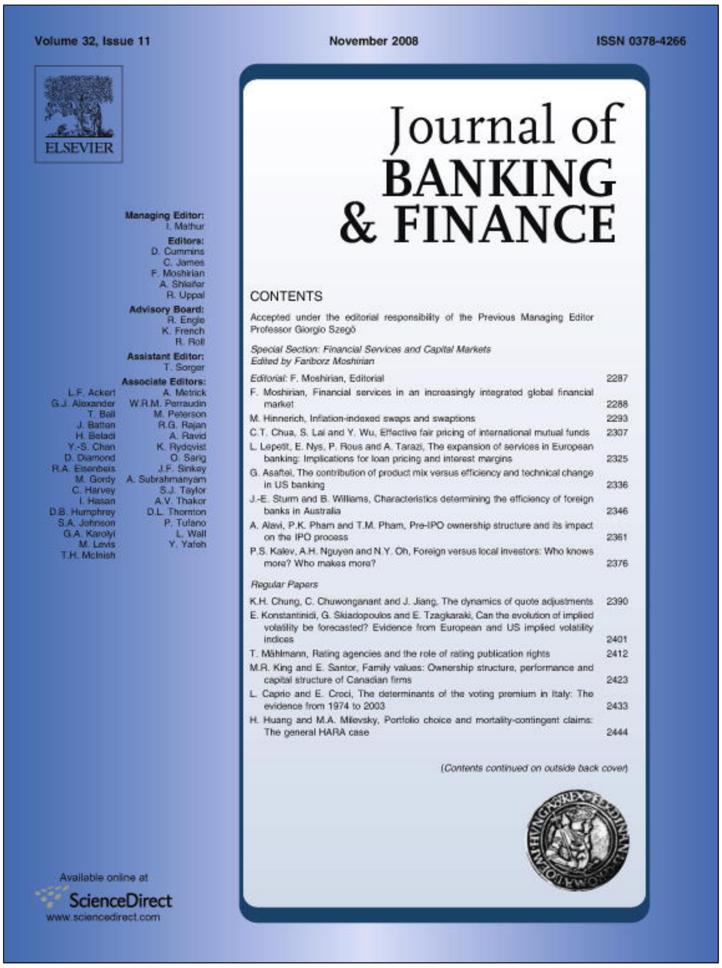
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# Accurate minimum capital risk requirements: A comparison of several approaches ☆

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ABSTRACT

In this paper we estimate, for several investment horizons, minimum capital risk requirements for short and long positions, using the unconditional distribution of three daily indexes futures returns and a set of short and long memory stochastic volatility and GARCH-type models. We consider the possibility that errors follow a *t*-Student distribution in order to capture the kurtosis of the returns' series. The results suggest that accurate modelling of extreme observations obtained for long and short trading investment positions is possible with an autoregressive stochastic volatility model. Moreover, modelling futures returns with a long memory stochastic volatility model produces, in general, excessive volatility persistence, and consequently, leads to large minimum capital risk requirement estimates. Finally, the models' predictive ability is assessed with the help of out-of-sample conditional tests.



## 1. Introduction

In recent years, financial markets across the world have reported an increase in volatility that has started to concern financial market regulators because of the large trading losses that some institutions such as banks might incur. With the Basle Accord of 1988, the first measure to tackle the problem was taken by demanding that financial institutions reserve a part of the capital to absorb a pre-specified percentage of these unforseen losses, which is known as minimum capital risk requirements (MCRRs) (see Brooks et al., 2000). From this moment, a need for quantitative techniques able to calculate these possible losses was created. The value-at-risk (VaR) has become a very popular tool that provides an estimate of the probability of likely losses to occur over a given time horizon due to changes in market prices. Several methods have been proposed to calculate the VaRs, such as the "delta-normal" method, the historical simulation that involves the estimation of the quantile of the portfolio returns, and the structured Monte Carlo simulation (see Dowd, 1998; Jorion, 2001). Although the Monte Carlo approach is powerful and flexible for generating VaR estimates because one can specify any stochastic process for the asset price, it is not free of important drawbacks. The first and most important one is related to the stochastic process that has been assumed for the price of the asset. In particular, if the underlying assumption is not correct, the calculated VaRs can be inaccurate. An alternative procedure that could overcome the first drawback and that is considered in our paper is to use bootstrap techniques rather than Monte Carlo simulation.

In this paper, we address an approach to the calculation of the MCRRs that follows the one adopted in many Internal Risk Management Models (see, for example, Hsieh, 1993; Brooks et al., 2000). These models are more important for European financial institutions since they are forced to respect the EC Capital Adequacy Directive (CAD). Thus, research in this area is justified given the importance of calculating accurate minimum capital risk requirements in order to avoid a wasting of valuable resources in those financial institutions that use or plan to use this approach in calculating minimum capital risk requirements.

The main purpose of the paper is to compare several short and long memory stochastic volatility and GARCH-type models (which we will refer to as conditional approach) and a new unconditional approach, based on moving block bootstrap techniques, not only in terms of accurate calculation of MCRRs, but also in terms of their out-of-sample predictive ability for futures short and long positions. It is important to remark that long memory volatility models have scarcely been tested before in the risk management literature. Some examples are the autoregressive long memory stochastic volatility model (ARLMSV) of Breidt et al. (1998) and Harvey (1998),

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the hyperbolic GARCH (HYGARCH) of Davidson (2004), the fractional integrated EGARCH (FIEGARCH) of Baillie et al. (1996) and the fractional integrated GARCH (FIGARCH) of Baillie (1996). These models specify volatility as a fractional integrated process with the purpose of capturing the slow decay of the autocorrelation functions of non-linear transformations of returns like squares and absolute values. Their importance in finance and economics literature is very relevant, not only theoretically, but also empirically, since they have already been applied gainfully to exchange rates, inflation rates, futures returns, options and other economic data. Some examples that illustrate their applicability are, among others, Baillie et al. (1996); Bollerslev and Mikkelsen (1996); Harvey (1998); Breidt et al. (1998); Baillie and Bollerslev (2000); Pérez and Ruiz (2001); Baillie et al. (2002); Jin and Frechette (2004); Taylor (2005); Baillie et al. (2007) and Cardamone and Folkinshteyn (2007). So far, there is no evidence on how these models perform when they are applied to the calculus of minimum capital risk requirements. In this way, our study fills a gap that we think is of particular interest for risk management practitioners.

A second contribution of this work is to allow the errors of some of these original models to follow a *t*-Student distribution in order to better fit the kurtosis of the data, as was suggested by Lehar et al. (2002). Our third contribution to this literature is to propose a much more accurate methodology for the calculation of the MCRRs in the context of unconditional approaches, when the returns are themselves dependent (even weakly) based on the moving block bootstrap of Künsch (1989) and Liu and Singh (1992). To sum up, our aim is to add new evidence from the futures market to the modelling of financial data by calculating appropriate MCRRs for three indexes futures, to highlight some volatility forecasting features of well-known specifications, since accurate volatility estimators for futures positions are essential for imposing optimal capital deposits, and to compare different approaches in order to better understand the risks associated with derivative positions.

We proceed by calculating the MCRRs for three indexes futures (the FTSE-100 index futures, the Russell 2000 index futures and the S&P 500 index futures) defined on long and short positions for 1, 5, 10, 30, 90 and 180 day horizons, using the new unconditional and conditional approaches. Regarding the first, we use the moving block bootstrap for computing the unconditional distribution of returns since, contrary to previous papers, we have found that the returns of the considered financial series are not independent and identically distributed (iid), not only due to the existence of non-linear dependence, but also due to a weak linear dependence structure of their own returns detected by the rejection of the null hypothesis of the corrected Ljung–Box test. This procedure is justified since we have calculated (for the FTSE-100 index futures) the MCRRs both with the iid bootstrap described in Efron and Tibshirani (1993) and the moving block bootstrap described in Lahiri (2003) and we have found huge differences in the MCRRs estimates, specially for long positions and larger investment horizons. Regarding the conditional approach, we have fitted several short and long memory stochastic volatility and GARCH-type models and we have found that models that introduce asymmetries between the conditional variance and the returns, such as the EGARCH, the GJR of Glosten et al. (1993) and the FIEGARCH, have been rejected by the BDS test (Brock et al., 1996) applied to the residuals. This is consistent with previous papers of Hsieh (1993) and Brooks et al. (2000) on futures data. The "best" models according to the BDS test take into account the volatility clustering, the fat tails of the returns distributions and the volatility persistence. Finally, the models' predictive ability is assessed by an exhaustive out-of-sample inference and by testing for conditional coverage.

The most important findings in this paper are: first, the autoregressive stochastic volatility model of Taylor (1986) with errors following a normal distribution performs better in terms of volatility forecasting and provides accurate MCRRs estimates; second, the long memory GARCHs (FIGARCH and HYGARCH) perform much better out-of-sample, specially for long positions, than the short memory GARCH with errors following both distributions, Gaussian and *t*-Student, for two out of the three futures series analyzed. However, the long memory stochastic volatility model, in general, generates excessive MCRRs due to the extreme volatility persistence that is produced by this model; and finally, the MCRRs based upon GARCH models are generally larger for short investment horizons and smaller for long investment horizons than the ones obtained with the alternative specifications due to the decreasing volatility forecastability registered by GARCH models when the forecasting horizon increases (see Christoffersen and Diebold, 2000).

