1857.6	0.000
17	0.328	0.218	1898.2	0.000
18	0.331	-0.032	1939.6	0.000
19	0.296	-0.088	1972.6	0.000
20	0.281	-0.036	2002.7	0.000
21	0.287	0.017	2034.1	0.000
22	0.321	0.082	2073.5	0.000
23	0.298	-0.110	2107.5	0.000
24	0.275	-0.053	2136.6	0.000
25	0.258	0.009	2162.3	0.000
26	0.264	0.019	2189.1	0.000
27	0.250	-0.029	2213.3	0.000
28	0.230	-0.017	2234.0	0.000
29	0.211	-0.013	2251.4	0.000
30	0.182	-0.002	2264.3	0.000
31	0.164	0.070	2274.9	0.000
32	0.143	-0.120	2283.0	0.000
33	0.126	0.030	2289.3	0.000

If we look at the graph of the series roma_1 (place_1), it can be observed that every month the series roma_1 has different constants, they oscillate around different levels from month to month. Further, we will see that this is true for all series: roma_1, roma_2, roma_3, roma_4. The reason of this could be the influence of the temperature on the carbon dioxide emissions in Roma. We will "clean" these series from these fluctuations by regressing the series roma_1 on the dummy variables of the month.

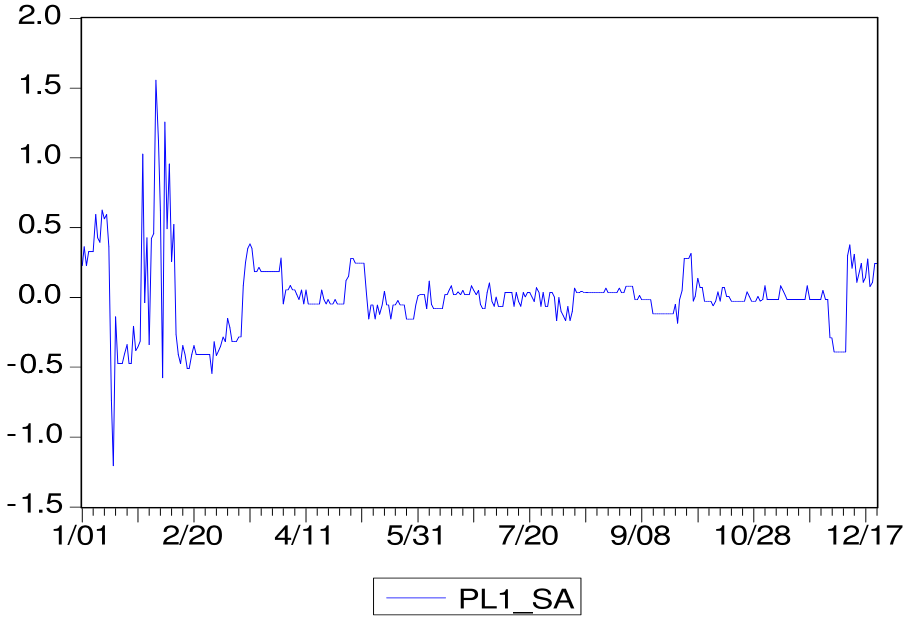
From the table below it is easy to sea that all twelve dummy variables are significant.

Dependent Variable: PLACE1
Method: Least Squares
Date: 07/13/07 Time: 18:50
Sample: 1/01/2002 12/22/2002

Variable	Coefficient	Std. Error	t-Statistic	Prob.
M1	1.672003	0.047724	35.03503	0.0000
M2	1.009921	0.050215	20.11180	0.0000
M3	0.716129	0.047724	15.00572	0.0000
M4	0.946667	0.048513	19.51382	0.0000
M5	0.654122	0.047724	13.70642	0.0000
M6	0.581852	0.048513	11.99382	0.0000
M7	0.563441	0.047724	11.80630	0.0000
M8	0.666229	0.047724	13.96011	0.0000
M9	0.617778	0.048513	12.73437	0.0000
M10	0.926882	0.047724	19.42181	0.0000
M11	0.915556	0.048513	18.87252	0.0000
M12	1.190404	0.056651	21.01311	0.0000

R-squared	0.582234	Mean dependent var	0.863722
Adjusted R-squared	0.568875	S.D. dependent var	0.404682
S.E. of regression	0.265715	Akaike info criterion	0.220339
Sum squared resid	24.28786	Schwarz criterion	0.350954
Log likelihood	-27.22030	Durbin-Watson stat	0.748879

The result of cleaning is the following series PL1_SA:



The PL1_SA seams to be stationary, the results of the unit root test are:

ADF Test Statistic	-5.149518	1% Critical Value*	-3.4509
		5% Critical Value	-2.8700
		10% Critical Value	-2.5712

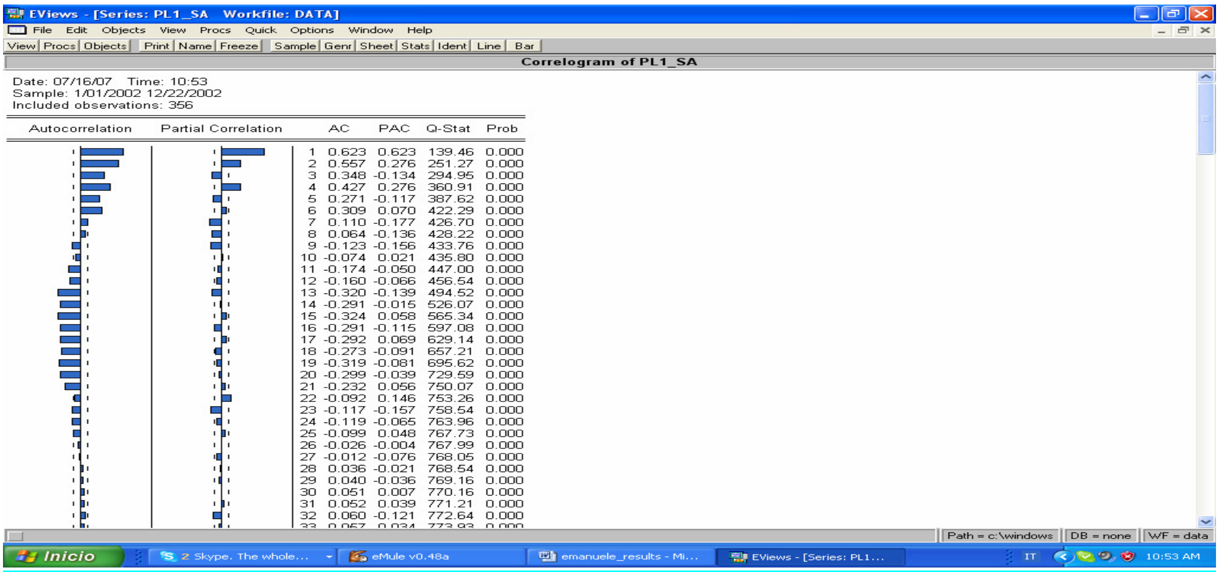
*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(PL1_SA)
Method: Least Squares
Date: 07/13/07 Time: 20:41
Sample(adjusted): 1/06/2002 12/22/2002
Included observations: 351 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
PL1_SA(-1)	-0.251612	0.048861	-5.149518	0.0000
D(PL1_SA(-1))	-0.196165	0.062922	-3.117580	0.0020
D(PL1_SA(-2))	0.018042	0.058538	0.308206	0.7581
D(PL1_SA(-3))	-0.225757	0.057223	-3.945212	0.0001
D(PL1_SA(-4))	0.110681	0.053474	2.069808	0.0392
C	-0.001478	0.009998	-0.147786	0.8826

R-squared	0.333037	Mean dependent var	-0.000242
Adjusted R-squared	0.323371	S.D. dependent var	0.227678
S.E. of regression	0.187282	Akaike info criterion	-0.495452
Sum squared resid	12.10077	Schwarz criterion	-0.429456
Log likelihood	92.95188	F-statistic	34.45408
Durbin-Watson stat	1.985083	Prob(F-statistic)	0.000000

The correlogram of residuals is:



Date: 07/16/07 Time: 10:53
Sample: 1/01/2002 12/22/2002
Included observations: 356

	AC	PAC	Q-Stat	Prob
1	0.623	0.623	139.46	0.000
2	0.557	0.276	251.27	0.000
3	0.348	-0.134	294.95	0.000
4	0.427	0.276	360.91	0.000
5	0.271	-0.117	387.62	0.000
6	0.309	0.070	422.29	0.000
7	0.110	-0.177	426.70	0.000
8	0.064	-0.136	428.22	0.000
9	-0.123	-0.156	433.76	0.000
10	-0.074	0.021	435.80	0.000
11	-0.174	-0.050	447.00	0.000
12	-0.160	-0.066	456.54	0.000
13	-0.320	-0.139	494.52	0.000
14	-0.291	-0.015	526.07	0.000
15	-0.324	0.058	565.34	0.000
16	-0.291	-0.115	597.08	0.000
17	-0.292	0.069	629.14	0.000
18	-0.273	-0.091	657.21	0.000
19	-0.319	-0.081	695.62	0.000
20	-0.299	-0.039	729.59	0.000
21	-0.232	0.056	750.07	0.000
22	-0.092	0.146	753.26	0.000
23	-0.117	-0.157	758.54	0.000
24	-0.119	-0.065	763.96	0.000
25	-0.099	0.048	767.73	0.000
26	-0.026	-0.004	767.99	0.000
27	-0.012	-0.076	768.05	0.000
28	0.036	-0.021	768.54	0.000
29	0.040	-0.036	769.16	0.000
30	0.051	0.007	770.16	0.000
31	0.052	0.039	771.21	0.000
32	0.060	-0.121	772.64	0.000

Now the ARMA model for PL1_SA series will be built so that the residuals will be uncorrelated. After the ARCH test will be made and using the results of the ARCH test, the GARCH model will be built if necessary.

The results of building AR(9) model for PL1_SA are shown below:

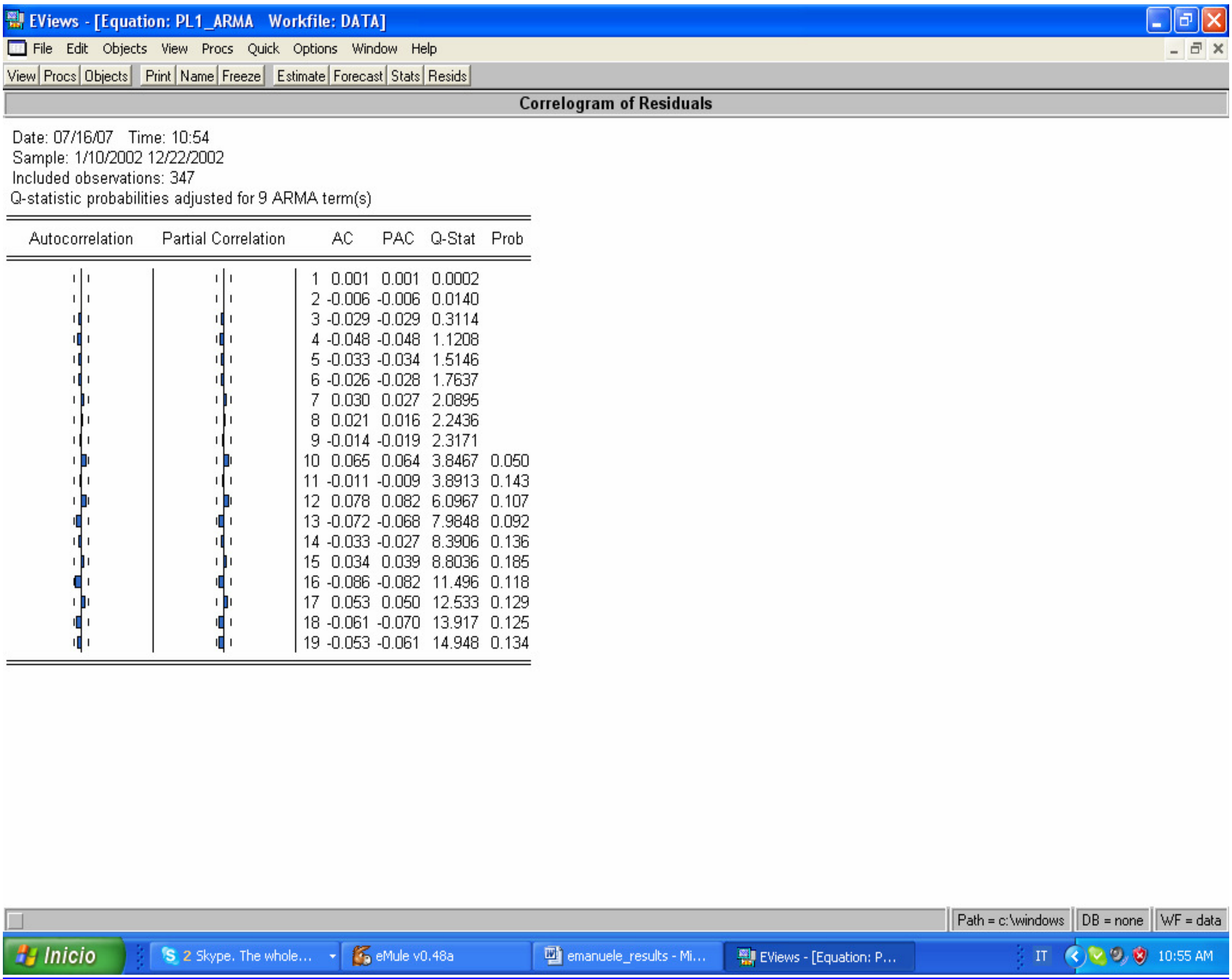
Dependent Variable: PL1_SA
Method: Least Squares
Date: 07/13/07 Time: 20:57
Sample(adjusted): 1/10/2002 12/22/2002
Included observations: 347 after adjusting endpoints
Convergence achieved after 3 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.009379	0.026164	-0.358486	0.7202
AR(1)	0.524751	0.053782	9.756980	0.0000
AR(2)	0.166014	0.060885	2.726669	0.0067
AR(3)	-0.157738	0.061201	-2.577398	0.0104
AR(4)	0.300535	0.061223	4.908862	0.0000
AR(5)	-0.095764	0.063329	-1.512172	0.1314
AR(6)	0.160877	0.061414	2.619543	0.0092
AR(7)	-0.063785	0.061357	-1.039564	0.2993
AR(8)	-0.048822	0.060684	-0.804536	0.4217
AR(9)	-0.157333	0.053490	-2.941354	0.0035

