APPENDIX of
Outliers, GARCH-type models and risk measures: A comparison of several approaches

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Methods under evaluation

Complete results of the simulation study for evaluating the effects of outliers on the estimation of the MCRRs. The methods under evaluation can be classified in three different categories:

[1] Correcting for significant outliers

- **M1** Filtering using the procedure by Grané and Veiga (2010) with hard-thresholding and then fitting the considered model,
- **M2** Filtering using the procedure by Grané and Veiga (2010) with soft-thresholding and next fitting the considered model,
- **M3** Filtering using the procedure by Franses and Ghijsels (1999) and then fitting a GARCH(1,1) model,

[2] Accommodating outliers using complex distributions and

- **M4** Fitting t-distributed GARCH(1,1) or GJR(1,1) models, with endogenous degrees of freedom,
- **M5** Fitting skewed t-distributed GARCH(1,1) or GJR(1,1) models, with endogenous degrees of freedom,

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[3] Accounting for outlier effects by robust estimation

M6 Fitting a Gaussian GARCH(1,1) using Muler and Yohai (2008)'s robust BM2 estimation method.

Results for **M1** method. MCRRs for 95% coverage probability as a percentage of the initial value of the simulated series corrected for outliers (standard deviation) and e_r stands for the relative error.

			GARCH(1,1)				GJR(1,1)			
			Long Position		Short Position		Long Position		Short Position	
		n	MCRR	e_r	MCRR	e_r	MCRR	e_r	MCRR	e_r
s1	1 ALO	500	1.599 (0.584)	0.018	1.627 (0.628)	0.018	2.150 (1.036)	0.026	2.200 (1.050)	0.029
	of size	1000	1.575 (0.530)	0.010	1.605 (0.552)	0.010	2.150 (1.096)	0.017	2.207 (1.200)	0.019
	$\omega_{AO} = 5\sigma_y$	5000	1.572 (0.505)	0.002	1.600 (0.525)	0.001	2.104 (0.904)	0.002	2.159 (0.963)	0.002
	1 ALO	500	1.653 (0.639)	0.052	1.684 (0.688)	0.054	2.258 (1.271)	0.077	2.323 (1.361)	0.087
	of size	1000	1.598 (0.537)	0.024	1.630 (0.561)	0.026	2.181 (1.078)	0.032	2.240 (1.151)	0.035
	$\omega_{AO} = 10\sigma_y$	5000	1.576 (0.510)	0.004	1.605 (0.531)	0.004	2.112 (0.905)	0.006	2.166 (0.963)	0.006
	1 ALO	500	1.722 (0.744)	0.096	1.757 (0.815)	0.106	2.419 (1.745)	0.154	2.512 (2.205)	0.175
	of size	1000	1.640 (0.573)	0.051	1.673 (0.602)	0.053	2.264 (1.293)	0.071	2.335 (1.462)	0.079
	$\omega_{AO} = 15\sigma_y$	5000	1.585 (0.522)	0.010	1.615 (0.546)	0.011	2.145 (1.119)	0.022	2.209 (1.302)	0.026
s2	2 ALOs	500	1.795 (0.718)	0.143	1.831 (0.771)	0.146	2.554 (2.166)	0.219	2.674 (2.862)	0.251
	of size	1000	1.687 (0.584)	0.081	1.723 (0.620)	0.084	2.407 (1.486)	0.139	2.487 (1.668)	0.149
	$\omega_{AO} = 15\sigma_y$	5000	1.585 (0.522)	0.010	1.615 (0.546)	0.011	2.201 (1.273)	0.049	2.271 (1.610)	0.054
s3	Patch 3 ALOs	500	1.824 (1.180)	0.161	1.875 (1.344)	0.173	2.572 (2.355)	0.227	2.699 (2.858)	0.262
	of size	1000	1.685 (0.793)	0.080	1.723 (0.881)	0.084	2.296 (1.455)	0.086	2.368 (1.666)	0.094
	$\omega_{AO} = 15\sigma_y$	5000	1.613 (0.697)	0.028	1.645 (0.766)	0.029	2.134 (0.930)	0.017	2.193 (0.992)	0.018
s4	1 AVO	500	5.528 (6.454)	2.519	6.712 (9.609)	3.200	2.988 (2.889)	0.426	3.218 (3.570)	0.505
	of size	1000	3.940 (5.677)	1.526	4.685 (8.387)	1.948	2.453 (1.671)	0.160	2.568 (1.866)	0.186
	$\omega_{AO} = 15\sigma_y$	5000	2.227 (3.106)	0.419	2.422 (4.117)	0.516	2.121 (1.017)	0.010	2.185 (1.103)	0.014
s5a	No outliers	500	1.571 (0.579)		1.598 (0.620)		2.096 (1.004)		2.138 (1.011)	
		1000	1.560 (0.530)		1.589 (0.551)		2.114 (0.993)		2.165 (1.047)	
		5000	1.569 (0.506)		1.598 (0.526)		2.099 (0.904)		2.154 (0.960)	