The remainder of the paper proceeds as follows. In Section 2, we present a description of data and its main statistical properties. In Section 3, we estimate several conditional heteroscedastic and stochastic volatility models and present the forecasting and the MCRRs methodologies. We present the moving block bootstrap in Section 4. In Section 5 we report on the models' forecasts and the comparison of their predictive ability through out-of-sample conditional tests and we conclude in Section 6.

## 2. Data analysis

In this study, we have calculated the MCRRs for three indexes futures: the FTSE-100 index futures, the S&P 500 index futures, and the Russell 2000 index futures. The data was collected from EconWin and spans the period of 2 August 1989–18 May 2005 for the FTSE-100 index futures, the period of 4 August 1989–16 October 2006 for the S&P 500 index futures and the period of 5 February 1993–15 December 2006 for the Russell 2000 index futures. We have deleted from the data set observations corresponding to non-trading days to avoid the incorporations of spurious zero returns, leaving 3980, 4366 and 3421 observations for the FTSE-100, S&P 500 and Russell indexes futures, respectively.

Fig. 1 depicts the financial returns, $y_t = \log p_t - \log p_{t-1}$, where $p_t$ is the price at time $t$ of the corresponding index futures. Fig. 2 shows the volatility evolutions of the three returns series considered. In Table 1, where we report some summary statistics, we observe that the three returns series are negatively skewed and have a kurtosis between 5.7349 and 8.3197. According to the Kiefer and Salmon (1983) test, the third and fourth moments are significantly different from zero and three, respectively. This makes us suspect that the normal distribution is not the best approximation to the returns' distributions.[1]

Next, we tested whether the returns are iid because a rejection of this hypothesis leads to a difference on how conditional and unconditional densities describe short term dynamics of prices (see Hsieh, 1993). To this end, we have applied the BDS test of Brock et al., 1996 to the returns series. The results are reported in Table 2. The null hypothesis of iid is rejected for all series at a 5% level of significance, as in Hsieh (1993) and Brooks et al. (2000).

Hsieh (1991) showed that the BDS test can detect many types of non iid causes including linear dependence, non-stationarity, chaos and non-linear stochastic processes. In order to understand the underlying reason, we calculated the autocorrelation functions of the returns and squared returns up to order 15 and we tested if

[1] The Kiefer and Salmon, 1983 test is given by

$$KS_N = (KS_S)^2 + (KS_K)^2,$$

where $KS_S = \sqrt{\frac{T}{6}}\left[\frac{1}{T}\sum_{t=1}^{T} y_t^{*3} - \frac{3}{T}\sum_{t=1}^{T} y_t^{*}\right]$, $KS_K = \sqrt{\frac{T}{24}}\left[\frac{1}{T}\sum_{t=1}^{T} y_t^{*4} - \frac{6}{T}\sum_{t=1}^{T} y_t^{*2} + 3\right]$ and $y_t^*$ are the standardized returns. If the distribution of $y_t^*$ is conditional $N(0,1)$ then $KS_S$ and $KS_K$ are asymptotically $N(0,1)$, and $KS_N$ is asymptotically $\chi^2(2)$.

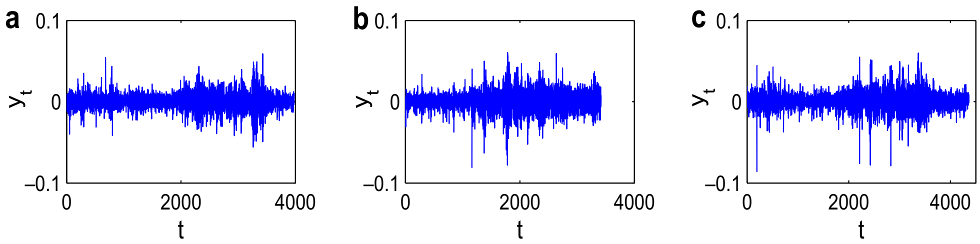


**Fig. 1.** Series of financial returns: (a) FTSE-100 index futures, (b) Russell index futures and (c) S&P 500 index futures.

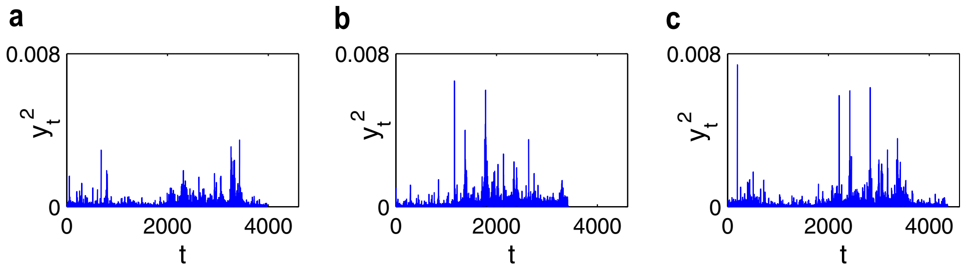


**Fig. 2.** Squared returns: (a) FTSE-100 index futures, (b) Russell index futures and (c) S&P 500 index futures.

**Table 1**
Summary statistics for the returns series

| Futures contracts | FTSE-100 | Russell | S&P 500 |
|---|---|---|---|
| Mean | 0.0002 | 0.0004 | 0.0004 |
| Variance | 0.0001 | 0.0001 | 0.0002 |
| Skewness | −0.0841* | −0.2588* | −0.2961* |
| Kurtosis | 5.8592* | 8.3197* | 5.7349* |
| $KS_S$ | −2.1645 | −7.0655 | −6.9767 |
| $KS_K$ | 36.7945 | 32.6280 | 71.9093 |

* Means that the third and fourth moments are significantly (at 5% level) different from zero and three, respectively.

**Table 2**
BDS test statistic for financial returns

| $\epsilon/\sigma$ | Contracts | Embedding dimensions | | | |
|---|---|---|---|---|---|
| | | 2 | 3 | 4 | 5 |
| 0.5 | FTSE-100 | 8.9 | 13.0 | 17.0 | 20.6 |
| | Russell | 15.4 | 22.9 | 30.3 | 40.9 |
| | SPF | 10.9 | 14.8 | 18.3 | 23.2 |
| 1.0 | FTSE-100 | 10.7 | 15.0 | 18.6 | 21.7 |
| | Russell | 14.4 | 21.3 | 25.9 | 31.4 |
| | SPF | 11.0 | 15.3 | 18.5 | 22.4 |
| 1.5 | FTSE-100 | 12.7 | 17.4 | 20.6 | 23.3 |
| | Russell | 14.0 | 20.7 | 24.1 | 27.1 |
| | SPF | 11.1 | 15.5 | 18.1 | 21.0 |
| 2.0 | FTSE-100 | 13.8 | 18.7 | 21.7 | 23.9 |
| | Russell | 13.8 | 20.7 | 23.7 | 25.9 |
| | SPF | 11.9 | 15.9 | 17.9 | 20.2 |

The critical values of the statistic for a two-tailed test are: 1.645 (10%), 1.960 (5%), 2.326 (2%), and 2.576 (1%).

they are statistically significant. Table 3 shows that the corrected Ljung–Box Q statistic and the individual significance tests evidence (at a 5% significance level) that both the returns and the squared observations are autocorrelated, although the autocorrelation is much stronger for the series of the squared returns. We used the heteroscedastic corrected version of the Ljung–Box statistic, when we applied it to the returns (Diebold, 1988).

After finding a non-linear dependence of the series, we then checked whether this non-linearity is in mean or in variance. We tested the null of zero conditional mean with the proposal of Hsieh (1989), Hsieh (1991). If the null hypothesis is true, the bicorrelation coefficients, $\rho(i,j) = E(y_t y_{t-i} y_{t-j})/[\mathrm{Var}(y_t)]^{3/2}$, are zero for all $i,j \geqslant 1$. These coefficients are asymptotically normal distributed with zero mean and variance $[(1/T)\sum y_t^2 y_{t-i}^2 y_{t-j}^2]/[(1/T)\sum y_t^2]^3$. Table 4 shows the results. None of the bicorrelation coefficients are statistically significant, which lead us to conclude that the non-linear dependence is in variance.