R-squared	0.520659	Mean dependent var	-0.009276
Adjusted R-squared	0.507858	S.D. dependent var	0.257871
S.E. of regression	0.180904	Akaike info criterion	-0.553305
Sum squared resid	11.02875	Schwarz criterion	-0.442374
Log likelihood	105.9984	F-statistic	40.67211
Durbin-Watson stat	1.983739	Prob(F-statistic)	0.000000

Inverted AR Roots	.89 -.20i	.89+.20i	.44+.74i	.44 -.74i
	-.11+.77i	-.11 -.77i	-.50 -.46i	-.50+.46i
	-.92			

The coefficients before constant and variables AR(5), AR(7), AR(8) are not significant. The Durbin-Watson statistic is almost 2. Looking at the correlogram of residuals, we can see that residuals are white noise.



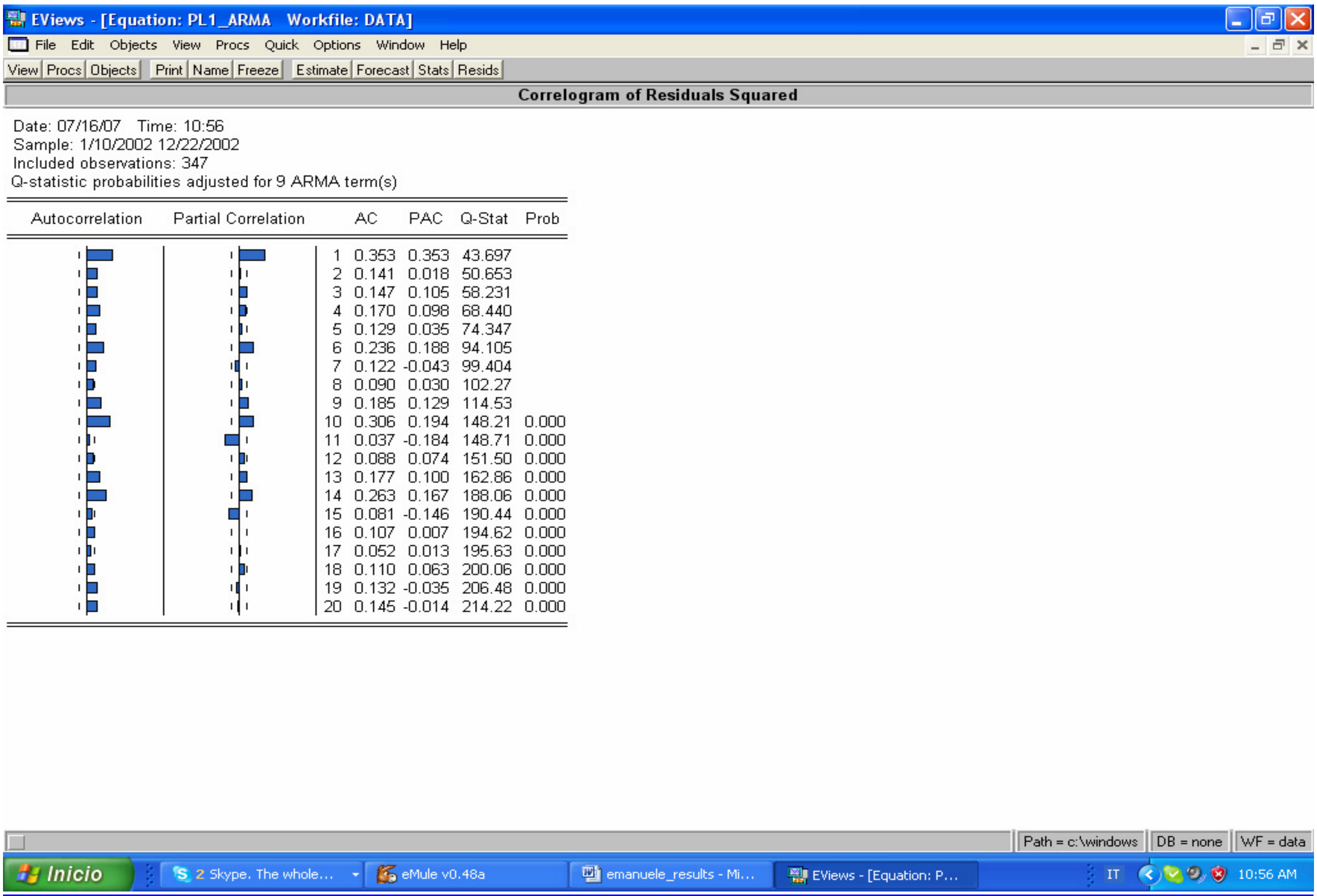
EViews - [Equation: PL1_ARMA Workfile: DATA]

Correlogram of Residuals

Date: 07/16/07 Time: 10:54
Sample: 1/10/2002 12/22/2002
Included observations: 347
Q-statistic probabilities adjusted for 9 ARMA term(s)

	AC	PAC	Q-Stat	Prob
1	0.001	0.001	0.0002	
2	-0.006	-0.006	0.0140	
3	-0.029	-0.029	0.3114	
4	-0.048	-0.048	1.1208	
5	-0.033	-0.034	1.5146	
6	-0.026	-0.028	1.7637	
7	0.030	0.027	2.0895	
8	0.021	0.016	2.2436	
9	-0.014	-0.019	2.3171	
10	0.065	0.064	3.8467	0.050
11	-0.011	-0.009	3.8913	0.143
12	0.078	0.082	6.0967	0.107
13	-0.072	-0.068	7.9848	0.092
14	-0.033	-0.027	8.3906	0.136
15	0.034	0.039	8.8036	0.185
16	-0.086	-0.082	11.496	0.118
17	0.053	0.050	12.533	0.129
18	-0.061	-0.070	13.917	0.125
19	-0.053	-0.061	14.948	0.134

To check that we found a good specification of the model, namely linear model AR(9), let us look at the correlogram of the residuals squared and the results of the ARCH test:



Correlogram of Residuals Squared

Date: 07/16/07 Time: 10:56
Sample: 1/10/2002 12/22/2002
Included observations: 347
Q-statistic probabilities adjusted for 9 ARMA term(s)

	AC	PAC	Q-Stat	Prob
1	0.353	0.353	43.697	
2	0.141	0.018	50.653	
3	0.147	0.105	58.231	
4	0.170	0.098	68.440	
5	0.129	0.035	74.347	
6	0.236	0.188	94.105	
7	0.122	-0.043	99.404	
8	0.090	0.030	102.27	
9	0.185	0.129	114.53	
10	0.306	0.194	148.21	0.000
11	0.037	-0.184	148.71	0.000
12	0.088	0.074	151.50	0.000
13	0.177	0.100	162.86	0.000
14	0.263	0.167	188.06	0.000
15	0.081	-0.146	190.44	0.000
16	0.107	0.007	194.62	0.000
17	0.052	0.013	195.63	0.000
18	0.110	0.063	200.06	0.000
19	0.132	-0.035	206.48	0.000
20	0.145	-0.014	214.22	0.000

ARCH Test:

F-statistic	49.21873	Probability	0.000000
Obs*R-squared	43.30842	Probability	0.000000

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 07/13/07 Time: 21:38
Sample(adjusted): 1/11/2002 12/22/2002
Included observations: 346 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.020164	0.006989	2.885288	0.0042
RESID^2(-1)	0.353377	0.050370	7.015606	0.0000
R-squared	0.125169	Mean dependent var		0.031423
Adjusted R-squared	0.122626	S.D. dependent var		0.135074
S.E. of regression	0.126522	Akaike info criterion		-1.291041
Sum squared resid	5.506667	Schwarz criterion		-1.268808
Log likelihood	225.3502	F-statistic		49.21873
Durbin-Watson stat	2.009351	Prob(F-statistic)		0.000000

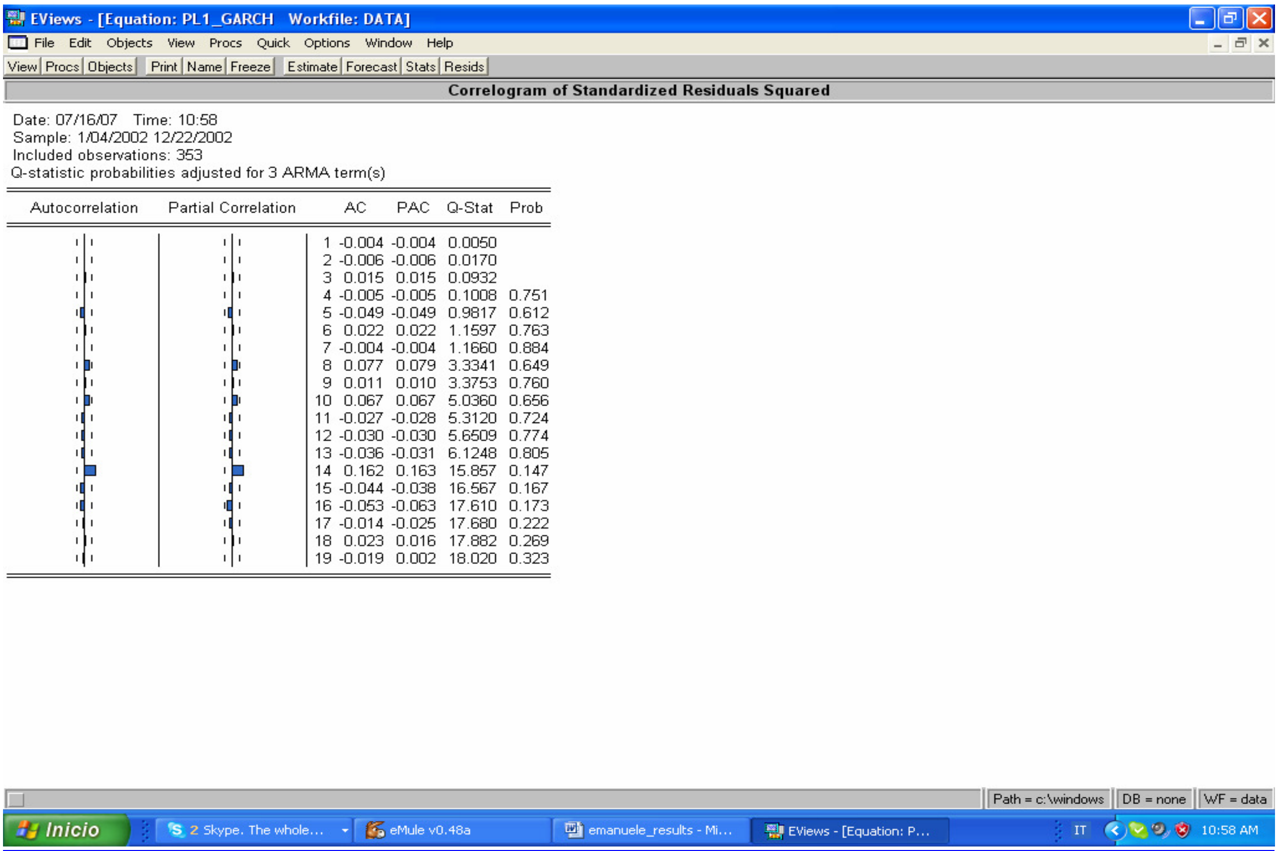
The results of the ARCH test and the correlogram of residuals squared show that GARCH(1,1) model should be used:

Dependent Variable: PL1_SA
Method: ML - ARCH
Date: 07/13/07 Time: 21:42
Sample(adjusted): 1/04/2002 12/22/2002
Included observations: 353 after adjusting endpoints
Convergence achieved after 35 iterations

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.009197	0.008380	1.097541	0.2724
AR(1)	0.547954	0.067856	8.075190	0.0000
AR(2)	0.158310	0.092429	1.712775	0.0868
AR(3)	-0.092231	0.067057	-1.375416	0.1690
		Variance Equation		
C	0.000264	4.33E-05	6.094861	0.0000
ARCH(1)	0.233224	0.047566	4.903220	0.0000
GARCH(1)	0.777626	0.022194	35.03756	0.0000
R-squared	0.427606	Mean dependent var		-0.002315
Adjusted R-squared	0.417680	S.D. dependent var		0.261396
S.E. of regression	0.199471	Akaike info criterion		-1.741639
Sum squared resid	13.76691	Schwarz criterion		-1.664967
Log likelihood	314.3994	F-statistic		43.07976
Durbin-Watson stat	2.078242	Prob(F-statistic)		0.000000
Inverted AR Roots	.48 -.03i	.48+.03i	-.41	

Looking at the correlogram of residuals one can conclude that the residuals are seemed to be white noise. The correlogram of the standardized residuals squared and the results of the ARCH LM test also confirm that adequate

GARCH model (where there is no autocorrelation of residuals squared) was chosen.