GARCH(1,1) model with parameter values $\{\alpha_0 = 0.01, \alpha_1 = 0.08, \beta_1 = 0.91\}$ and GJR(1,1) model with parameter values $\{\alpha_0 = 0.02, \alpha_1 = 0.03, \beta_1 = 0.91, \gamma_1 = 0.10\}$.

Results for **M2** method. MCRRs for 95% coverage probability as a percentage of the initial value of the simulated series corrected for outliers (standard deviation) and e_r stands for the relative error.

			GARCH(1,1)				GJR(1,1)			
			Long Position		Short Position		Long Position		Short Position	
		n	MCRR	e_r	MCRR	e_r	MCRR	e_r	MCRR	e_r
s1	1 ALO	500	1.601 (0.581)	0.019	1.630 (0.626)	0.020	2.153 (1.036)	0.027	2.203 (1.049)	0.030
	of size	1000	1.576 (0.528)	0.010	1.606 (0.550)	0.011	2.153 (1.098)	0.018	2.209 (1.201)	0.020
	$\omega_{AO} = 5\sigma_y$	5000	1.572 (0.504)	0.002	1.600 (0.524)	0.001	2.104 (0.903)	0.002	2.159 (0.962)	0.002
	1 ALO	500	1.671 (0.667)	0.064	1.704 (0.720)	0.066	2.278 (1.305)	0.087	2.345 (1.404)	0.097
	of size	1000	1.617 (0.547)	0.037	1.649 (0.574)	0.038	2.197 (1.095)	0.039	2.257 (1.172)	0.042
	$\omega_{AO} = 10\sigma_y$	5000	1.581 (0.507)	0.008	1.611 (0.528)	0.008	2.119 (0.907)	0.010	2.174 (0.965)	0.009
	1 ALO	500	1.744 (0.819)	0.110	1.781 (0.903)	0.115	2.455 (1.813)	0.171	2.554 (2.284)	0.195
	of size	1000	1.673 (0.607)	0.072	1.708 (0.644)	0.075	2.292 (1.336)	0.084	2.365 (1.510)	0.092
	$\omega_{AO} = 15\sigma_y$	5000	1.603 (0.527)	0.022	1.632 (0.552)	0.021	2.163 (1.134)	0.030	2.228 (1.318)	0.034
s2	2 ALOs	500	1.946 (1.006)	0.239	1.994 (1.087)	0.248	2.679 (2.054)	0.278	2.792 (2.392)	0.306
	of size	1000	1.775 (0.812)	0.138	1.821 (0.908)	0.146	2.489 (1.598)	0.177	2.581 (1.804)	0.192
	$\omega_{AO} = 15\sigma_y$	5000	1.603 (0.527)	0.022	1.632 (0.552)	0.021	2.219 (1.213)	0.057	2.288 (1.474)	0.062
s3	Patch 3 ALOs	500	1.856 (1.268)	0.181	1.909 (1.434)	0.195	2.612 (2.496)	0.246	2.747 (3.054)	0.285
	of size	1000	1.703 (0.831)	0.092	1.743 (0.933)	0.097	2.303 (1.502)	0.089	2.377 (1.734)	0.098
	$\omega_{AO} = 15\sigma_y$	5000	1.624 (0.721)	0.035	1.658 (0.798)	0.038	2.144 (0.943)	0.021	2.202 (1.008)	0.022
s4	1 AVO	500	5.527 (6.733)	2.518	6.805 (10.490)	3.258	3.005 (2.925)	0.434	3.243 (3.643)	0.517
	of size	1000	3.960 (5.692)	1.538	4.712 (8.369)	1.965	2.470 (1.693)	0.168	2.588 (1.893)	0.195
	$\omega_{AO} = 15\sigma_y$	5000	2.334 (3.433)	0.488	2.580 (5.006)	0.615	2.147 (1.039)	0.023	2.214 (1.130)	0.028
s5a	No outliers	500	1.571 (0.579)		1.598 (0.620)		2.096 (1.004)		2.138 (1.011)	
		1000	1.560 (0.530)		1.589 (0.551)		2.114 (0.993)		2.165 (1.047)	
		5000	1.569 (0.506)		1.598 (0.526)		2.099 (0.904)		2.154 (0.960)	

GARCH(1,1) model with parameter values $\{\alpha_0 = 0.01, \alpha_1 = 0.08, \beta_1 = 0.91\}$ and GJR(1,1) model with parameter values $\{\alpha_0 = 0.02, \alpha_1 = 0.03, \beta_1 = 0.91, \gamma_1 = 0.10\}$.