## 3. Conditional approach: GARCH and stochastic volatility modelling

### 3.1. Model selection and estimation

Given the conclusions of Section 2, we need to model carefully the conditional variance of the returns series to obtain accurate MCRR estimates. Rather than choosing a model a priori, we have estimated several models in an attempt to choose the best specifications for each series. In the conditional heteroscedasticity context, we first estimated the GARCH(1,1), the FIGARCH(1,1) and the HYGARCH(1,1) with normal and *t*-Student errors. We estimated the FIGARCH model, despite its drawbacks, because we would like to compare its forecasting performance to that of its alternative, the HYGARCH model. Finally, we also considered the GJR(1,1), the EGARCH(1,1) and the FIEGARCH(1,1) since Brooks and Persand (2003) found that allowing for asymmetric responses of volatility to positive and negative returns can improve the VaR estimates. The criteria chosen to select the models are based on their capacity to capture the non-linear dependence in returns. With this purpose, we applied the BDS test to the standardized residuals. Note that in this case, we have calculated new critical values because the test favors the null of iid, when we applied it to the standardized residuals of GARCH-type models. Therefore, we have simulated 2000 data series from each model with a sample size similar to the original one (imposing the same coefficient estimates), fitted each model on the simulated data and ran the BDS test on the residuals. The models have been estimated by quasi-maximum likelihood (QML) with the Ox GARCH 4.2 package of Laurent and Peters (2006).

**Table 3**
Autocorrelations of returns and squared returns

| Lag length | FTSE-100 returns | Squared returns | Russell returns | Squared returns | S&P 500 returns | Squared returns |
|---|---|---|---|---|---|---|
| 1 | 0.015 | 0.214* | −0.002 | 0.188* | −0.038* | 0.193* |
| 2 | −0.028 | 0.304* | −0.045* | 0.326* | −0.055* | 0.168* |
| 3 | −0.075* | 0.251* | 0.012 | 0.206* | −0.017 | 0.149* |
| 4 | 0.037* | 0.230* | 0.010 | 0.269* | 0.011 | 0.118* |
| 5 | −0.026 | 0.249* | −0.018 | 0.214* | −0.039* | 0.158* |
| 6 | −0.041* | 0.258* | −0.028 | 0.251* | −0.018 | 0.113* |
| 7 | −0.024 | 0.201* | −0.010 | 0.186* | −0.037* | 0.139* |
| 8 | 0.045* | 0.282* | −0.003 | 0.189* | −0.002 | 0.122* |
| 9 | 0.033 | 0.164* | −0.004 | 0.232* | 0.014 | 0.101* |
| 10 | −0.025 | 0.234* | −0.033 | 0.205* | 0.002 | 0.116* |
| 11 | 0.003 | 0.242* | 0.043* | 0.190* | −0.001 | 0.110* |
| 12 | 0.003 | 0.191* | 0.055* | 0.183* | 0.047* | 0.102* |
| 13 | 0.037* | 0.227* | 0.017 | 0.193* | 0.023 | 0.081* |
| 14 | −0.019 | 0.146* | −0.030 | 0.144* | 0.011 | 0.088* |
| 15 | 0.028 | 0.166* | 0.009 | 0.240* | −0.022 | 0.089* |
| Q(15) | 31.959* | 3113.5* | 18.494 | 2458.6* | 25.429* | 1058.7* |

The last line contains the values of the heteroscedastic corrected version of the Ljung–Box Q statistic.
* Means that the correlation of order $m$ with $m = 1, \ldots, 15$ is significant at a 5% significance level.

**Table 4**
Bicorrelation coefficients of the futures returns

| $\rho(i,j)$ | FTSE-100 | Russell | S&P 500 |
|---|---|---|---|
| $\rho(1,1)$ | 0.08 | 0.04 | 0.24 |
| $\rho(1,2)$ | −0.06 | −0.03 | 0.03 |
| $\rho(2,2)$ | 0.14 | 0.10 | 0.05 |
| $\rho(1,3)$ | 0.00 | 0.06 | 0.07 |
| $\rho(2,3)$ | −0.02 | −0.02 | 0.02 |
| $\rho(3,3)$ | 0.07 | 0.10 | 0.12 |
| $\rho(1,4)$ | 0.04 | 0.04 | 0.03 |
| $\rho(2,4)$ | 0.13 | 0.05 | 0.03 |
| $\rho(3,4)$ | 0.03 | −0.10 | −0.07 |
| $\rho(4,4)$ | 0.01 | 0.08 | 0.13 |
| $\rho(1,5)$ | −0.08 | −0.04 | −0.01 |
| $\rho(2,5)$ | −0.03 | 0.07 | 0.05 |
| $\rho(3,5)$ | 0.02 | 0.01 | −0.01 |
| $\rho(4,5)$ | −0.01 | −0.01 | 0.00 |
| $\rho(5,5)$ | −0.01 | −0.04 | 0.12 |

**Table 5**
BDS test statistic for the standardized residuals (*significant at a 5% significance level)

| Russell | $\epsilon/\sigma$ | Embedding dimensions | | | |
|---|---|---|---|---|---|
| | | 2 | 3 | 4 | 5 |
| GARCH-Gauss | 0.5 | 0.183 | 1.357 | 1.509 | 1.841 |
| | 1.0 | −0.371 | 1.028 | 1.231 | 1.541 |
| | 1.5 | −0.961 | 0.254 | 0.514 | 0.799 |
| | 2.0 | −1.539 | −0.609 | −0.191 | 0.038 |
| GARCH-Stud | 0.5 | 0.550 | 1.847 | 2.096 | 2.397* |
| | 1.0 | −0.057 | 1.441 | 1.700 | 2.045 |
| | 1.5 | −0.580 | 0.710 | 1.042 | 1.353 |
| | 2.0 | −1.090 | −0.088 | 0.400 | 0.646 |
| FIGARCH-Stud | 0.5 | 1.029 | 1.534 | 1.412 | 1.461 |
| | 1.0 | 0.672 | 1.386 | 1.069 | 1.038 |
| | 1.5 | 0.251 | 0.786 | 0.566 | 0.556 |
| | 2.0 | −0.188 | 0.195 | 0.141 | 0.110 |

The critical values can be obtained from the authors upon request.

Finally, the selected models, in the context of conditional heteroscedasticity, are the ones for which we do not reject the null hypothesis of iid standardized residuals, which means that these models have captured the non-linear dependence in the returns. These models are: the GARCH-Gauss (normal errors), the GARCH-Stud ($t$-Student errors), the FIGARCH-Stud and the HYGARCH-Gauss (this model is only significant for the FTSE-100 index futures). Table 5 presents the results of the BDS test for the standardized residuals obtained from fitting the selected models to the Russell index futures.[2]

The HYGARH model proposed by Davidson (2004) is given by

$$y_t = \mu + \varepsilon_t = \mu + \sigma_t \epsilon_t,$$

where $\varepsilon_t$ is the prediction error, $\sigma_t^2$ is the variance of $y_t$ given information at time $t-1$, $\sigma_t > 0$, $\epsilon_t \sim \text{NID}(0,1)$ or a $t$-Student distribution and

$$\sigma_t^2 = \omega + \theta(L)\varepsilon_t^2,$$

where

$$\theta(L) = 1 - \frac{\alpha^*(L)}{\beta(L)}(1 + \psi((1-L)^d - 1)),$$

$\alpha^*(L) = 1 - \sum_{i=1}^{q} \alpha_i^* L^i$, $\beta(L) = 1 - \sum_{i=1}^{p} \beta_i L^i$, $\omega > 0$, $\psi \geqslant 0$ and $d \geqslant 0$. For values of $d \in [0, 1/2)$ the conditional variance is stationary. The model simplifies to a GARCH($p,q$) and to a FIGARCH($p,d,q$) for $\psi = 0$ and $\psi = 1$, respectively. For $0 < \psi < 1$, we have a nested model that is able to generate long memory as $d$ increases.

From Table 6, we observe that the volatility persistence implied by the GARCH-type models, that depends on the sum of $\alpha$ and $\beta$ for the GARCH models, is quite high. As an example and for the FTSE-100 index futures, we have obtained values of implied volatility of 0.988 and 0.991 for the GARCH-Gauss and GARCH-Stud, respectively.

In the context of stochastic volatility, natural competitors to the GARCH, FIGARCH and HYGARCH models are the autoregressive stochastic volatility model (denoted ARSV) of Taylor (1986) and the autoregressive fractional integrated stochastic volatility model (denoted ARLMSV) that extends the models of Breidt et al. (1998) and Harvey (1998). The first is a short memory model while the second has as a short memory component and a long memory component (a fractional integrated process is specified for the volatility). These models have been estimated with the Whittle estimation method. Following the same procedure as before, we applied the BDS test to the residuals of the ARSV and the ARLMSV models and have observed that the null hypothesis of iid is not rejected for the ARSV residuals in all series except for the Russell index futures. Regarding the ARLMSV model, it seems that the model fits the FTSE-100 index futures returns very well.

[2] The results for the other series are similar to the ones presented here, except for the GARCH model that performs slightly worse for the FTSE-100 index futures. All test results are available on request from the authors.