EViews - [Equation: PL1_GARCH Workfile: DATA]

Correlogram of Standardized Residuals Squared

Date: 07/16/07 Time: 10:58
Sample: 1/04/2002 12/22/2002
Included observations: 353
Q-statistic probabilities adjusted for 3 ARMA term(s)

	AC	PAC	Q-Stat	Prob
1	-0.004	-0.004	0.0050	
2	-0.006	-0.006	0.0170	
3	0.015	0.015	0.0932	
4	-0.005	-0.005	0.1008	0.751
5	-0.049	-0.049	0.9817	0.612
6	0.022	0.022	1.1597	0.763
7	-0.004	-0.004	1.1660	0.884
8	0.077	0.079	3.3341	0.649
9	0.011	0.010	3.3753	0.760
10	0.067	0.067	5.0360	0.656
11	-0.027	-0.028	5.3120	0.724
12	-0.030	-0.030	5.6509	0.774
13	-0.036	-0.031	6.1248	0.805
14	0.162	0.163	15.857	0.147
15	-0.044	-0.038	16.567	0.167
16	-0.053	-0.063	17.610	0.173
17	-0.014	-0.025	17.680	0.222
18	0.023	0.016	17.882	0.269
19	-0.019	0.002	18.020	0.323

ARCH Test:

F-statistic	0.004891	Probability	0.944285
Obs*R-squared	0.004919	Probability	0.944087

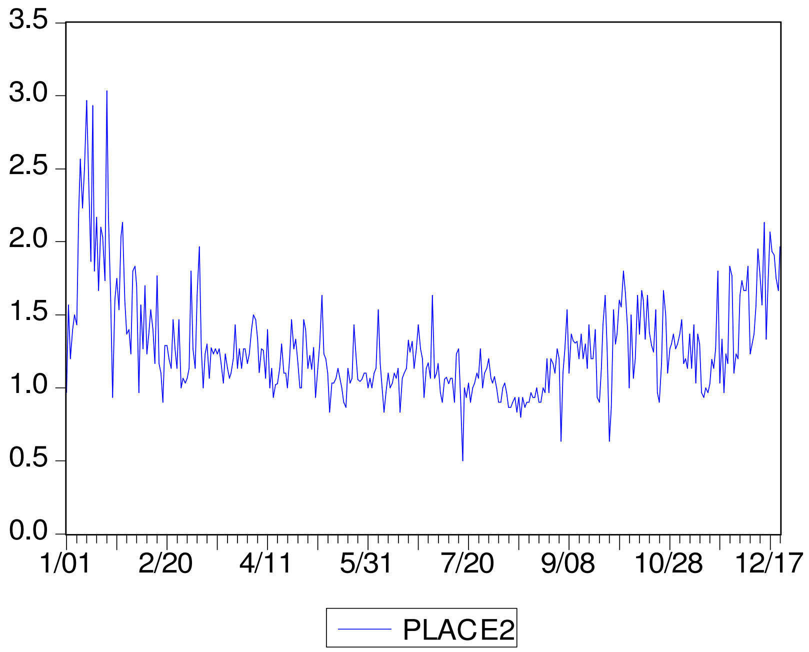
Test Equation:
Dependent Variable: STD_RESID^2
Method: Least Squares
Date: 07/16/07 Time: 13:46
Sample(adjusted): 1/05/2002 12/22/2002
Included observations: 352 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.010706	0.156271	6.467643	0.0000
STD_RESID^2(-1)	-0.003738	0.053453	-0.069935	0.9443

R-squared	0.000014	Mean dependent var	1.006941
Adjusted R-squared	-0.002843	S.D. dependent var	2.748496
S.E. of regression	2.752400	Akaike info criterion	4.868490
Sum squared resid	2651.498	Schwarz criterion	4.890442
Log likelihood	-854.8542	F-statistic	0.004891
Durbin-Watson stat	1.999998	Prob(F-statistic)	0.944285

PLACE_2 (ROMA_2)

The original data of series roma_2:



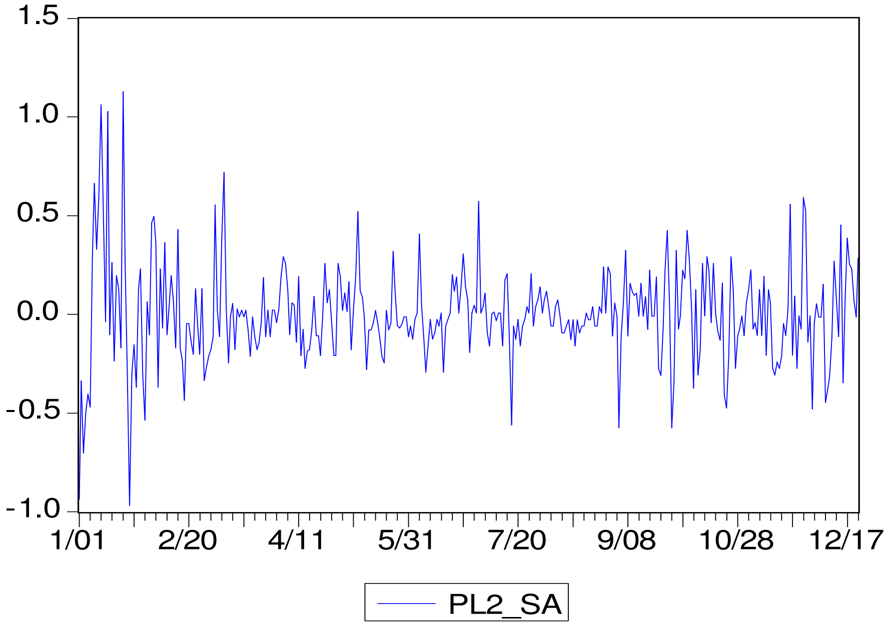
The results of the unit root test again showed that this place_2 is not stationary. Again, as it was made with the place_1 we will "clean" these series from these fluctuations by regressing the series place_2 on dummy variables of the month. The results of this regression are:

Dependent Variable: PLACE2
Method: Least Squares
Date: 07/13/07 Time: 11:08
Sample: 1/01/2002 12/22/2002
Included observations: 356

Variable	Coefficient	Std. Error	t-Statistic	Prob.
M1	1.903763	0.045372	41.95874	0.0000
M2	1.336111	0.047741	27.98661	0.0000
M3	1.245341	0.045372	27.44717	0.0000
M4	1.208210	0.046122	26.19580	0.0000
M5	1.112545	0.045372	24.52037	0.0000
M6	1.126481	0.046122	24.42381	0.0000
M7	1.060036	0.045372	23.36308	0.0000
M8	0.960394	0.045372	21.16699	0.0000
M9	1.209259	0.046122	26.21855	0.0000
M10	1.374791	0.045372	30.30025	0.0000
M11	1.240741	0.046122	26.90112	0.0000
M12	1.680303	0.053859	31.19806	0.0000

R-squared	0.507081	Mean dependent var	1.278881
Adjusted R-squared	0.491319	S.D. dependent var	0.354200
S.E. of regression	0.252622	Akaike info criterion	0.119283
Sum squared resid	21.95337	Schwarz criterion	0.249898
Log likelihood	-9.232307	Durbin-Watson stat	1.406185

All the dummy variables are significant. Now, the following series PL2_SA was received:



The results of the unit root test for the new series PL2_SA show that there is stationarity:

ADF Test Statistic	-8.863673	1% Critical Value*	-3.4509
		5% Critical Value	-2.8700
		10% Critical Value	-2.5712

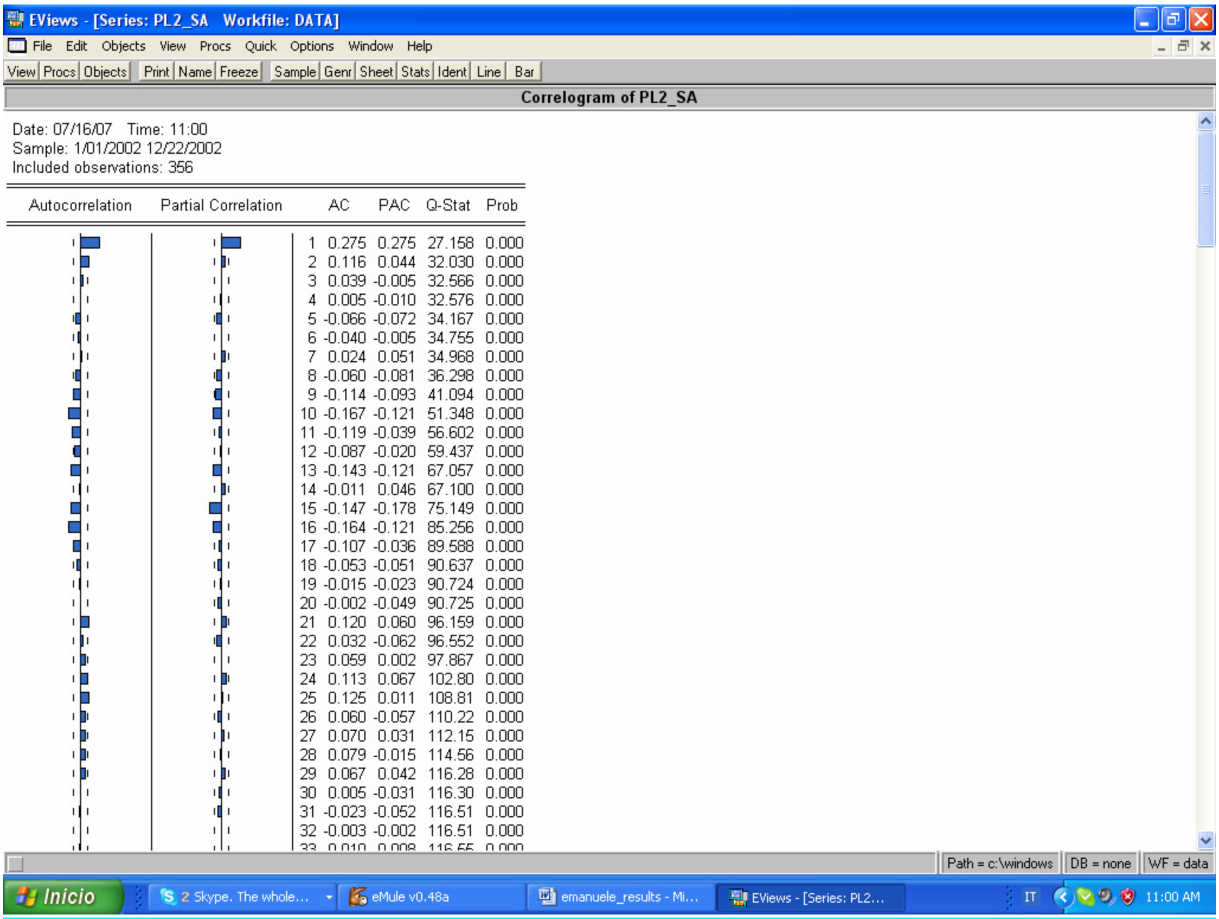
*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(PL2_SA)
Method: Least Squares
Date: 07/13/07 Time: 22:09
Sample(adjusted): 1/06/2002 12/22/2002
Included observations: 351 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
PL2_SA(-1)	-0.815094	0.091959	-8.863673	0.0000
D(PL2_SA(-1))	0.056341	0.083760	0.672653	0.5016
D(PL2_SA(-2))	0.069280	0.074562	0.929157	0.3535
D(PL2_SA(-3))	0.064513	0.065116	0.990741	0.3225
D(PL2_SA(-4))	0.067902	0.052093	1.303489	0.1933
C	0.006498	0.012442	0.522281	0.6018