Results for **M3** method. MCRRs for 95% coverage probability as a percentage of the initial value of the simulated series (standard deviation) and e_r stands for the relative error.

			GARCH(1,1)			
			Long Position		Short Position	
		n	MCRR	e_r	MCRR	e_r
s1	1 ALO of size	500	1.558 (0.557)	-0.008	1.585 (0.593)	-0.008
	$\omega_{AO} = 5\sigma_y$	1000	1.573 (0.556)	0.008	1.604 (0.588)	0.009
	1 ALO of size	500	1.676 (1.576)	0.067	1.737 (2.332)	0.087
	$\omega_{AO} = 10\sigma_y$	1000	1.595 (0.743)	0.022	1.627 (0.789)	0.024
	1 ALO of size	500	1.780 (1.293)	0.133	1.829 (1.529)	0.145
	$\omega_{AO} = 15\sigma_y$	1000	1.616 (0.763)	0.036	1.651 (0.843)	0.039
s2	2 ALOs of size	500	1.915 (1.986)	0.219	2.002 (2.524)	0.253
	$\omega_{AO} = 15\sigma_y$	1000	1.622 (1.263)	0.040	1.698 (1.960)	0.069
s5a	No outliers	500	1.571 (0.579)		1.598 (0.620)	
		1000	1.560 (0.530)		1.589 (0.551)	

GARCH(1,1) model with parameter values $\{\alpha_0 = 0.01, \alpha_1 = 0.08, \beta_1 = 0.91\}$

Results for **M4** method. MCRRs for 95% coverage probability as a percentage of the initial value of the simulated series (standard deviation) and e_r stands for the relative error.

			GARCH(1,1)				GJR(1,1)			
			Long Position		Short Position		Long Position		Short Position	
		n	MCRR	e_r	MCRR	e_r	MCRR	e_r	MCRR	e_r
s1	1 ALO	500	1.618 (0.590)	0.030	1.648 (0.639)	0.031	2.173 (1.058)	0.037	2.224 (1.077)	0.040
	of size	1000	1.582 (0.529)	0.014	1.612 (0.553)	0.014	2.145 (1.001)	0.015	2.197 (1.050)	0.015
	$\omega_{AO} = 5\sigma_y$	5000	1.573 (0.511)	0.003	1.602 (0.534)	0.003	2.105 (0.904)	0.003	2.159 (0.961)	0.002
	1 ALO	500	1.761 (0.772)	0.121	1.798 (0.854)	0.125	2.384 (1.309)	0.137	2.454 (1.373)	0.148
	of size	1000	1.662 (0.580)	0.065	1.696 (0.609)	0.067	2.262 (1.119)	0.070	2.322 (1.187)	0.073
	$\omega_{AO} = 10\sigma_y$	5000	1.589 (0.535)	0.013	1.619 (0.562)	0.013	2.130 (0.924)	0.015	2.186 (0.983)	0.015
	1 ALO	500	1.971 (1.093)	0.255	2.023 (1.259)	0.266	2.674 (1.669)	0.276	2.773 (1.820)	0.297
	of size	1000	1.790 (0.701)	0.147	1.831 (0.743)	0.152	2.442 (1.360)	0.155	2.518 (1.516)	0.163
	$\omega_{AO} = 15\sigma_y$	5000	1.619 (0.579)	0.032	1.650 (0.615)	0.033	2.175 (0.966)	0.036	2.233 (1.034)	0.037
s2	2 ALOs	500	2.213 (1.177)	0.409	2.276 (1.329)	0.424	3.122 (2.259)	0.485	3.272 (2.685)	0.530
	of size	1000	1.989 (1.007)	0.275	2.047 (1.149)	0.288	2.799 (1.845)	0.325	2.914 (2.146)	0.346
	$\omega_{AO} = 15\sigma_y$	5000	1.689 (0.633)	0.076	1.724 (0.669)	0.079	2.292 (1.146)	0.092	2.362 (1.252)	0.097
s3	Patch 3 ALOs	500	2.111 (1.790)	0.343	2.204 (2.464)	0.379	2.845 (2.608)	0.359	3.015 (3.519)	0.410
	of size	1000	1.835 (0.936)	0.176	1.880 (1.030)	0.183	2.523 (1.731)	0.193	2.621 (2.143)	0.211
	$\omega_{AO} = 15\sigma_y$	5000	1.633 (0.707)	0.041	1.667 (0.778)	0.043	2.194 (1.090)	0.045	2.258 (1.211)	0.043
s4	1 AVO	500	10.041 (10.127)	5.391	13.902 (18.014)	7.700	3.551 (3.864)	0.694	4.004 (5.788)	0.872
	of size	1000	6.620 (8.462)	3.244	8.672 (14.501)	4.458	2.729 (2.130)	0.291	2.905 (2.654)	0.342
	$\omega_{AO} = 15\sigma_y$	5000	2.982 (4.289)	0.901	3.397 (6.421)	1.126	2.175 (1.042)	0.036	2.245 (1.134)	0.042
s5a	No outliers	500	1.571 (0.579)		1.598 (0.620)		2.096 (1.004)		2.138 (1.011)	
	(fitting the	1000	1.560 (0.530)		1.589 (0.551)		2.114 (0.993)		2.165 (1.047)	
	same model)	5000	1.569 (0.506)		1.598 (0.526)		2.099 (0.904)		2.154 (0.960)	
s5b	No outliers	500	1.569 (0.579)		1.596 (0.620)		2.094 (1.003)		2.136 (1.010)	
	(corresp.	1000	1.559 (0.530)		1.588 (0.552)		2.113 (0.990)		2.162 (1.045)	
	t-model)	5000	1.568 (0.504)		1.597 (0.526)		2.098 (0.904)		2.152 (0.961)	