**Table 6**
Estimates and standard errors (in parenthesis) of the GARCH-type selected models

| | Parameters | | | | | | |
|---|---|---|---|---|---|---|---|
| | $\mu$ | $\gamma$ | $\alpha$ | $\beta$ | *DF* | *d* | $ln(\psi)$ |
| *GARCH-Gauss* | | | | | | | |
| FTSE-100 | 0.0003 | 0.013 | 0.076 | 0.912 | | | |
| | (0.0001) | (0.003) | (0.009) | (0.011) | | | |
| Russell | 0.0007 | 0.019 | 0.115 | 0.875 | | | |
| | (0.0002) | (0.005) | (0.013) | (0.013) | | | |
| S&P 500 | 0.0005 | 0.008 | 0.057 | 0.936 | | | |
| | (0.0001) | (0.002) | (0.008) | (0.009) | | | |
| *GARCH-Stud* | | | | | | | |
| FTSE-100 | 0.0004 | 0.010 | 0.070 | 0.921 | 14.030 | | |
| | (0.0001) | (0.003) | (0.009) | (0.010) | (2.539) | | |
| Russell | 0.0008 | 0.014 | 0.104 | 0.890 | 17.009 | | |
| | (0.0002) | (0.004) | (0.013) | (0.013) | (4.061) | | |
| S&P 500 | 0.0006 | 0.005 | 0.054 | 0.943 | 6.271 | | |
| | (0.0001) | (0.002) | (0.008) | (0.009) | (0.576) | | |
| *FIGARCH-Stud* | | | | | | | |
| FTSE-100 | 0.0004 | 1.197 | 0.142 | 0.587 | 13.575 | 0.4801 | |
| | (0.0001) | (0.393) | (0.041) | (0.068) | (2.325) | (0.050) | |
| Russell | 0.0008 | 0.737 | n.s. | 0.266 | 18.640 | 0.3374 | |
| | (0.0002) | (0.158) | | (0.039) | (4.287) | (0.033) | |
| S&P 500 | 0.0006 | 0.752 | 0.227 | 0.608 | 6.576 | 0.420 | |
| | (0.0001) | (0.239) | (0.041) | (0.058) | (0.541) | (0.045) | |
| *HYGARCH-Gauss* | | | | | | | |
| FTSE-100 | 0.0003 | 0.039 | 0.145 | 0.658 | | 0.594 | −0.041 |
| | (0.0001) | (0.012) | (0.040) | (0.076) | | (0.093) | (0.020) |

n.s. stands for non significant at any relevant significance level.

The ARLMSV model is given by the following expressions:

$$y_t = \sigma\epsilon_t \exp\left(\frac{h_t}{2}\right), \quad (1)$$

$$(1-\phi L)(1-L)^d h_t = \eta_t. \quad (2)$$

In Eq. (1), $\sigma$ denotes a scale parameter, $\sigma_t = \exp(h_t/2)$ is the volatility of $y_t$ (the returns at time $t$), $\epsilon_t$ is NID$(0,1)$ and $\eta_t$ is NID$(0,\sigma_\eta^2)$, where $\sigma_\eta^2$ is the variance of $\eta_t$. The ARSV model is obtained from Eqs. (1) and (2) by imposing the restriction $d=0$.

Table 7 reports the parameter estimates for the stochastic volatility models. The estimate of $\sigma$ is $\hat{\sigma}$=$\exp\{0.5\hat{\mu}+0.5E(\log\epsilon_t^2)\}$, where $\hat{\mu}$ is the sample mean of $\log(y_t^2)$, and assuming the normality of errors we have that $E(\log\epsilon_t^2)=-1.27$ (see Zaffaroni, 2005). We have also observed that the volatility persistence implied is very high, inducing the effects of shocks to the conditional variance to take time to dissipate. We must remember that in the context of stochastic volatility, the persistence is given by $\phi$, while the memory is given by $d$, as in the case of the fractionally integrated GARCHs.

### 3.2. Forecasting

The main aim of this subsection is to highlight the volatility forecasting methodology. With the idea that a GARCH$(1,1)$ model can be written by recursive substitution as an ARCH$(\infty)$, the multi-step forecast of the conditional variance based upon the available information at $t$ is

$$\sigma_{t+k|t}^2 = \sigma^2 + (\alpha+\beta)^{k-1}(\sigma_{t+1|t}^2 - \sigma^2),$$

where $\sigma^2$ (the unconditional variance) is equal to $\sigma^2 = \gamma(1-\alpha-\beta)^{-1}$ and it is assumed that $(\alpha+\beta)<1$ in order to guarantee that $\sigma^2$ exists and the multi-step forecast of the conditional variance converges to the unconditional variance at an exponential rate fixed by $\alpha+\beta$ (see Andersen et al., 2006). If, instead of a GARCH$(1,1)$, we have a FIGARCH$(1,d,1)$, the actual conditional variance forecasts are given by

$$\sigma_{t+k|t+k-1}^2 = \gamma(1-\beta)^{-1} + \lambda(L)\sigma_{t+k-1|t+k-2}^2$$

with $\sigma_{t+k|t+k-1}^2 \equiv \epsilon_t^2$ for $k<0$, $\lambda(L) \equiv 1-(1-\beta L)^{-1}(1-\alpha L-\beta L)(1-L)^d$, whose coefficients are computed from the following expressions:

$$\lambda_1 = \alpha + d, \quad \lambda_j = \beta\lambda_{j-1} + \left[\frac{j-1-d}{j} - (\alpha+\beta)\right]\delta_{j-1}, \quad j=2,3,\ldots$$

and $\delta_j \equiv \delta_{j-1}(j-1-d)/j$. Note that the $\delta_j$'s are the coefficients in the Maclaurin series expansion of $(1-L)^d$ (see Andersen et al., 2006).

With respect to stochastic volatility, first, we have to estimate $h_t$ based on the full sample. To this end, we use a state-space smoothing algorithm (Kalman filter) that leads to the minimum mean square linear estimator (MMSLE) of $h_t$ (see Harvey and Shephard, 1993; Harvey, 1998). The method is based on transforming Eq. (1) to obtain

$$\mathbf{w} = k\mathbf{1} + \mathbf{h} + \xi, \quad (3)$$

where $\mathbf{w}$ is a $T\times 1$-vector that contains the observations of $\log y_t^2$, $t=1,\ldots,T$, $\mathbf{1}$ is a $T\times 1$-vector of ones, $\xi$ is a $T\times 1$-vector containing $\log\epsilon_t^2 - E(\log\epsilon_t^2)$, $t=1,\ldots,T$, and $k=\log\sigma^2+E(\log\epsilon_t^2)$. Under the assumptions that $h_t$ is stationary and $h_t$ and $\xi_t$ are uncorrelated, the covariance matrix of $\mathbf{w}$ is $\mathbf{V}=\mathbf{V}_h+\mathbf{V}_\xi$, where $\mathbf{V}_h$ and $\mathbf{V}_\xi$ are the covariance matrices of $h_t$ and $\xi_t$, respectively. Hence, the MMSLE of $h_t$, in matrix notation, is given by

$$\tilde{\mathbf{h}} = \mathbf{V}_h\mathbf{V}^{-1}\mathbf{w} + k(\mathbf{I}-\mathbf{V}_h\mathbf{V}^{-1})\mathbf{1}, \quad (4)$$

where $\mathbf{I}$ is the identity matrix $T\times T$. Moreover, since the $\xi_t$'s are serially uncorrelated, $\mathbf{V}_\xi = \sigma_\xi^2\mathbf{I}$, where $\sigma_\xi^2$ is the variance of $\xi_t$. Then, Eq. (4) can be written as

$$\tilde{\mathbf{h}} = (\mathbf{I}-\sigma_\xi^2\mathbf{V}^{-1})\mathbf{w} + k\sigma_\xi^2\mathbf{V}^{-1}\mathbf{1},$$

and $k$ can be estimated by the sample mean of $\log y_t^2$ (see Harvey, 1998).[3] Yajima (1988) showed that there is only a slight loss of efficiency if the mean is used instead of the GLS estimator. Moreover, since the matrix $\mathbf{V}$ is a Toeplitz matrix, we have implemented the Trench algorithm described in Zohar (1969) to invert it.

Forecasting $\log y_t^2$, for $t=T+1,\ldots,T+l$, implies for the stationary case that

$$\tilde{\mathbf{w}}_l = \mathbf{R}\mathbf{V}^{-1}\mathbf{w} + k(\tilde{\mathbf{I}}-\mathbf{R}\mathbf{V}^{-1})\mathbf{1},$$

where $\tilde{\mathbf{I}}$ is an $l\times T$-matrix defined in blocks in the following way:

$$\tilde{\mathbf{I}} = \left(\begin{array}{ccccc|ccc} 1 & 0 & 0 & \ldots & 0 & 0 & \ldots & 0 \\ 0 & 1 & 0 & \ldots & 0 & 0 & \ldots & 0 \\ \vdots & & \ddots & & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & 1 & 0 & \ldots & 0 \end{array}\right),$$

$\tilde{\mathbf{w}}_l$ is a $l\times 1$-vector containing the forecasts of $\log y_t^2$ for $t=$

[3] Harvey (1998) showed that if $h_t$ is not stationary, we should differentiate Eq. (3) and then estimators of the first differences of $h_t$ can be calculated from an analogous equation to Eq. (4).