R-squared	0.382456	Mean dependent var	0.001966
Adjusted R-squared	0.373506	S.D. dependent var	0.294350
S.E. of regression	0.232982	Akaike info criterion	-0.058763
Sum squared resid	18.72684	Schwarz criterion	0.007233
Log likelihood	16.31294	F-statistic	42.73286
Durbin-Watson stat	1.998415	Prob(F-statistic)	0.000000

The correlogram of residuals is:



	Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob
1			0.275	0.275	27.158	0.000
2			0.116	0.044	32.030	0.000
3			0.039	-0.005	32.566	0.000
4			0.005	-0.010	32.576	0.000
5			-0.066	-0.072	34.167	0.000
6			-0.040	-0.005	34.755	0.000
7			0.024	0.051	34.968	0.000
8			-0.060	-0.081	36.298	0.000
9			-0.114	-0.093	41.094	0.000
10			-0.167	-0.121	51.348	0.000
11			-0.119	-0.039	56.602	0.000
12			-0.087	-0.020	59.437	0.000
13			-0.143	-0.121	67.057	0.000
14			-0.011	0.046	67.100	0.000
15			-0.147	-0.178	75.149	0.000
16			-0.164	-0.121	85.256	0.000
17			-0.107	-0.036	89.588	0.000
18			-0.053	-0.051	90.637	0.000
19			-0.015	-0.023	90.724	0.000
20			-0.002	-0.049	90.725	0.000
21			0.120	0.060	96.159	0.000
22			0.032	-0.062	96.552	0.000
23			0.059	0.002	97.867	0.000
24			0.113	0.067	102.80	0.000
25			0.125	0.011	108.81	0.000
26			0.060	-0.057	110.22	0.000
27			0.070	0.031	112.15	0.000
28			0.079	-0.015	114.56	0.000
29			0.067	0.042	116.28	0.000
30			0.005	-0.031	116.30	0.000
31			-0.023	-0.052	116.51	0.000
32			-0.003	-0.002	116.51	0.000
33			0.010	0.008	116.55	0.000

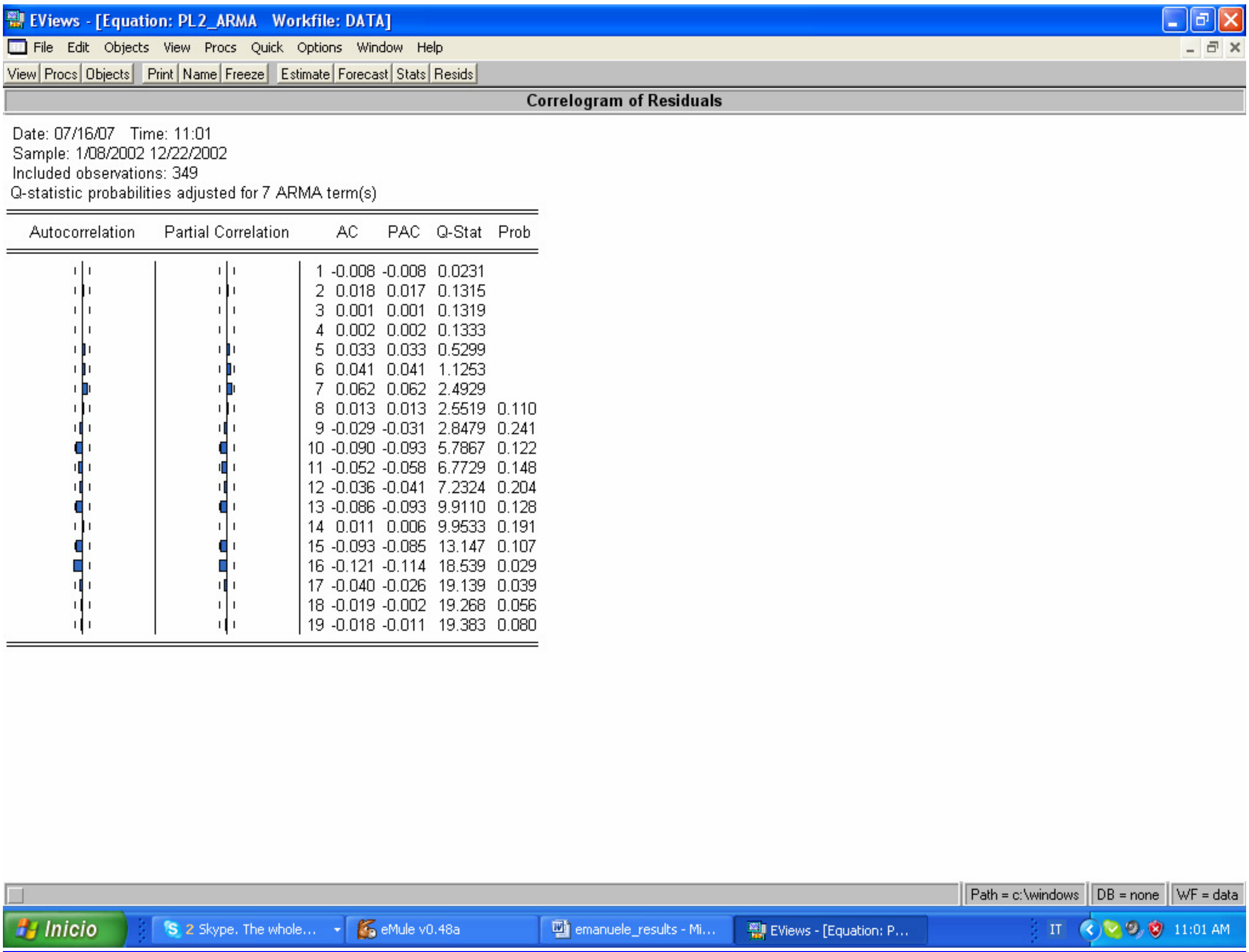
The results of the AR(1) model are:

Dependent Variable: PL2_SA
Method: Least Squares
Date: 07/13/07 Time: 22:14
Sample(adjusted): 1/02/2002 12/22/2002
Included observations: 355 after adjusting endpoints
Convergence achieved after 3 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.003954	0.017194	0.229983	0.8182
AR(1)	0.276108	0.050135	5.507332	0.0000

R-squared	0.079124	Mean dependent var	0.002640
Adjusted R-squared	0.076515	S.D. dependent var	0.243983
S.E. of regression	0.234463	Akaike info criterion	-0.057423
Sum squared resid	19.40538	Schwarz criterion	-0.035608
Log likelihood	12.19253	F-statistic	30.33070
Durbin-Watson stat	2.028503	Prob(F-statistic)	0.000000
Inverted AR Roots	.28		

The correlogram of residuals show that the residuals are white noise:



Date: 07/16/07 Time: 11:01
Sample: 1/08/2002 12/22/2002
Included observations: 349
Q-statistic probabilities adjusted for 7 ARMA term(s)

	AC	PAC	Q-Stat	Prob
1	-0.008	-0.008	0.0231	
2	0.018	0.017	0.1315	
3	0.001	0.001	0.1319	
4	0.002	0.002	0.1333	
5	0.033	0.033	0.5299	
6	0.041	0.041	1.1253	
7	0.062	0.062	2.4929	
8	0.013	0.013	2.5519	0.110
9	-0.029	-0.031	2.8479	0.241
10	-0.090	-0.093	5.7867	0.122
11	-0.052	-0.058	6.7729	0.148
12	-0.036	-0.041	7.2324	0.204
13	-0.086	-0.093	9.9110	0.128
14	0.011	0.006	9.9533	0.191
15	-0.093	-0.085	13.147	0.107
16	-0.121	-0.114	18.539	0.029
17	-0.040	-0.026	19.139	0.039
18	-0.019	-0.002	19.268	0.056
19	-0.018	-0.011	19.383	0.080

The results of the ARCH LM test show that GARCH model should not be used for the series PL2_SA:

ARCH Test:

F-statistic	1.909019	Probability	0.167949
Obs*R-squared	1.909510	Probability	0.167018

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 07/13/07 Time: 22:19
Sample(adjusted): 1/03/2002 12/22/2002
Included observations: 354 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.050790	0.007381	6.881511	0.0000
RESID^2(-1)	0.073435	0.053149	1.381673	0.1679

R-squared	0.005394	Mean dependent var	0.054799
Adjusted R-squared	0.002569	S.D. dependent var	0.127854
S.E. of regression	0.127690	Akaike info criterion	-1.272792
Sum squared resid	5.739250	Schwarz criterion	-1.250931
Log likelihood	227.2841	F-statistic	1.909019
Durbin-Watson stat	1.980439	Prob(F-statistic)	0.167949

PLACE_3 (ROMA_3)

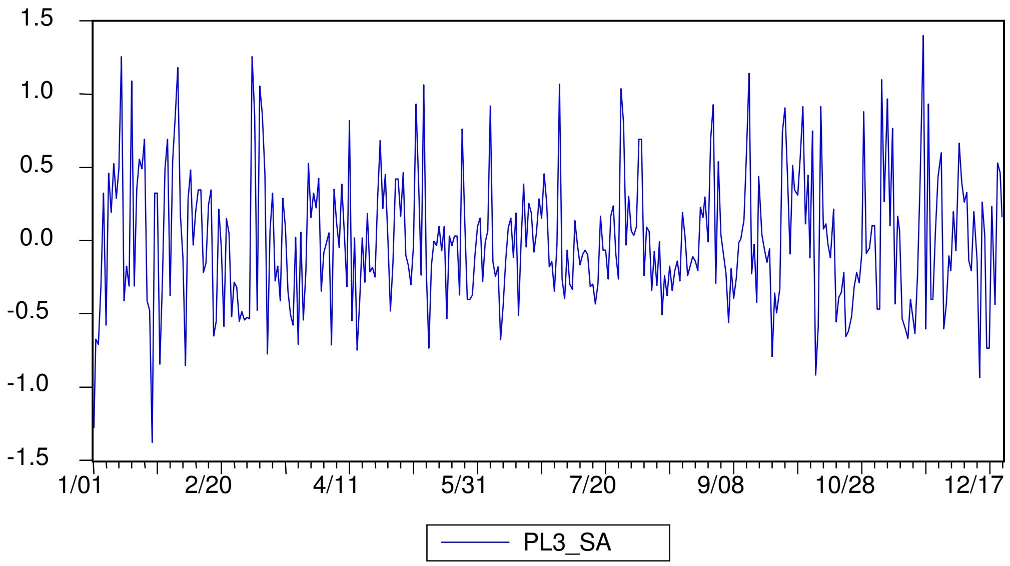
The same approach with the regression of the series on the dummy variables of the months is used for the series place_3. The results of this regression are:

Dependent Variable: PLACE3
Method: Least Squares
Date: 07/13/07 Time: 10:50
Sample: 1/01/2002 12/22/2002
Included observations: 356

Variable	Coefficient	Std. Error	t-Statistic	Prob.
M1	2.377419	0.083520	28.46516	0.0000
M2	1.787235	0.087881	20.33705	0.0000
M3	1.810842	0.083520	21.68146	0.0000
M4	1.550000	0.084901	18.25658	0.0000
M5	1.470251	0.083520	17.60351	0.0000
M6	1.347407	0.084901	15.87035	0.0000
M7	1.466129	0.083520	17.55416	0.0000
M8	1.242115	0.083520	14.87201	0.0000
M9	1.627531	0.084901	19.16977	0.0000
M10	1.654839	0.083520	19.81361	0.0000
M11	1.635556	0.084901	19.26429	0.0000
M12	2.237879	0.099143	22.57226	0.0000

R-squared	0.322808	Mean dependent var	1.670675
Adjusted R-squared	0.301154	S.D. dependent var	0.556265
S.E. of regression	0.465021	Akaike info criterion	1.339660
Sum squared resid	74.38825	Schwarz criterion	1.470276
Log likelihood	-226.4595	Durbin-Watson stat	1.609630

Again, all the dummy variables are significant. The evolution of the "cleaned" series PL3_SA can be found in the figure below:



The PL3_SA is stationary, it is easy to see both from the graph and from the unit root test:

ADF Test Statistic	-7.936220	1% Critical Value*	-3.4509
		5% Critical Value	-2.8700
		10% Critical Value	-2.5712