GARCH(1,1) model with parameter values $\{\alpha_0 = 0.01, \alpha_1 = 0.08, \beta_1 = 0.91\}$ and GJR(1,1) model with parameter values $\{\alpha_0 = 0.02, \alpha_1 = 0.03, \beta_1 = 0.91, \gamma_1 = 0.10\}$.

Results for **M5** method. MCRRs for 95% coverage probability as a percentage of the initial value of the simulated series (standard deviation) and e_r stands for the relative error.

			GARCH(1,1)				GJR(1,1)			
			Long Position		Short Position		Long Position		Short Position	
		n	MCRR	e_r	MCRR	e_r	MCRR	e_r	MCRR	e_r
s1	1 ALO	500	1.614 (0.575)	0.027	1.645 (0.654)	0.029	2.188 (1.079)	0.044	2.241 (1.079)	0.048
	of size	1000	1.577 (0.527)	0.011	1.605 (0.548)	0.010	2.161 (1.024)	0.022	2.210 (1.052)	0.021
	$\omega_{AO} = 5\sigma_y$	5000	1.580 (0.523)	0.007	1.609 (0.546)	0.007	2.097 (0.882)	-0.001	2.151 (0.937)	-0.001
	1 ALO	500	1.759 (0.762)	0.120	1.797 (0.871)	0.125	2.358 (1.249)	0.125	2.434 (1.307)	0.138
	of size	1000	1.662 (0.582)	0.065	1.695 (0.609)	0.067	2.277 (1.138)	0.077	2.336 (1.197)	0.079
	$\omega_{AO} = 10\sigma_y$	5000	1.585 (0.516)	0.010	1.615 (0.540)	0.011	2.138 (0.942)	0.019	2.193 (1.001)	0.018
	1 ALO	500	1.968 (1.084)	0.253	2.021 (1.270)	0.265	2.758 (2.062)	0.316	2.877 (2.324)	0.346
	of size	1000	1.790 (0.705)	0.147	1.829 (0.743)	0.151	2.462 (1.331)	0.165	2.538 (1.452)	0.172
	$\omega_{AO} = 15\sigma_y$	5000	1.612 (0.548)	0.027	1.643 (0.576)	0.028	2.205 (1.066)	0.051	2.265 (1.149)	0.052
s2	2 ALOs	500	2.214 (1.190)	0.409	2.270 (1.337)	0.421	3.104 (2.242)	0.481	3.268 (2.631)	0.529
	of size	1000	1.989 (1.011)	0.275	2.047 (1.147)	0.288	2.795 (1.851)	0.322	2.914 (2.132)	0.346
	$\omega_{AO} = 15\sigma_y$	5000	1.689 (0.633)	0.076	1.725 (0.670)	0.079	2.292 (1.148)	0.092	2.361 (1.250)	0.096
s3	Patch 3 ALOs	500	2.107 (1.782)	0.341	2.200 (2.453)	0.602	2.863 (2.758)	0.364	3.056 (3.614)	0.429
	of size	1000	1.832 (0.933)	0.174	1.875 (1.015)	0.180	2.522 (1.736)	0.193	2.623 (2.141)	0.212
	$\omega_{AO} = 15\sigma_y$	5000	1.633 (0.708)	0.041	1.667 (0.776)	0.043	2.194 (1.090)	0.045	2.257 (1.211)	0.048
s4	1 AVO	500	9.796 (9.901)	5.236	13.778(17.774)	7.622	3.500 (3.843)	0.684	4.054 (5.854)	0.896
	of size	1000	6.506 (8.193)	3.171	8.553 (13.976)	4.383	2.704 (2.119)	0.279	2.932 (2.674)	0.354
	$\omega_{AO} = 15\sigma_y$	5000	2.995 (4.268)	0.909	3.417 (6.318)	1.138	2.171 (1.039)	0.034	2.251 (1.137)	0.045
s5a	No outliers	500	1.571 (0.579)		1.598 (0.620)		2.096 (1.004)		2.138 (1.011)	
	(fitting the	1000	1.560 (0.530)		1.589 (0.551)		2.114 (0.993)		2.165 (1.047)	
	same model)	5000	1.569 (0.506)		1.598 (0.526)		2.099 (0.904)		2.154 (0.960)	
s5b	No outliers	500	1.568 (0.570)		1.596 (0.628)		2.098 (1.008)		2.136 (0.998)	
	(corresp.	1000	1.558 (0.531)		1.587 (0.551)		2.113 (0.996)		2.158 (1.032)	
	skew-t-model)	5000	1.567 (0.504)		1.597 (0.526)		2.097 (0.903)		2.151 (0.958)	