**Table 7**
Estimates of the stochastic volatility selected models

| | Parameters | | | |
|---|---|---|---|---|
| | $\phi$ | $\sigma$ | $\sigma_\eta^2$ | *d* |
| *ARSV* | | | | |
| FTSE-100 | 0.994 | 0.003 | 0.007 | |
| Russell | 0.995 | 0.003 | 0.010 | |
| S&P 500 | 0.996 | 0.002 | 0.007 | |
| *ARLMSV* | | | | |
| FTSE-100 | 0.968 | 0.003 | 0.001 | 0.467 |
| Russell | 0.810 | 0.003 | 0.004 | 0.660 |
| S&P 500 | −0.795 | 0.002 | 0.098 | 0.874 |

$T+1,\ldots,T+l$, and $\mathbf{R}$ is a $l \times T$-matrix of covariances between $\mathbf{w}_l$ and $\mathbf{w}$. The forecasts of $\sigma^2_{T+j}$ for $j=1,\ldots,l$, are obtained by taking the exponential of the elements of $\mathbf{w}_l$ and multiplying them by $\tilde{\sigma}^2 = T^{-1}\sum_{t=1}^{T}\tilde{y}_t^2$, where $\tilde{y}_t = y_t \exp(-\tilde{h}_t/2)$.

### 3.3. MCRR methodology

Capital risk requirements, given by the percentage of the initial value of the position for 95% coverage, are estimated for 1, 5, 10, 30, 90 and 180 day investment horizons. To this end, we have generated 20,000 paths of future values of the price series with the help of the parameter estimates, the disturbances obtained by sampling with replacement from the iid standardized residuals (iid bootstrap), and the multi-step ahead volatility forecasts. The maximum loss over a given holding period is then obtained by computing

$$Q = (P_0 - P_1)n,$$

where $n$ is the number of contracts, $P_0$ is the initial value of the position and $P_1$ is the lowest simulated price (for a long position) or the highest simulated price (for a short position) over the period. We have assumed that the futures position is opened on the final day of the sample (see Brooks et al., 2000; Brooks, 2002). If the number of contracts is one, without loss of generality, we can write $\frac{Q}{P_0} = (1 - \frac{P_1}{P_0})$ for a long position, and $\frac{Q}{P_0} = (\frac{P_1}{P_0} - 1)$ for a short position. Note that, since $P_0$ is constant, the distribution of $Q$ only depends on the distribution of $P_1$.

In this paper, we have proceeded as in Hsieh (1993) assuming that simulated prices are lognormally distributed since this hypothesis is frequent in the finance literature. Consequently, the maximum loss for a long position over the simulated days is given by $Q/P_0 = 1 - \exp(c_\alpha s + m)$, where $c_\alpha$ is the $\alpha \times 100\%$ percentile of the standard normal distribution and $s$ and $m$ are the standard deviation and mean of the $\ln(P_1/P_0)$, respectively. Analogously, the maximum loss for a short position over the simulated days is given by $Q/P_0 = \exp(c_{1-\alpha}s + m) - 1$, where $c_{1-\alpha}$ is the $(1-\alpha) \times 100\%$ percentile of the standard normal distribution (see Brooks, 2002).

The confidence intervals for the MCRRs are obtained as the 95% percentile intervals estimated by iid bootstrap. For each model we estimated the parameters, we forecasted the volatility and we kept the standardized residuals. Each value of the MCRR is obtained from 200 re-samples of the standardized residuals, proceeding as described above, and the confidence intervals have been computed from 1000 estimated MCRR values. We have chosen the percentile intervals because it is possible to obtain a better balance in the left and right sides using the empirical distribution of the MCRRs instead of the underlying normal distribution (Efron and Tibshirani, 1993, chapter 13). The confidence intervals not only allow us to determine if the differences in the MCRRs are significant for the conditional and unconditional approaches, but they also give us an idea about the sample dispersion in the MCRR estimates.

## 4. Unconditional approach: moving block bootstrap

In order to compute the unconditional density for the VaRs estimators, instead of using the iid bootstrap technique of Efron and Tibshirani (1993), as was done by Hsieh (1993) and Brooks et al. (2000), we have applied the moving block bootstrap (see Lahiri, 2003) on the observed price changes directly. We have seen in Section 2 that the return series are not iid, mainly due to the existence of non-linear dependence. In fact, the autocorrelation functions of the squared returns are strongly significant. On the other hand, we have also found that the returns of the three series present a weak dependence structure confirmed by the rejection of the null hypothesis of the corrected Ljung–Box test. These two findings lead to the rejection of the iid hypothesis and to the inadequacy of the iid bootstrap in obtaining the empirical distribution of the returns series.

For selecting the block size we have run a pilot experiment, following the algorithm described below. First, we simulated a series of size $T$ from a GARCH model (we have seen in Section 3 that this model generates residuals that are iid for the three series, and consequently, it is a good specification for the financial returns) and we obtained the “true values” of the 2.5% and 97.5% percentiles (estimators of the VaR for long and short positions, respectively) as the mean values of 10,000 realizations of the simulated series. Second, we performed a moving block bootstrap with block size equal to $b$. For this, we selected $M$ realizations of the simulated series and a block size $b$. For each realization, we split it in size $b$ blocks and reconstructed it $B$ times to obtain the percentiles of the realization. Using the $M$ computed values of the 2.5% and 97.5% percentiles we obtained a confidence interval for each one of them. Then, we evaluated the coverage of the confidence intervals obtained in the second step, using the “true values” of the percentiles computed previously. Finally, we repeated this procedure for different values of the block size $b$ and we selected the value of $b$ for which the coverage of the confidence intervals was optimal. We have used values of $T = 2049$, $M = 1000$, $B = 200$ and $b = 2^k$ for $k = 0,\ldots,11$ and the best results have been obtained for $b = 2$, which is a common value for the block size when the inference problem involves higher-level parameters (see Lahiri, 2003).

Once the block size has been fixed to $b = 2$, the estimation of the MCRRs (see Tables 8–10) has been carried out over 20,000 block bootstrap replicates of each returns series and the confidence intervals shown in Tables 11–13 have been obtained as the 95%

**Table 8**
Minimum capital risk requirements for 95% coverage probability as a percent of the initial value of the FTSE-100 index futures

| No. of days | GARCH-Gauss | GARCH *t*-Stud | FIGARCH *t*-Stud | HYGARCH-Gauss | ARSV | ARLMSV | Uncond |
|---|---|---|---|---|---|---|---|
| *Long position* | | | | | | | |
| 1 | 1.00 | 0.98 | 0.88 | 0.89 | 0.90 | 0.77 | 1.26 |
| 5 | 2.18 | 2.16 | 2.05 | 1.95 | 2.02 | 1.73 | 2.74 |
| 10 | 3.01 | 2.99 | 2.96 | 2.71 | 2.88 | 2.46 | 3.82 |
| 30 | 4.77 | 4.77 | 5.27 | 4.40 | 5.09 | 4.34 | 6.41 |
| 90 | 6.46 | 6.62 | 9.28 | 6.70 | 9.15 | 7.74 | 10.08 |
| 180 | 6.80 | 7.10 | 12.98 | 8.11 | 13.32 | 11.09 | 12.94 |
| *Short position* | | | | | | | |
| 1 | 1.06 | 1.05 | 0.95 | 0.96 | 0.94 | 0.80 | 1.35 |
| 5 | 2.38 | 2.37 | 2.28 | 2.17 | 2.14 | 1.82 | 3.14 |
| 10 | 3.41 | 3.42 | 3.42 | 3.14 | 3.10 | 2.64 | 4.62 |
| 30 | 6.06 | 6.16 | 6.75 | 5.79 | 5.79 | 4.90 | 8.78 |
| 90 | 10.37 | 10.89 | 14.03 | 11.06 | 11.63 | 9.69 | 17.55 |
| 180 | 14.16 | 15.39 | 22.80 | 16.92 | 19.12 | 15.48 | 28.07 |

**Table 9**
Minimum capital risk requirements for 95% coverage probability as a percent of the initial value of the Russell index futures

| No. of days | GARCH-Gauss | GARCH *t*-Stud | FIGARCH *t*-Stud | ARSV | ARLMSV | Uncond |
|---|---|---|---|---|---|---|
| *Long position* | | | | | | |
| 1 | 1.21 | 1.23 | 1.29 | 1.09 | 2.00 | 1.57 |
| 5 | 2.56 | 2.60 | 2.89 | 2.39 | 4.50 | 3.56 |
| 10 | 3.55 | 3.62 | 4.24 | 3.44 | 6.34 | 5.00 |
| 30 | 5.41 | 5.60 | 7.42 | 5.93 | 11.02 | 8.22 |
| 90 | 7.11 | 7.67 | 12.45 | 10.44 | 18.52 | 12.62 |
| 180 | 7.50 | 8.34 | 16.42 | 15.08 | 25.41 | 16.17 |
| *Short position* | | | | | | |
| 1 | 1.32 | 1.34 | 1.40 | 1.13 | 2.06 | 1.66 |
| 5 | 2.93 | 2.99 | 3.24 | 2.50 | 4.61 | 3.74 |
| 10 | 4.32 | 4.42 | 4.96 | 3.65 | 6.70 | 5.57 |
| 30 | 8.14 | 8.46 | 10.10 | 6.95 | 12.21 | 10.78 |
| 90 | 15.37 | 16.51 | 20.74 | 13.95 | 22.70 | 21.93 |
| 180 | 23.82 | 26.27 | 33.51 | 23.20 | 33.96 | 35.68 |