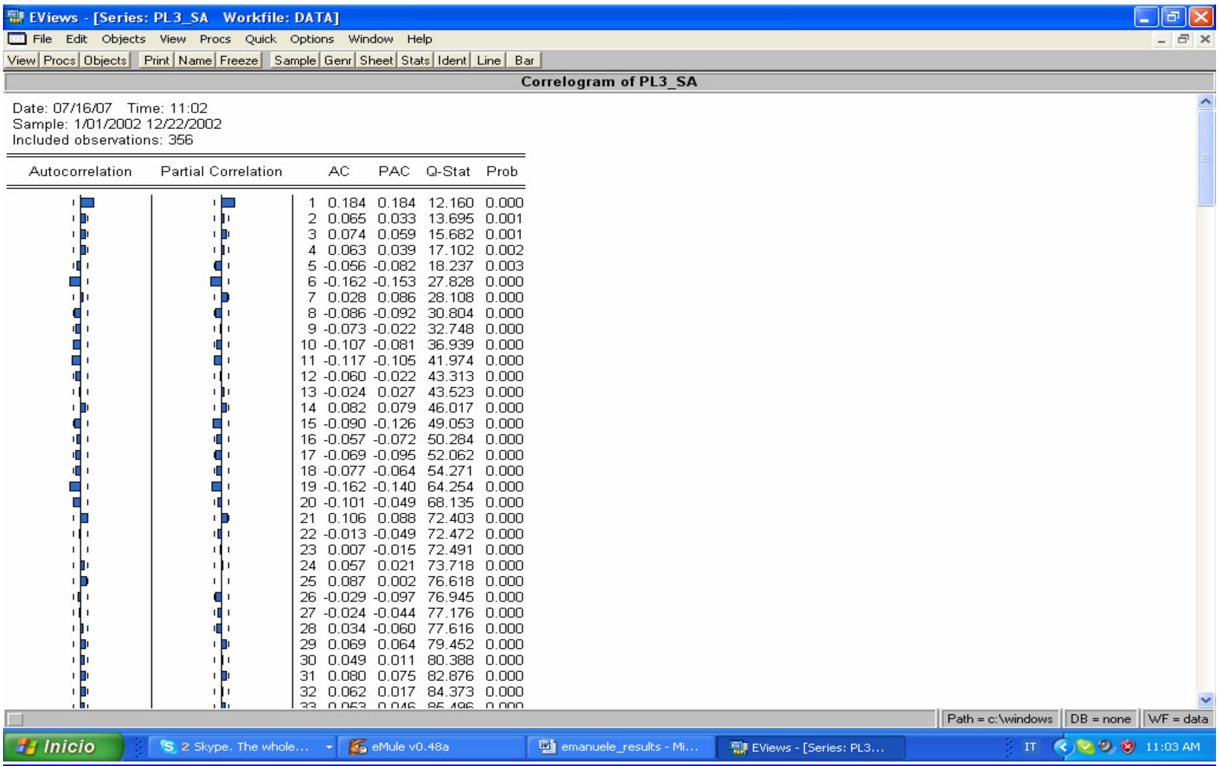
*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(PL3_SA)
Method: Least Squares
Date: 07/13/07 Time: 22:31
Sample(adjusted): 1/06/2002 12/22/2002
Included observations: 351 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
PL3_SA(-1)	-0.778169	0.098053	-7.936220	0.0000
D(PL3_SA(-1))	-0.057083	0.090930	-0.627767	0.5306
D(PL3_SA(-2))	-0.036246	0.081199	-0.446377	0.6556
D(PL3_SA(-3))	0.020666	0.068932	0.299802	0.7645
D(PL3_SA(-4))	0.083827	0.053055	1.580005	0.1150
C	0.005662	0.023791	0.237983	0.8120

R-squared	0.423412	Mean dependent var	-0.000457
Adjusted R-squared	0.415055	S.D. dependent var	0.582642
S.E. of regression	0.445615	Akaike info criterion	1.238222
Sum squared resid	68.50748	Schwarz criterion	1.304218
Log likelihood	-211.3079	F-statistic	50.66946
Durbin-Watson stat	2.011016	Prob(F-statistic)	0.000000

The correlogram of residuals is:



Date: 07/16/07 Time: 11:02
Sample: 1/01/2002 12/22/2002
Included observations: 356

	AC	PAC	Q-Stat	Prob
1	0.184	0.184	12.160	0.000
2	0.065	0.033	13.695	0.001
3	0.074	0.059	15.682	0.001
4	0.063	0.039	17.102	0.002
5	-0.056	-0.082	18.237	0.003
6	-0.162	-0.153	27.828	0.000
7	0.028	0.086	28.108	0.000
8	-0.086	-0.092	30.804	0.000
9	-0.073	-0.022	32.748	0.000
10	-0.107	-0.081	36.939	0.000
11	-0.117	-0.105	41.974	0.000
12	-0.060	-0.022	43.313	0.000
13	-0.024	0.027	43.523	0.000
14	0.082	0.079	46.017	0.000
15	-0.090	-0.126	49.053	0.000
16	-0.057	-0.072	50.284	0.000
17	-0.069	-0.095	52.062	0.000
18	-0.077	-0.064	54.271	0.000
19	-0.162	-0.140	64.254	0.000
20	-0.101	-0.049	68.135	0.000
21	0.106	0.088	72.403	0.000
22	-0.013	-0.049	72.472	0.000
23	0.007	-0.015	72.491	0.000
24	0.057	0.021	73.718	0.000
25	0.087	0.002	76.618	0.000
26	-0.029	-0.097	76.945	0.000
27	-0.024	-0.044	77.176	0.000
28	0.034	-0.060	77.616	0.000
29	0.069	0.064	79.452	0.000
30	0.049	0.011	80.388	0.000
31	0.080	0.075	82.876	0.000
32	0.062	0.017	84.373	0.000
33	0.053	0.046	85.496	0.000

The model AR(6) was chosen, the results of this model are:

Dependent Variable: PL3_SA
Method: Least Squares
Date: 07/13/07 Time: 03:13
Sample(adjusted): 1/07/2002 12/22/2002
Included observations: 350 after adjusting endpoints
Convergence achieved after 3 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.008131	0.025799	0.315160	0.7528
AR(1)	0.156153	0.053159	2.937486	0.0035
AR(2)	0.028128	0.053665	0.524135	0.6005
AR(3)	0.061404	0.053698	1.143499	0.2536
AR(4)	0.062987	0.053687	1.173215	0.2415
AR(5)	-0.065732	0.053790	-1.222013	0.2225
AR(6)	-0.154813	0.052823	-2.930776	0.0036
R-squared	0.067764	Mean dependent var		0.009232
Adjusted R-squared	0.051457	S.D. dependent var		0.451767
S.E. of regression	0.439990	Akaike info criterion		1.215668
Sum squared resid	66.40177	Schwarz criterion		1.292826
Log likelihood	-205.7418	F-statistic		4.155438
Durbin-Watson stat	1.964755	Prob(F-statistic)		0.000478
Inverted AR Roots	.69+.35i	.69 -.35i	-.03 -.76i	-.03+.76i
	-.59+.32i		-.59 -.32i	

It is easy to see that only the coefficients before AR(1) and AR(6) are significant. The residuals seem to be white noise.
The results of the ARCH LM test and correlogram of residuals squared show that GARCH model is not necessary in this case:

ARCH Test:

F-statistic	3.477359	Probability	0.063058
Obs*R-squared	3.462701	Probability	0.062768

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 07/13/07 Time: 22:38
Sample(adjusted): 1/08/2002 12/22/2002
Included observations: 349 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.170950	0.018247	9.368886	0.0000
RESID^2(-1)	0.099660	0.053444	1.864768	0.0631
R-squared	0.009922	Mean dependent var		0.189910
Adjusted R-squared	0.007069	S.D. dependent var		0.284056
S.E. of regression	0.283051	Akaike info criterion		0.319333
Sum squared resid	27.80084	Schwarz criterion		0.341425
Log likelihood	-53.72353	F-statistic		3.477359
Durbin-Watson stat	2.015487	Prob(F-statistic)		0.063058

PLACE_4 (ROMA_4)

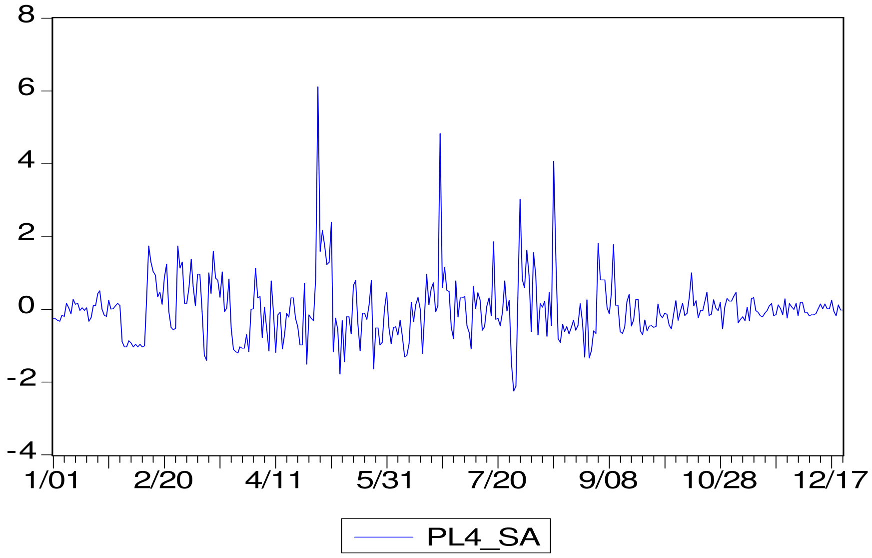
The results of the regression of the Place 4 on the dummy variables of the months are the following:

Dependent Variable: PLACE4
Method: Least Squares
Date: 07/13/07 Time: 10:51
Sample: 1/01/2002 12/22/2002
Included observations: 356

Variable	Coefficient	Std. Error	t-Statistic	Prob.
M1	1.996834	0.156180	12.78546	0.0000
M2	2.929696	0.164334	17.82769	0.0000
M3	4.035484	0.156180	25.83865	0.0000
M4	4.314444	0.158762	27.17559	0.0000
M5	2.344086	0.156180	15.00886	0.0000
M6	2.574444	0.158762	16.21577	0.0000
M7	2.414158	0.156180	15.45752	0.0000
M8	1.708961	0.156180	10.94224	0.0000
M9	1.126749	0.158762	7.097105	0.0000
M10	1.436858	0.156180	9.200005	0.0000
M11	1.276049	0.158762	8.037510	0.0000
M12	1.085466	0.185394	5.854919	0.0000

R-squared	0.587778	Mean dependent var	2.294073
Adjusted R-squared	0.574597	S.D. dependent var	1.333234
S.E. of regression	0.869574	Akaike info criterion	2.591500
Sum squared resid	260.1187	Schwarz criterion	2.722116
Log likelihood	-449.2870	Durbin-Watson stat	1.220791

Now we can look at the graph of the transformed series PL4_SA:



The results of the unit root test show stationarity:

ADF Test Statistic	-7.363903	1% Critical Value*	-3.4509
		5% Critical Value	-2.8700
		10% Critical Value	-2.5712

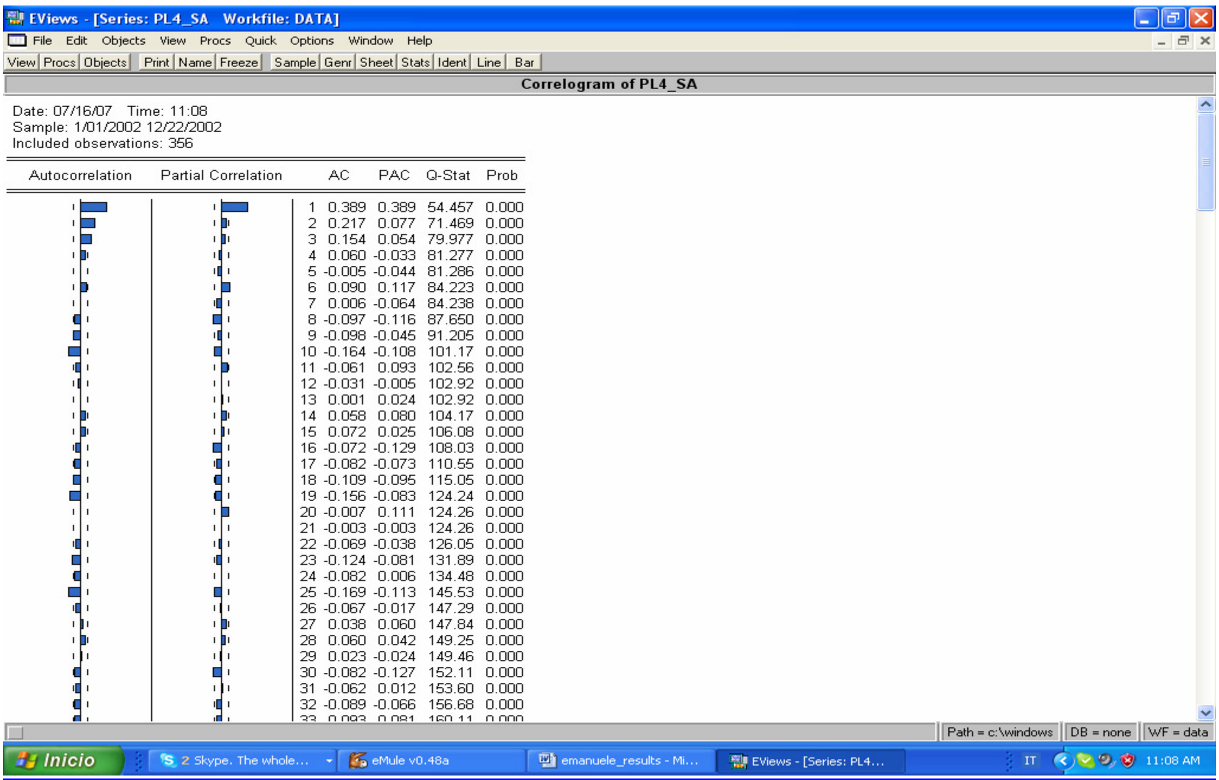
*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(PL4_SA)
Method: Least Squares
Date: 07/13/07 Time: 22:43
Sample(adjusted): 1/06/2002 12/22/2002
Included observations: 351 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
PL4_SA(-1)	-0.575665	0.078174	-7.363903	0.0000
D(PL4_SA(-1))	-0.069383	0.076244	-0.910018	0.3634
D(PL4_SA(-2))	-0.006611	0.071154	-0.092914	0.9260
D(PL4_SA(-3))	0.061492	0.063905	0.962235	0.3366
D(PL4_SA(-4))	0.043796	0.053775	0.814431	0.4160
C	0.002365	0.042417	0.055755	0.9556