GARCH(1,1) model with parameter values $\{\alpha_0 = 0.01, \alpha_1 = 0.08, \beta_1 = 0.91\}$ and GJR(1,1) model with parameter values $\{\alpha_0 = 0.02, \alpha_1 = 0.03, \beta_1 = 0.91, \gamma_1 = 0.10\}$.

Results for **M6** method. Monte Carlo MCRRs for 95% coverage probability as a percentage of the initial value of the simulated series (standard deviation) and e_r stands for the relative error.

			GARCH(1,1)			
			Long Position		Short Position	
		n	MCRR	e_r	MCRR	e_r
s1	1 outlier	500	1.636 (0.628)	0.041	1.665 (0.673)	0.042
	of size	1000	1.598 (0.543)	0.024	1.630 (0.565)	0.026
	$\omega_{AO} = 5\sigma_y$	5000	1.575 (0.502)	0.004	1.604 (0.523)	0.004
	1 outlier	500	1.817 (0.822)	0.157	1.855 (0.895)	0.161
	of size	1000	1.669 (0.569)	0.070	1.704 (0.598)	0.072
	$\omega_{AO} = 10\sigma_y$	5000	1.605 (0.533)	0.023	1.635 (0.556)	0.023
	1 outlier	500	2.016 (0.908)	0.283	2.064 (1.032)	0.292
	of size	1000	1.825 (0.752)	0.170	1.868 (0.802)	0.176
	$\omega_{AO} = 15\sigma_y$	5000	1.623 (0.528)	0.034	1.653 (0.551)	0.034
s2	2 outliers	500	2.411 (1.193)	0.535	2.483 (1.319)	0.554
	of size	1000	2.006 (0.763)	0.286	2.055 (0.800)	0.293
	$\omega_{AO} = 15\sigma_y$	5000	1.676 (0.572)	0.068	1.710 (0.601)	0.070
s3	no outliers	500	1.571 (0.579)		1.598 (0.620)	
		1000	1.560 (0.530)		1.589 (0.551)	
		5000	1.569 (0.506)		1.598 (0.526)	

GARCH(1,1) model with parameter values $\{\alpha_0 = 0.01, \alpha_1 = 0.08, \beta_1 = 0.91\}$

References

Franses, P. and H. Ghijsels (1999). Additive outliers, GARCH and forecasting volatility. *International Journal of Forecasting 15*, 1–9.

Grané, A. and H. Veiga (2010). Wavelet-based detection of outliers in financial time series. *Computational Statistics and Data Analysis 54*, 2580–2593.

Muler, N. and V. J. Yohai (2008). Robust estimates for GARCH models. *Journal of Statistical Planning and Inference 138*, 2918–2940.