**Table 10**
Minimum capital risk requirements for 95% coverage probability as a percent of the initial value of the S&P 500 index futures

| No. days | GARCH-Gauss | GARCH *t*-Stud | FIGARCH *t*-Stud | ARSV | ARLMSV | Uncond |
|---|---|---|---|---|---|---|
| *Long position* | | | | | | |
| 1 | 0.89 | 0.85 | 0.92 | 0.61 | 3.85 | 1.31 |
| 5 | 1.89 | 1.82 | 1.96 | 1.35 | 11.89 | 2.82 |
| 10 | 2.64 | 2.53 | 2.81 | 1.96 | 16.81 | 3.96 |
| 30 | 4.08 | 3.93 | 4.87 | 3.49 | 27.77 | 6.32 |
| 90 | 5.54 | 5.49 | 8.37 | 6.51 | 43.63 | 9.49 |
| 180 | 5.96 | 6.11 | 11.20 | 10.11 | 55.31 | 11.68 |
| *Short position* | | | | | | |
| 1 | 0.96 | 0.93 | 1.00 | 0.62 | 4.08 | 1.45 |
| 5 | 2.20 | 2.15 | 2.29 | 1.39 | 14.08 | 3.23 |
| 10 | 3.27 | 3.23 | 3.50 | 2.05 | 20.98 | 4.79 |
| 30 | 6.20 | 6.27 | 7.23 | 3.85 | 41.00 | 9.39 |
| 90 | 12.08 | 12.80 | 15.79 | 7.86 | 85.86 | 19.79 |
| 180 | 18.83 | 20.88 | 26.74 | 13.45 | 148.95 | 32.84 |

percentile intervals estimated by block-bootstrap. Each value of the MCRR is obtained from 200 re-samples of the returns series being considered proceeding as described above, and the confidence intervals are computed from 1000 estimated MCRR values. Fig. 3 shows the difference in the estimates of the MCRRs obtained with the iid bootstrap and the moving block bootstrap, specially for a long position. This difference reinforces the adequacy of the moving block bootstrap in our case.

## 5. Empirical results

### 5.1. MCRRs and confidence intervals

All series show larger MCRRs for short positions than for long positions, specially, as the investment horizon increases.

As an example, for the FTSE-100 index futures and according to the Gaussian GARCH(1,1), approximately 1%, 2.18% and 3.01% of the value of a long position (as a percentage of the initial value

**Table 11**
Approximate 95% central confidence intervals for the minimum capital risk requirements for 95% coverage probability as a percent of the initial value of the FTSE-100 index futures

| No. of days | GARCH-Gauss | GARCH *t*-Stud | FIGARCH *t*-Stud | HYGARCH-Gauss | ARSV | ARLMSV | Uncond |
|---|---|---|---|---|---|---|---|
| *Long position* | | | | | | | |
| 1 | [0.85, 1.14] | [0.86, 1.15] | [0.76, 1.02] | [0.77, 1.04] | [0.79, 1.03] | [0.67, 0.88] | [1.07, 1.48] |
| 5 | [1.89, 2.40] | [1.91, 2.43] | [1.78, 2.29] | [1.70, 2.17] | [1.78, 2.23] | [1.52, 1.90] | [2.41, 3.07] |
| 10 | [2.64, 3.34] | [2.66, 3.37] | [2.59, 3.31] | [2.39, 3.04] | [2.53, 3.19] | [2.16, 2.73] | [3.35, 4.26] |
| 30 | [4.22, 5.30] | [4.22, 5.29] | [4.65, 5.86] | [3.90, 4.90] | [4.54, 5.62] | [3.87, 4.80] | [5.63, 7.09] |
| 90 | [5.89, 7.34] | [5.74, 7.12] | [8.21, 10.31] | [5.91, 7.46] | [8.19, 10.10] | [6.94, 8.56] | [8.96, 11.26] |
| 180 | [6.27, 7.93] | [5.99, 7.51] | [11.49, 14.39] | [7.10, 9.03] | [12.00, 14.64] | [9.98, 12.20] | [11.41, 14.37] |
| *Short position* | | | | | | | |
| 1 | [0.90, 1.20] | [0.91, 1.21] | [0.81, 1.08] | [0.82, 1.10] | [0.81, 1.07] | [0.69, 0.91] | [1.14, 1.59] |
| 5 | [2.11, 2.60] | [2.12, 2.61] | [2.03, 2.51] | [1.93, 2.38] | [1.90, 2.36] | [1.62, 2.02] | [2.73, 3.66] |
| 10 | [3.05, 3.77] | [3.04, 3.77] | [3.05, 3.77] | [2.80, 3.46] | [2.75, 3.42] | [2.34, 2.91] | [4.08, 5.31] |
| 30 | [5.57, 6.73] | [5.47, 6.62] | [6.11, 7.42] | [5.24, 6.33] | [5.22, 6.40] | [4.43, 5.41] | [7.86, 9.86] |
| 90 | [9.79, 11.86] | [9.90, 11.27] | [12.62, 15.38] | [9.93, 12.02] | [10.43, 12.82] | [8.69, 10.68] | [15.72, 19.34] |
| 180 | [14.04, 16.74] | [12.88, 15.34] | [20.54, 25.00] | [15.37, 18.39] | [17.15, 21.10] | [13.90, 17.07] | [25.47, 30.91] |

**Table 12**
Approximate 95% central confidence intervals for the minimum capital risk requirements for 95% coverage probability as a percent of the initial value of the Russell index futures

| No. of days | GARCH-Gauss | GARCH *t*-Stud | FIGARCH *t*-Stud | ARSV | ARLMSV | Uncond |
|---|---|---|---|---|---|---|
| *Long position* | | | | | | |
| 1 | [0.99, 1.40] | [1.00, 1.42] | [1.06, 1.50] | [0.91, 1.26] | [1.68, 2.41] | [1.24, 1.97] |
| 5 | [2.25, 2.88] | [2.29, 2.93] | [2.55, 3.26] | [2.12, 2.69] | [3.92, 5.09] | [2.97, 4.05] |
| 10 | [3.10, 3.96] | [3.17, 4.04] | [3.72, 4.75] | [3.03, 3.84] | [5.65, 7.14] | [4.28, 5.71] |
| 30 | [4.79, 6.04] | [4.96, 6.25] | [6.62, 8.29] | [5.30, 6.57] | [9.94, 12.21] | [7.14, 9.18] |
| 90 | [6.23, 8.02] | [6.75, 8.72] | [11.11, 14.00] | [9.36, 11.65] | [16.72, 20.40] | [11.20, 14.29] |
| 180 | [6.53, 8.31] | [7.30, 9.35] | [14.55, 18.32] | [13.44, 16.63] | [23.22, 27.61] | [14.27, 18.05] |
| *Short position* | | | | | | |
| 1 | [1.14, 1.46] | [1.16, 1.49] | [1.22, 1.56] | [0.97, 1.27] | [1.77, 2.40] | [1.39, 2.01] |
| 5 | [2.64, 3.21] | [2.69, 3.27] | [2.92, 3.56] | [2.24, 2.78] | [4.04, 5.16] | [3.28, 4.21] |
| 10 | [3.89, 4.70] | [3.98, 4.82] | [4.46, 5.43] | [3.27, 4.04] | [5.89, 7.37] | [4.95, 6.22] |
| 30 | [7.35, 8.81] | [7.65, 9.16] | [9.11, 10.97] | [6.21, 7.62] | [10.89, 13.61] | [9.71, 11.84] |
| 90 | [13.92, 16.69] | [14.99, 17.99] | [18.70, 2.80] | [12.47, 15.40] | [20.06, 25.26] | [19.9, 24.34] |
| 180 | [22.02, 25.51] | [24.33, 28.32] | [30.38, 36.94] | [20.76, 25.41] | [30.14, 37.59] | [32.42, 39.46] |