R-squared	0.313834	Mean dependent var	0.000412
Adjusted R-squared	0.303890	S.D. dependent var	0.952471
S.E. of regression	0.794677	Akaike info criterion	2.395185
Sum squared resid	217.8716	Schwarz criterion	2.461181
Log likelihood	-414.3549	F-statistic	31.55880
Durbin-Watson stat	1.989678	Prob(F-statistic)	0.000000

The correlogram of residuals is:



Date: 07/16/07 Time: 11:08
Sample: 1/01/2002 12/22/2002
Included observations: 356

Lag	AC	PAC	Q-Stat	Prob
1	0.389	0.389	54.457	0.000
2	0.217	0.077	71.469	0.000
3	0.154	0.054	79.977	0.000
4	0.060	-0.033	81.277	0.000
5	-0.005	-0.044	81.286	0.000
6	0.090	0.117	84.223	0.000
7	0.006	-0.064	84.238	0.000
8	-0.097	-0.116	87.650	0.000
9	-0.098	-0.045	91.205	0.000
10	-0.164	-0.108	101.17	0.000
11	-0.061	0.093	102.56	0.000
12	-0.031	-0.005	102.92	0.000
13	0.001	0.024	102.92	0.000
14	0.058	0.080	104.17	0.000
15	0.072	0.025	106.08	0.000
16	-0.072	-0.129	108.03	0.000
17	-0.082	-0.073	110.55	0.000
18	-0.109	-0.095	115.05	0.000
19	-0.156	-0.083	124.24	0.000
20	-0.007	0.111	124.26	0.000
21	-0.003	-0.003	124.26	0.000
22	-0.069	-0.038	126.05	0.000
23	-0.124	-0.081	131.89	0.000
24	-0.082	0.006	134.48	0.000
25	-0.169	-0.113	145.53	0.000
26	-0.067	-0.017	147.29	0.000
27	0.038	0.060	147.84	0.000
28	0.060	0.042	149.25	0.000
29	0.023	-0.024	149.46	0.000
30	-0.082	-0.127	152.11	0.000
31	-0.062	0.012	153.60	0.000
32	-0.089	-0.066	156.68	0.000

The results of the AR(10) model are:

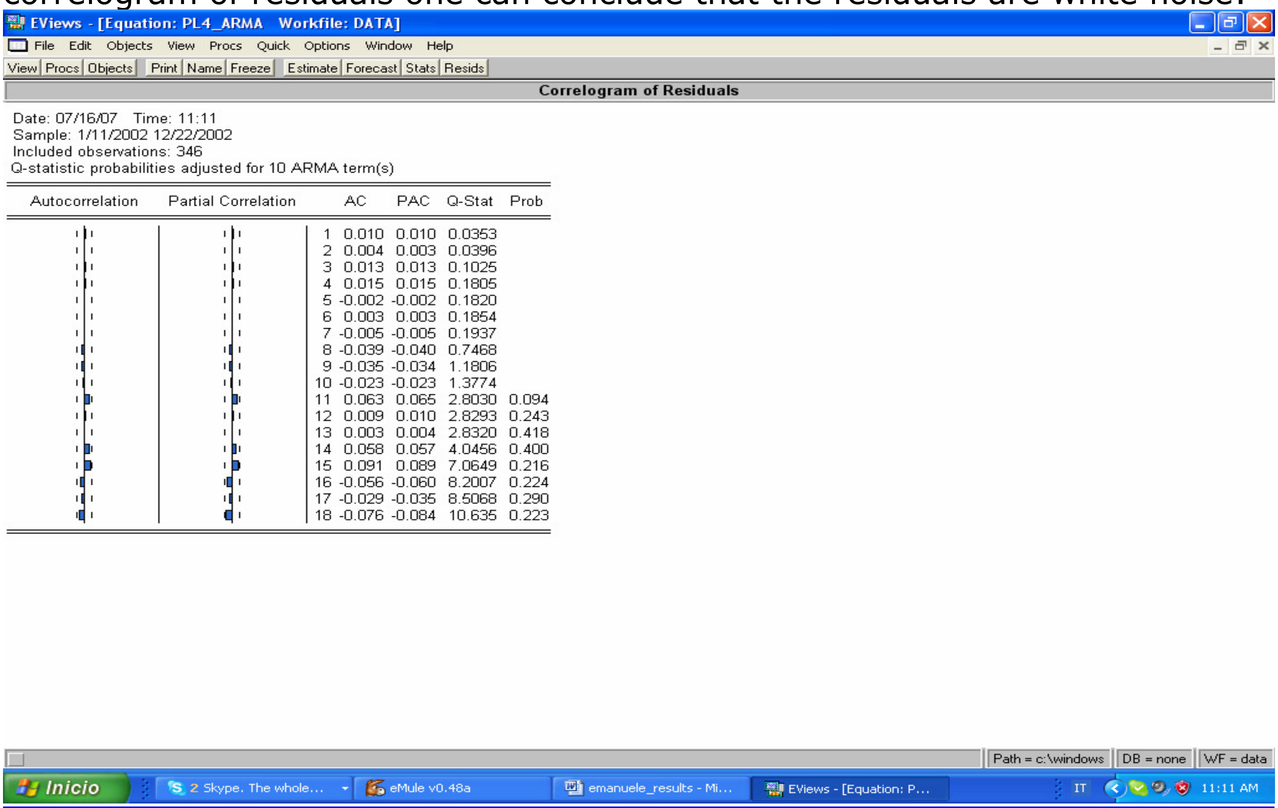
Dependent Variable: PL4_SA
Method: Least Squares
Date: 07/13/07 Time: 22:48
Sample(adjusted): 1/11/2002 12/22/2002
Included observations: 346 after adjusting endpoints
Convergence achieved after 3 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.002089	0.060695	0.034416	0.9726
AR(1)	0.350350	0.054304	6.451629	0.0000
AR(2)	0.063968	0.057569	1.111152	0.2673
AR(3)	0.053428	0.057458	0.929861	0.3531
AR(4)	-0.010613	0.057526	-0.184493	0.8537
AR(5)	-0.083949	0.056966	-1.473673	0.1415
AR(6)	0.147065	0.056977	2.581140	0.0103
AR(7)	-0.011441	0.057530	-0.198877	0.8425
AR(8)	-0.092034	0.057459	-1.601729	0.1102
AR(9)	-0.006531	0.057572	-0.113442	0.9097
AR(10)	-0.108179	0.054289	-1.992673	0.0471

R-squared	0.198774	Mean dependent var	0.003377
Adjusted R-squared	0.174857	S.D. dependent var	0.867436
S.E. of regression	0.787956	Akaike info criterion	2.392526
Sum squared resid	207.9928	Schwarz criterion	2.514811
Log likelihood	-402.9070	F-statistic	8.310946
Durbin-Watson stat	1.979873	Prob(F-statistic)	0.000000

Inverted AR Roots	.84 -.25i	.84+.25i	.49+.68i	.49 -.68i
	-.01+.69i	-.01 -.69i	-.40+.72i	-.40 -.72i
	-.76 -.22i	-.76+.22i		

Coefficients before AR(1), AR(6), AR(10) are significant. Looking at the correlogram of residuals one can conclude that the residuals are white noise:


EViews - [Equation: PL4_ARMA Workfile: DATA]

Correlogram of Residuals

Date: 07/16/07 Time: 11:11
Sample: 1/11/2002 12/22/2002
Included observations: 346
Q-statistic probabilities adjusted for 10 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.010	0.010	0.0353	
		2	0.004	0.003	0.0396	
		3	0.013	0.013	0.1025	
		4	0.015	0.015	0.1805	
		5	-0.002	-0.002	0.1820	
		6	0.003	0.003	0.1854	
		7	-0.005	-0.005	0.1937	
		8	-0.039	-0.040	0.7468	
		9	-0.035	-0.034	1.1806	
		10	-0.023	-0.023	1.3774	
		11	0.063	0.065	2.8030	0.094
		12	0.009	0.010	2.8293	0.243
		13	0.003	0.004	2.8320	0.418
		14	0.058	0.057	4.0456	0.400
		15	0.091	0.089	7.0649	0.216
		16	-0.056	-0.060	8.2007	0.224
		17	-0.029	-0.035	8.5068	0.290
		18	-0.076	-0.084	10.635	0.223

The results of the ARCH LM test and the correlogram of residuals squared show that there is no need to use GARCH model:

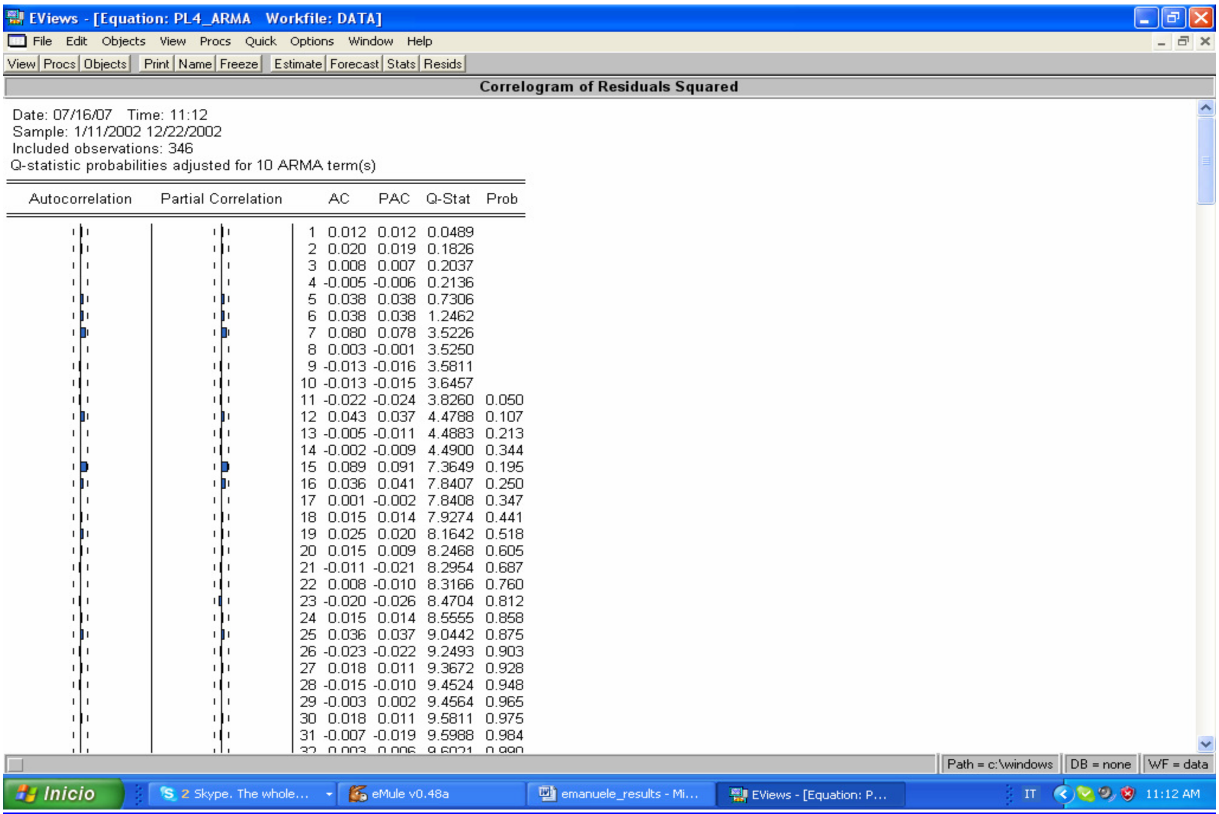
ARCH Test:

F-statistic	0.048117	Probability	0.826503
Obs*R-squared	0.048391	Probability	0.825887

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 07/13/07 Time: 22:52
Sample(adjusted): 1/12/2002 12/22/2002
Included observations: 345 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.595738	0.128898	4.621784	0.0000
RESID^2(-1)	0.011843	0.053991	0.219356	0.8265

R-squared	0.000140	Mean dependent var	0.602878
Adjusted R-squared	-0.002775	S.D. dependent var	2.313369
S.E. of regression	2.316576	Akaike info criterion	4.523838
Sum squared resid	1840.718	Schwarz criterion	4.546119
Log likelihood	-778.3621	F-statistic	0.048117
Durbin-Watson stat	2.000465	Prob(F-statistic)	0.826503



EViews - [Equation: PL4_ARMA Workfile: DATA]

Correlogram of Residuals Squared

Date: 07/16/07 Time: 11:12
Sample: 1/11/2002 12/22/2002
Included observations: 346
Q-statistic probabilities adjusted for 10 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.012	0.012	0.0489	
		2	0.020	0.019	0.1826	
		3	0.008	0.007	0.2037	
		4	-0.005	-0.006	0.2136	
		5	0.038	0.038	0.7306	
		6	0.038	0.038	1.2462	
		7	0.080	0.078	3.5226	
		8	0.003	-0.001	3.5250	
		9	-0.013	-0.016	3.5811	
		10	-0.013	-0.015	3.6457	
		11	-0.022	-0.024	3.8260	0.050
		12	0.043	0.037	4.4788	0.107
		13	-0.005	-0.011	4.4883	0.213
		14	-0.002	-0.009	4.4900	0.344
		15	0.089	0.091	7.3649	0.195
		16	0.036	0.041	7.8407	0.250
		17	0.001	-0.002	7.8408	0.347
		18	0.015	0.014	7.9274	0.441
		19	0.025	0.020	8.1642	0.518
		20	0.015	0.009	8.2468	0.605
		21	-0.011	-0.021	8.2954	0.687
		22	0.008	-0.010	8.3166	0.760
		23	-0.020	-0.026	8.4704	0.812
		24	0.015	0.014	8.5555	0.858
		25	0.036	0.037	9.0442	0.875
		26	-0.023	-0.022	9.2493	0.903
		27	0.018	0.011	9.3672	0.928
		28	-0.015	-0.010	9.4524	0.948
		29	-0.003	0.002	9.4564	0.965
		30	0.018	0.011	9.5811	0.975
		31	-0.007	-0.019	9.5988	0.984
		32	0.003	0.006	9.6021	0.990

Vector Autoregression (VAR) model for "Roma" series series

Theoretical background:
Vector autoregression (VAR) is an econometric model used to capture the evolution and the interdependencies between multiple time series, generalizing the univariate AR models. All the variables in a VAR are treated symmetrically by including for each variable an equation explaining its evolution based on its own lags and the lags of all the other variables in the model. Based on this feature, Christopher Sims advocates the use of VAR models as a theory-free method to estimate economic relationships, thus being an alternative to the "incredible identification restrictions" in structural models.

Definition

A VAR model describes the evolution of a set of *n* variables (called ***endogenous variables***) measured over the same sample period ($t = 1, ..., T$) as a linear function of only their past evolution. The variables are collected in a $n \times 1$ vector y_t, which has as the i^{th} element $y_{i,t}$ the time *t* observation of variable y_i. For example, if the i^{th} variable is GDP, then $y_{i,t}$ is the value of GDP at t.

A ***(reduced) p-th order VAR***, denoted ***VAR(p)***, is

$$y_t = c + A_1 y_{t-1} + A_2 y_{t-2} + \cdots + A_p y_{t-p} + e_t,$$

where *c* is a $n \times 1$ vector of constants (**intercept**), A_i is a $n \times n$ matrix (for every $i = 1, ..., p$) and e_t is a $n \times 1$ vector of error terms satisfying

$$\mathrm{E}(e_t) = 0$$

$$\mathrm{E}(e_t e'_{t-k}) = 0$$

$$\mathrm{E}(e_t e'_t) = \Omega$$

The *k*-periods back observation y_{t-k} is called the *k*-th ***lag*** of *y*. Thus, a p-th order VAR is also called a ***VAR with p lags***.

VAR for Place_1, Place_2, Place_3, Place_4:

The VAR model was built for different p – where p is the order of the VAR or a number of lags in the VAR. We have four endogeneous variables in our model, namely Place_1, Place_2, Place_3 and Place_4. The final model was chosen using the Akaike Iinformation Criteria. The model with 2 lags seemed to be the best. So, the following VAR model was considered:

PLACE1 = C(1,1)*PLACE1(-1) + C(1,2)*PLACE1(-2) + C(1,3)*PLACE2(-1) + C(1,4)*PLACE2(-2) + C(1,5)*PLACE3(-1) + C(1,6)*PLACE3(-2) + C(1,7)*PLACE4(-1) + C(1,8)*PLACE4(-2) + C(1,9)

PLACE2 = C(2,1)*PLACE1(-1) + C(2,2)*PLACE1(-2) + C(2,3)*PLACE2(-1) + C(2,4)*PLACE2(-2) + C(2,5)*PLACE3(-1) + C(2,6)*PLACE3(-2) + C(2,7)*PLACE4(-1) + C(2,8)*PLACE4(-2) + C(2,9)

PLACE3 = C(3,1)*PLACE1(-1) + C(3,2)*PLACE1(-2) + C(3,3)*PLACE2(-1) + C(3,4)*PLACE2(-2) + C(3,5)*PLACE3(-1) + C(3,6)*PLACE3(-2) + C(3,7)*PLACE4(-1) + C(3,8)*PLACE4(-2) + C(3,9)

PLACE4 = C(4,1)*PLACE1(-1) + C(4,2)*PLACE1(-2) + C(4,3)*PLACE2(-1) + C(4,4)*PLACE2(-2) + C(4,5)*PLACE3(-1) + C(4,6)*PLACE3(-2) + C(4,7)*PLACE4(-1) + C(4,8)*PLACE4(-2) + C(4,9)

The results of the model are presented below:

Date: 07/14/07 Time: 18:48
Sample(adjusted): 1/03/2002 12/22/2002
Included observations: 354 after adjusting endpoints
Standard errors & t-statistics in parentheses

	PLACE1	PLACE2	PLACE3	PLACE4
PLACE1(-1)	0.551963	0.174062	0.152010	-0.069002
	(0.05259)	(0.06557)	(0.12542)	(0.23493)
	(10.4958)	(2.65459)	(1.21203)	(-0.29371)
PLACE1(-2)	0.340976	0.069709	0.051494	0.081401
	(0.05355)	(0.06677)	(0.12772)	(0.23924)
	(6.36705)	(1.04398)	(0.40318)	(0.34025)
PLACE2(-1)	-0.130889	0.390013	0.200888	0.125580
	(0.05063)	(0.06312)	(0.12074)	(0.22616)
	(-2.58544)	(6.17871)	(1.66386)	(0.55528)
PLACE2(-2)	0.116323	0.098273	0.246818	-0.385035
	(0.04971)	(0.06198)	(0.11855)	(0.22206)
	(2.34013)	(1.58560)	(2.08201)	(-1.73394)
PLACE3(-1)	0.046545	-0.014547	0.205345	0.033547
	(0.02685)	(0.03348)	(0.06405)	(0.11997)
	(1.73320)	(-0.43445)	(3.20622)	(0.27963)
PLACE3(-2)	-0.035646	0.079935	0.048153	0.020235
	(0.02660)	(0.03317)	(0.06345)	(0.11885)
	(-1.33987)	(2.40976)	(0.75894)	(0.17026)
PLACE4(-1)	0.000161	-0.015458	-0.008815	0.528857
	(0.01162)	(0.01449)	(0.02771)	(0.05190)
	(0.01388)	(-1.06703)	(-0.31812)	(10.1890)
PLACE4(-2)	-0.008629	0.004864	0.002612	0.274544
	(0.01165)	(0.01453)	(0.02779)	(0.05206)
	(-0.74051)	(0.33478)	(0.09398)	(5.27383)
C	0.109805	0.361161	0.516764	0.679883
	(0.04993)	(0.06225)	(0.11908)	(0.22305)
	(2.19921)	(5.80140)	(4.33979)	(3.04816)
R-squared	0.759370	0.530403	0.304170	0.575828
Adj. R-squared	0.753790	0.519514	0.288035	0.565992
Sum sq. resids	13.39871	20.83001	76.20777	267.3914
S.E. equation	0.197071	0.245717	0.469992	0.880368
F-statistic	136.0922	48.70904	18.85138	58.54366
Log likelihood	77.21830	-0.880544	-230.4617	-452.6410
Akaike AIC	-0.385414	0.055822	1.352891	2.608141
Schwarz SC	-0.287042	0.154194	1.451263	2.706513
Mean dependent	0.857491	1.278950	1.672204	2.297241
S.D. dependent	0.397164	0.354482	0.557007	1.336335
Determinant Residual Covariance		0.000234		
Log Likelihood		-529.6687		
Akaike Information Criteria		**3.195869**		
Schwarz Criteria		3.589356		

Therefore, VAR model with substituted coefficients is:

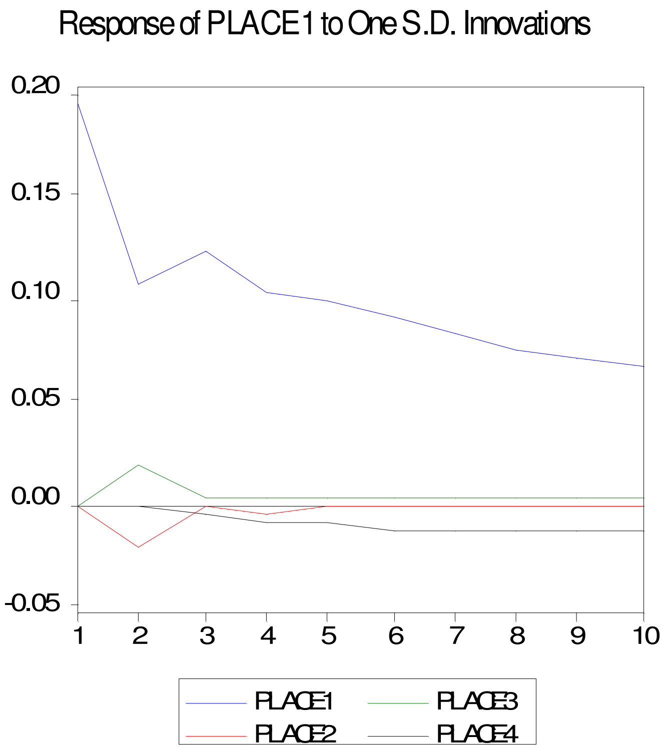
PLACE1 = 0.5519630866*PLACE1(-1) + 0.34097562*PLACE1(-2) - 0.1308887824*PLACE2(-1) + 0.1163227792*PLACE2(-2) + 0.04654480392*PLACE3(-1) - 0.03564620008*PLACE3(-2) + 0.0001612730357*PLACE4(-1) - 0.008629322165*PLACE4(-2) + 0.1098049974

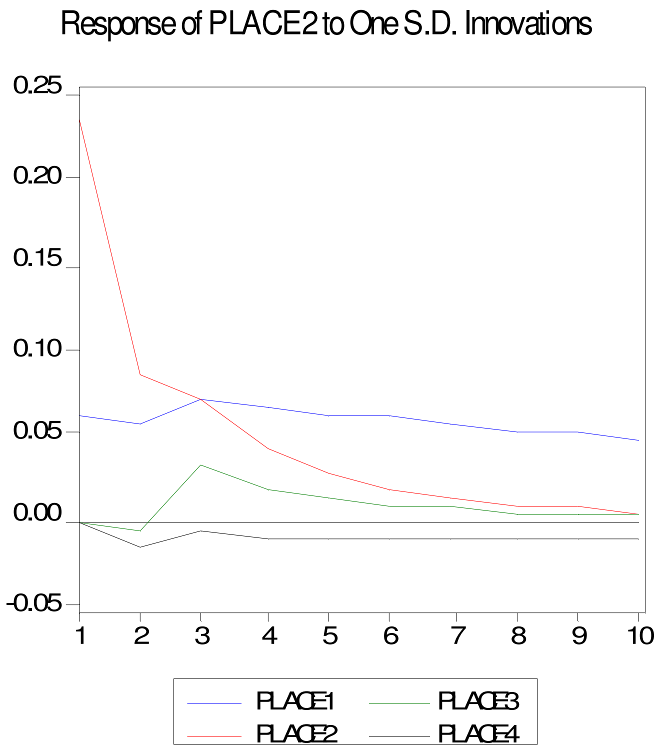
PLACE2 = 0.174061703*PLACE1(-1) + 0.06970926065*PLACE1(-2) + 0.3900134474*PLACE2(-1) + 0.09827261244*PLACE2(-2) - 0.01454710597*PLACE3(-1) + 0.0799352489*PLACE3(-2) - 0.01545803919*PLACE4(-1) + 0.004864295046*PLACE4(-2) + 0.3611613433