**Table 13**
Approximate 95% central confidence intervals for the minimum capital risk requirements for 95% coverage probability as a percent of the initial value of the S&P 500 index futures

| No. of days | GARCH-Gauss | GARCH *t*-Stud | FIGARCH *t*-Stud | ARSV | ARLMSV | Uncond |
|---|---|---|---|---|---|---|
| *Long position* | | | | | | |
| 1 | [0.72, 1.21] | [0.69, 1.17] | [0.74, 1.27] | [0.51, 0.78] | [2.93, 5.19] | [1.06, 1.73] |
| 5 | [1.60, 2.26] | [1.54, 2.18] | [1.66, 2.36] | [1.16, 1.57] | [9.71, 14.11] | [2.38, 3.40] |
| 10 | [2.26, 3.03] | [2.17, 2.93] | [2.41, 3.26] | [1.72, 2.21] | [14.40, 19.39] | [3.38, 4.56] |
| 30 | [3.54, 4.66] | [3.40, 4.51] | [4.23, 5.55] | [3.10, 3.90] | [24.95, 30.53] | [5.55, 7.16] |
| 90 | [4.76, 6.21] | [4.73, 6.21] | [7.19, 9.33] | [5.79, 7.20] | [39.95, 47.41] | [8.33, 10.73] |
| 180 | [5.06, 6.68] | [5.22, 6.92] | [9.70, 12.50] | [9.03, 11.08] | [50.86, 58.76] | [10.19, 13.19] |
| *Short position* | | | | | | |
| 1 | [0.83, 1.22] | [0.80, 1.19] | [0.86, 1.30] | [0.53, 0.76] | [3.06, 5.59] | [1.12, 1.77] |
| 5 | [1.94, 2.43] | [1.91, 2.38] | [2.03, 2.54] | [1.22, 1.57] | [11.14, 17.54] | [2.80, 3.58] |
| 10 | [2.91, 3.58] | [2.88, 3.53] | [3.12, 3.83] | [1.81, 2.26] | [17.64, 25.13] | [4.23, 5.36] |
| 30 | [5.64, 6.77] | [5.72, 6.84] | [6.57, 7.92] | [3.46, 4.24] | [35.18, 47.01] | [8.40, 10.33] |
| 90 | [11.08, 13.04] | [11.83, 13.83] | [14.53, 17.28] | [7.09, 8.67] | [73.68, 98.27] | [17.88, 21.42] |
| 180 | [17.42, 19.98] | [19.42, 22.28] | [24.47, 28.78] | [12.11, 14.70] | [123.87, 169.07] | [29.85, 35.62] |

of the position) will be enough to cover 95% of the expected losses if the position is held for 1, 5 and 10 days, respectively. The MCRRs for a short position are approximately 1.06%, 2.38% and 3.41%, respectively. This finding could be explained by the existence of a positive drift in the returns over the sample period, indicating that the series are not symmetric about zero. In fact, the mean for all series is positive over the sample period. The FTSE-100 index futures MCRRs are smaller than the ones obtained by Brooks et al. (2000) for the same series, possibly because our sample period does not include such extreme events like the stock market crash of October 1987.

Moreover, the MCCRs derived from block bootstrap are in general larger than those obtained from the conditional approach. This may occur because the level of volatility at the beginning of the

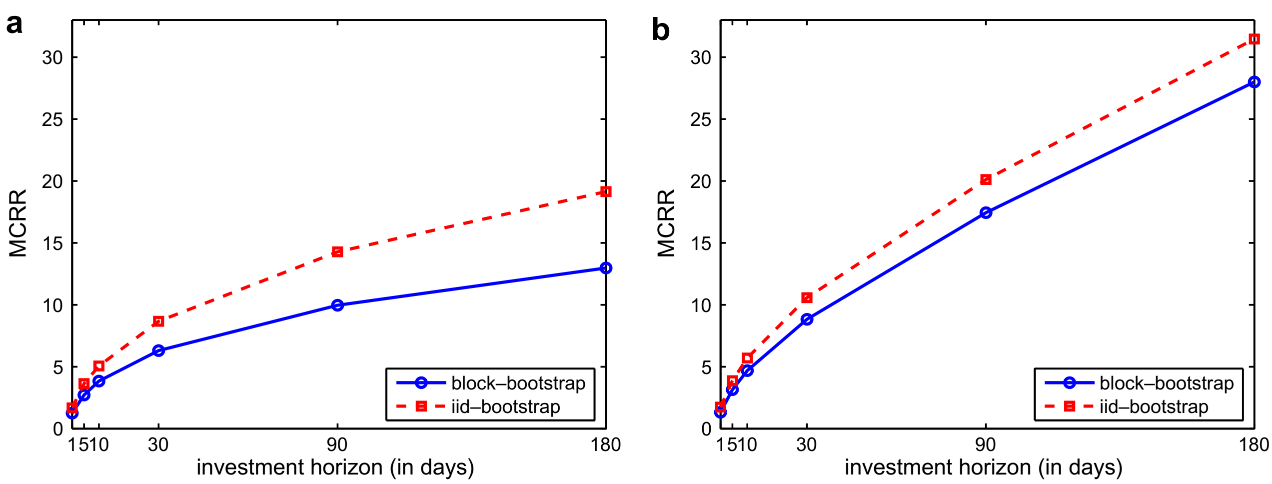


**Fig. 3.** Comparison of the moving block bootstrap and the iid bootstrap methods in computing the capital requirement for 95% coverage probability as a percent of the initial value of the FTSE-100 Index Futures for (a) long position and (b) short position.

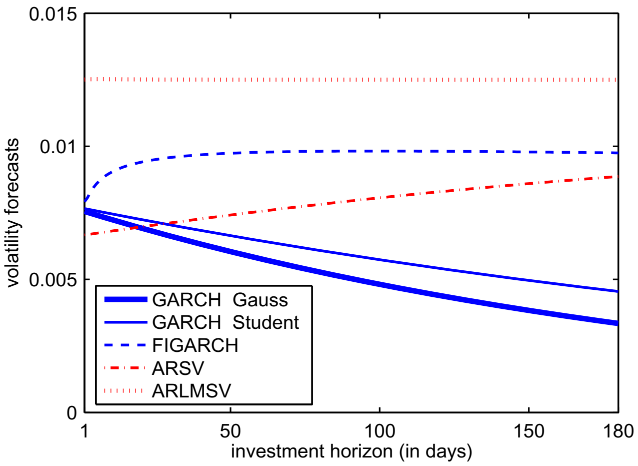


**Fig. 4.** Russell index futures volatility forecasts.

MCRR's calculation period is low in relation to its historical level (see Fig. 2). Therefore, the conditional approach gives us lower volatility forecasts than the historical average. As the holding period increases from 1 to 180 days, the MCRR estimates converge to those of the unconditional approach, except the ones obtained with the ARLMSV model for the returns of the Russell and S&P 500 indexes futures. Those seem to diverge from the unconditionally estimated MCRRs as the horizon increases (see Tables 8–10). This happens because the ARLMSV model generates excessive volatility persistence for these two returns series. Note that the estimates of $d$ (the long memory parameter) in these two cases lead to a non-stationary model.

We also observe that the MCRRs calculated with the Gaussian GARCH are in general higher for short investment horizons and smaller for larger investment horizons in comparison to the ones calculated with other specifications. Furthermore, the MCRRs based upon the FIGARCH model (for the Russell and the S&P 500 indexes futures) are larger than the ones calculated based upon the alternative models. This is due to the low volatility forecastability of the GARCH model in larger forecasting horizons and the high volatility forecasted by the FIGARCH model. In fact, from Fig. 4 we observe that GARCH models forecast high values for the volatility at the beginning of the out-of-sample period that decrease exponentially with the forecasting horizon.

Tables 11–13 show the 95% confidence intervals for the MCRRs based upon the unconditional and the conditional approaches. The results show that the amplitude of the intervals increase with the investment horizon, which makes the MCRR estimates for longer horizons less reliable. Except for the FTSE-100 index futures series, the confidence intervals for 5 or more investment days for both the GARCH and the HYGARCH models never overlap with the ones obtained with the unconditional density (see Brooks et al., 2000). This indicates that there is a huge difference between the MCRRs obtained using these models and the ones obtained with the unconditional density. This is not the case for the other conditional specifications.

### 5.2. Models' predictive ability

For a full evaluation of the results, we performed out-of-sample conditional tests on the MCRRs calculated with the selected models. By definition, the failure rate of a model is the number of times the estimated MCRRs are inferior to the returns (in absolute value). If the MCRR model is correctly specified, the failure rate should be equal to the pre-specified MCRR level (in our case, 5%).[4] Therefore, we calculated the MCRRs for one day horizon for both long and short positions and then checked if these MCRRs have been exceeded by price movements in day $t+1$. We rolled this process forward and we calculated the MCRRs for 756 days. In Table 14 we present the number of violations of the MCRR estimates generated by the models and by sampling with the moving block bootstrap from the unconditional distribution of returns. Both for the S&P 500 index futures and short positions the number of violations (in percentage) almost never exceeds the 5% nominal value. This indicates that the models generate "slight" excessive MCRRs for this series and this position. On the contrary, for the FTSE-100 and Russell indexes futures and long positions the models tend to over reject. Looking at Figs. 1 and 2, it is not strange that this happens since the out-of-sample period corresponds to a very volatile period and the series are negatively skewed. The best performance is for the ARSV model that registers failure rates closer to the nominal 5% level, specially, for short positions. Regarding the ARLMSV model, the results show that the model generates failure rates inferior to the theoretical ones, and consequently, excessive MCRRs. For the Russell and S&P 500 indexes futures, we have not calculated the failure rate due to its bad performance in calculating the MCRR estimates.