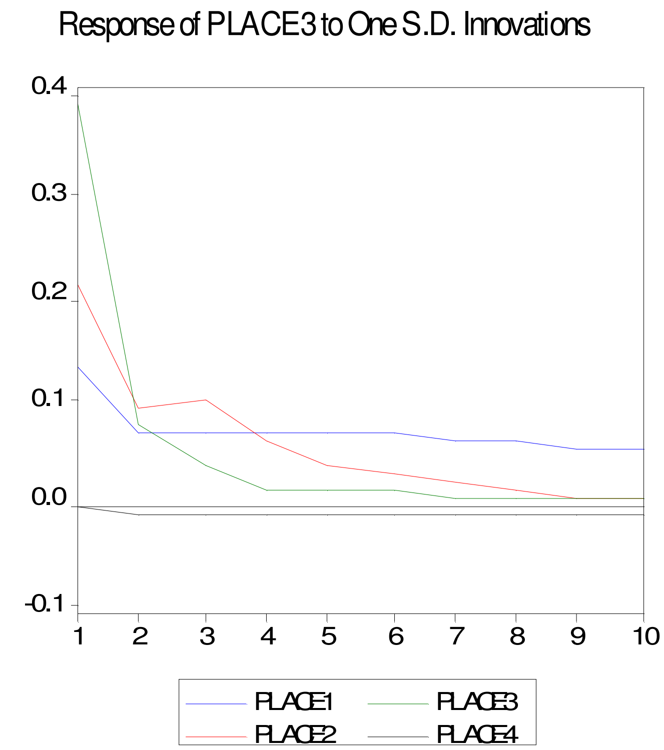
PLACE3 = 0.1520103912*PLACE1(-1) + 0.05149362355*PLACE1(-2) + 0.2008876741*PLACE2(-1) + 0.2468178363*PLACE2(-2) + 0.2053452228*PLACE3(-1) + 0.04815326724*PLACE3(-2) - 0.008814945369*PLACE4(-1) + 0.002611945406*PLACE4(-2) + 0.5167636868

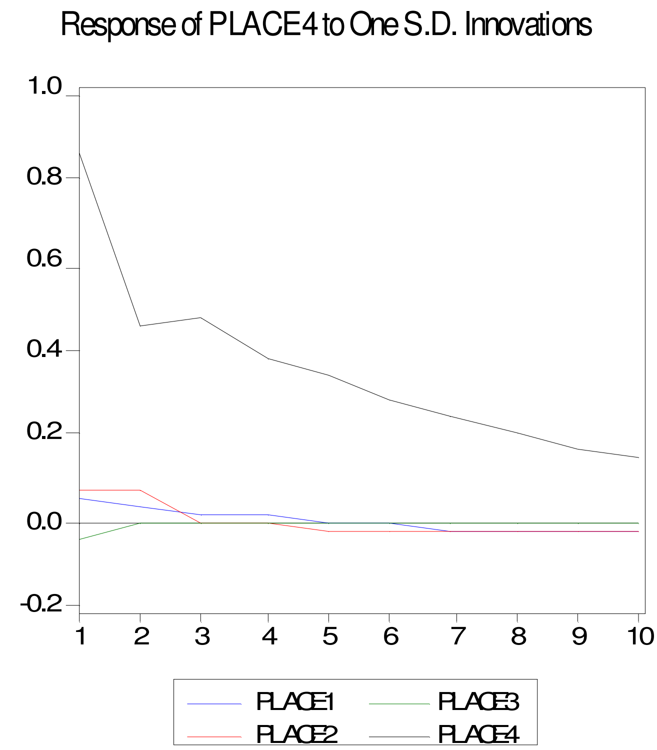
PLACE4 = - 0.06900152809*PLACE1(-1) + 0.0814007968*PLACE1(-2) + 0.1255799382*PLACE2(-1) - 0.3850349024*PLACE2(-2) + 0.03354667015*PLACE3(-1) + 0.02023534816*PLACE3(-2) + 0.5288572968*PLACE4(-1) + 0.2745437302*PLACE4(-2) + 0.6798832701

The responses of Place_1, Place_2, Place_3 and Place_4 to one standard deviation innovations of Place_1, Place_2, Place_3 and Place_4 are:

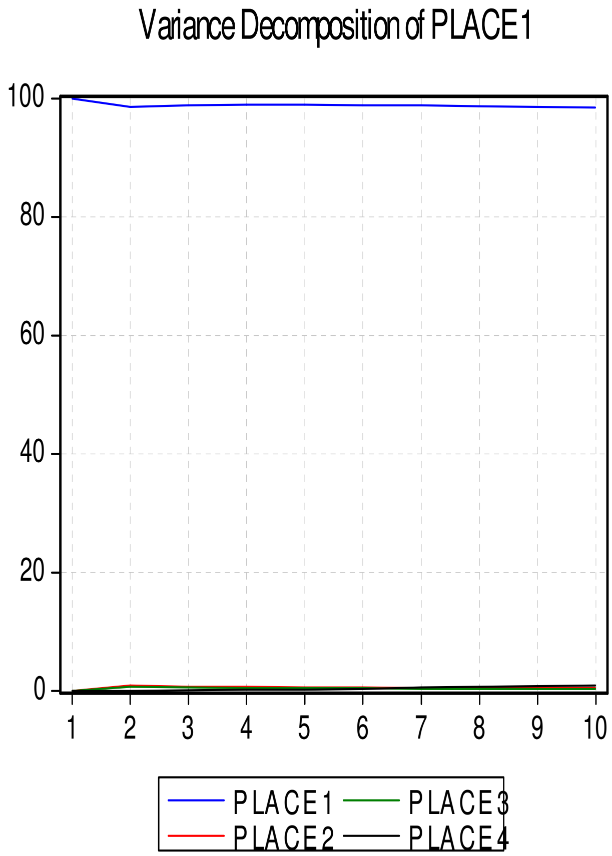

Response of PLACE1 to One S.D. Innovations
0.20
0.15
0.10
0.05
0.00
-0.05
1 2 3 4 5 6 7 8 9 10
PLACE1 PLACE3
PLACE2 PLACE4

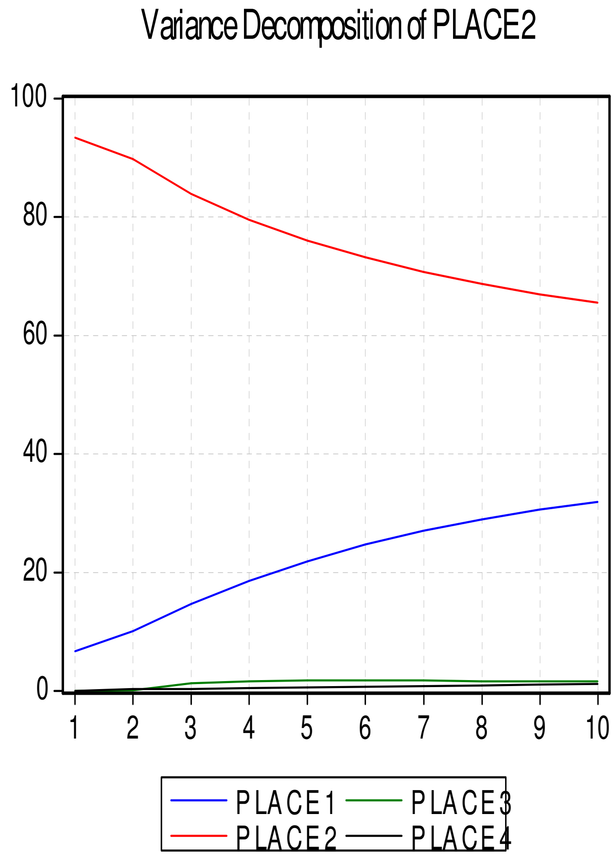

Response of PLACE2 to One S.D. Innovations
0.25
0.20
0.15
0.10
0.05
0.00
-0.05
1 2 3 4 5 6 7 8 9 10
PLACE1 PLACE3
PLACE2 PLACE4

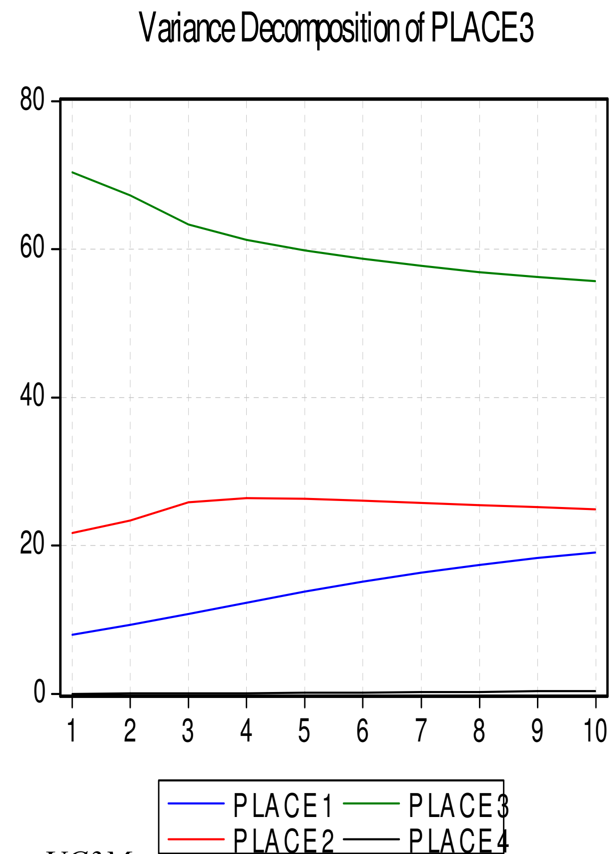

Response of PLACE3 to One S.D. Innovations
0.4
0.3
0.2
0.1
0.0
-0.1
1 2 3 4 5 6 7 8 9 10
PLACE1 PLACE3
PLACE2 PLACE4

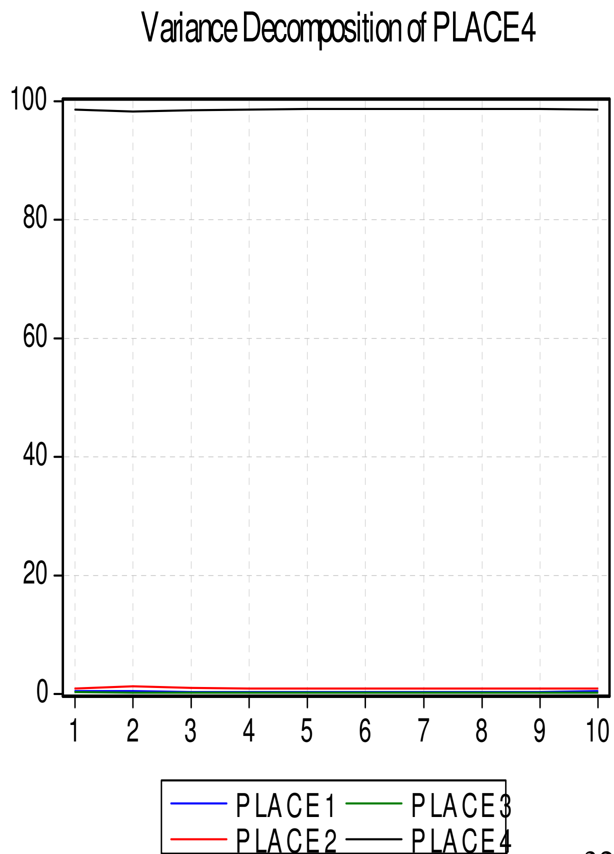

Response of PLACE4 to One S.D. Innovations
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
1 2 3 4 5 6 7 8 9 10
PLACE1 PLACE3
PLACE2 PLACE4

The impulse variance decomposition of Place_1, Place_2, Place_3, Place_4 are:









Conclusions:

in the paper the carbon dioxide emissions in Roma city were modeled using time series analysis. The models for 4 different places were built. For the first place the GARCH model was chosen like the most adequate while for other 3 places the ARMA models were chosen. At the end The VAR model was built for these four places.

6. References

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