Since the calculation of the empirical failure rate defines a sequence of ones (MCRR violation) and zeros (no MCRR violation), we can test if the theoretical failure rate, $f$, is equal to 5%, i.e., $H_0: f = 5\%$ vs. $H_1: f \neq 5\%$. Standard evaluation of the failure rate proceeds by simply comparing the percentage of exceedances to the true failure rate. But, as it was pointed out in the works of West (1996) and McCracken (2000) when parameters are estimated, parameter uncertainty can play a role in out-of-sample inference. According to Christoffersen (1998), testing for conditional coverage is important in the presence of higher order dynamics and he proposes a procedure that is composed of three tests.[5]

The first tests for the unconditional coverage (denoted $LR_{\mathrm{uc}}$), the second for the independence part of the conditional coverage hypothesis (denoted $LR_{\mathrm{ind}}$) and the third is a joint test of coverage and independence (denoted $LR_{\mathrm{cc}}$). With this complete procedure it is possible to check if the dynamics or the error distribution is misspecified or both. Table 15 reports the results of the likelihood ratio tests for conditional coverage. They show evidence that the ARSV model is the only one to pass the three tests for the three series, and that among the GARCH-type models the FIGARCH has the worst performance, and the GARCH with errors following a $t$-Student distribution improves upon the Gaussian GARCH for the S&P 500 index futures returns.

[4] For a long position the failure rate is obtained as the percentage of negative returns smaller than the one day ahead MCRRs calculated for long positions. Analogously, for a short position the failure rate is estimated as the percentage of positive returns larger than the one day ahead MCRRs calculated for short positions (see Giot and Laurent, 2003, 2004).

[5] The unconditional coverage test is a standard likelihood ratio test given by

$$LR_{\mathrm{uc}} = -2log[L(p; I_1, I_2, \ldots, I_T)/L(\hat{\pi}; I_1, I_2, \ldots, I_T)] \overset{\mathrm{asy}}{\sim} \chi^2(1),$$

where $\{I_t\}_{t=1}^{T}$ is the indicator sequence, $p$ is the theoretical coverage, $\hat{\pi} = n_1/(n_0+n_1)$ is the maximum likelihood estimate of the alternative failure rate $\pi$, $n_0$ is the number of zeros and $n_1$ is the number of ones in the sequence $\{I_t\}_{t=1}^{T}$. The likelihood ratio test of independence is

$$LR_{\mathrm{ind}} = -2log[L(\hat{\Pi}_2; I_1, I_2, \ldots, I_T)/L(\hat{\Pi}_1; I_1, I_2, \ldots, I_T)] \overset{\mathrm{asy}}{\sim} \chi^2(1),$$

where $\hat{\Pi}_1 = \begin{bmatrix} n_{00}/(n_{00}+n_{01}) & n_{01}/(n_{00}+n_{01}) \\ n_{10}/(n_{10}+n_{11}) & n_{11}/(n_{10}+n_{11}) \end{bmatrix}$, $\hat{\Pi}_2 = \begin{bmatrix} 1-\hat{\pi}_2 & \hat{\pi}_2 \\ 1-\hat{\pi}_2 & \hat{\pi}_2 \end{bmatrix}$, $n_{ij}$ is the number of observations with value $i$ followed by $j$ and $\hat{\pi}_2 = (n_{01}+n_{11})/(n_{00}+n_{10}+n_{01}+n_{11})$.The joint test of coverage and independence is given by

$$LR_{\mathrm{cc}} = -2\log[L(p; I_1, I_2, \ldots, I_T)/L(\hat{\Pi}_1; I_1, I_2, \ldots, I_T)] \overset{\mathrm{asy}}{\sim} \chi^2(1).$$

**Table 14**
Estimates of the failure rate (proportions of exceedances) obtained one step ahead

| | FTSE-100 | | Russell | | S&P 500 | |
|---|---|---|---|---|---|---|
| | Long position (%) | Short position (%) | Long position (%) | Short position (%) | Long position (%) | Short position (%) |
| Unconditional | 10.6 | 8.6 | 10.2 | 7.5 | 4.4 | 3.0 |
| GARCH-Gauss | 6.1 | 3.6 | 6.9 | 4.1 | 4.0 | 3.6 |
| GARCH *t*-Stud | 6.0 | 3.8 | 6.9 | 4.0 | 4.5 | 3.7 |
| FIGARCH *t*-Stud | 5.0 | 3.2 | 6.6* | 3.7* | 3.6* | 3.2* |
| HYGARCH-Gauss | 5.6 | 3.6 | | | | |
| ARSV | 6.6 | 5.2 | 6.2* | 4.9* | 3.7* | 4.1* |
| ARLMSV | 3.3 | 2.8 | | | | |

The MCRR's are computed to cover the 95% of expected losses. The * means that we have not calculated the failure rate for these models.

**Table 15**
*p*-Values for the null hypotheses $f = \alpha$, with $\alpha = 5\%$

| | FTSE-100 | | | Russell | | | S&P 500 | | |
|---|---|---|---|---|---|---|---|---|---|
| | $LR_{uc}$ | $LR_{ind}$ | $LR_{cc}$ | $LR_{uc}$ | $LR_{ind}$ | $LR_{cc}$ | $LR_{uc}$ | $LR_{ind}$ | $LR_{cc}$ |
| | *Long position* | | | | | | | | |
| Unconditional | 0.000 | 0.101 | 0.000 | 0.000 | 0.024 | 0.000 | 0.413 | 0.062 | 0.120 |
| GARCH-Gauss | 0.185 | 0.202 | 0.173 | 0.025 | 0.080 | 0.016 | 0.178 | 0.142 | 0.132 |
| GARCH *t*-Stud | 0.243 | 0.173 | 0.188 | 0.025 | 0.080 | 0.016 | 0.519 | 0.076 | 0.162 |
| FIGARCH *t*-Stud | 0.973 | 0.046 | 0.129 | 0.052* | 0.351* | 0.091* | 0.058* | 0.080* | 0.035* |
| HYGARCH-Gauss | 0.491 | 0.103 | 0.197 | | | | | | |
| ARSV | 0.052 | 0.149 | 0.050 | 0.138* | 0.234* | 0.154* | 0.087* | 0.380* | 0.152* |
| ARLMSV | 0.023 | 0.254 | 0.038 | | | | | | |
| | *Short position* | | | | | | | | |
| Unconditional | 0.000 | 0.291 | 0.000 | 0.003 | 0.043 | 0.001 | 0.008 | 0.229 | 0.014 |
| GARCH-Gauss | 0.058 | 0.165 | 0.060 | 0.242 | 0.103 | 0.128 | 0.058 | 0.157 | 0.059 |
| GARCH *t*-Stud | 0.126 | 0.939 | 0.298 | 0.178 | 0.115 | 0.112 | 0.087 | 0.142 | 0.076 |
| FIGARCH *t*-Stud | 0.014 | 0.758 | 0.045 | 0.087* | 0.142* | 0.076* | 0.014* | 0.209* | 0.021* |
| HYGARCH-Gauss | 0.058 | 0.941 | 0.160 | | | | | | |
| ARSV | 0.842 | 0.978 | 0.929 | 0.893* | 0.051* | 0.140* | 0.242* | 0.103* | 0.128* |
| ARLMSV | 0.002 | 0.273 | 0.005 | | | | | | |

$LR_{uc}$, $LR_{ind}$, $LR_{cc}$, stand for the *LR* test of unconditional coverage, the *LR* test of independence and the joint test of coverage and independence, respectively. The * means that we have not calculated the failure rate for these models.

## 6. Conclusion

This paper compares three different approaches (unconditional density computed using moving block bootstrap, conditional heteroscedastic and stochastic volatility models) to calculate minimum capital risk requirements for long and short positions for three indexes futures. We calculated the MCRRs for 1, 5, 10, 30, 90 and 180 day investment horizons and we found that the autoregressive stochastic volatility model of Taylor (1986) with errors following a normal distribution does produce more accurate estimates of MCRRs and that it is the only one whose failure rate in the out-of-sample test is closer to the nominal 5%. For these reasons, we conclude that the ARSV model seems reasonable in modelling the three studied indexes futures. Second, the long memory GARCHs (FIGARCH and HYGARCH) perform much better out-of-sample, specially for long positions, than the short memory GARCH with errors following both distributions, Gaussian and *t*-Student. It is important to mention that the HYGARCH is slightly better than the other models considered in modelling the FTSE-100 index futures. Nevertheless, we found that, in general, the long memory stochastic volatility model generates excessive MCRRs since it produces extreme volatility persistence. Third, short memory GARCH models with errors following a *t*-Student distribution improve in terms of failure rate, upon short memory GARCH models with errors following the Gaussian distribution. Finally, we also found that the volatility forecastability decreases with the increase of the investment horizons, which is reflected by the range of the MCRRs confidence intervals (see Christoffersen and Diebold, 2000, for similar conclusions). This reinforces the importance of volatility modelling in obtaining accurate MCRRs and, therefore, in avoiding a wasting of valuable resources in those financial institutions that use or plan to use these models to calculate minimum capital risk requirements.

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