STATISTICS II
EXERCISES FOR LESSON 1
ACADEMIC YEAR 2009/10

Point estimation

1. A simple random sample of prices for a certain fruit in 9 different shops has produced the following values

$$2.5 \quad 2.1 \quad 1.8 \quad 2.1 \quad 2.3 \quad 2.2 \quad 2.2 \quad 2.0 \quad 1.9$$

   You are asked to compute:

   a) The mean, variance and standard deviation of the sample.

   b) Which estimators used to answer the preceding question are unbiased (for the corresponding population values)?

   c) Use an unbiased estimation procedure to estimate the variance of the sample mean.

   d) Use an unbiased estimator to estimate the proportion of shops where the price of the fruit is lower than 2.1 euros.

2. A market study has been conducted on 64 persons. They have been asked if they would be interested in purchasing two products. 36 have given an affirmative answer for the first product, while 32 have shown an interest in the second product and 26 are interested in both.

   You are asked to:

   a) Use an unbiased estimation procedure to estimate the percentage of consumers interested in purchasing both products.

   b) Use an unbiased estimation procedure to estimate the percentage of consumers interested in purchasing the first product but not the second one.

   c) Indicate the variance of the estimator used to answer this second question and estimate its value from the sample.

3. Let X_1, X_2, X_3 denote a simple random sample for a population with mean μ and standard deviation σ. Define from this sample the following two estimators for the population mean:

$$\hat{\mu}_1 = \frac{X_1 + 2X_2 + 2X_3}{5}, \quad \hat{\mu}_2 = \frac{X_1 + 3X_2 + X_3}{5}$$

   You are asked to:

   a) Verify that both estimators are unbiased.

   b) Determine which one of the estimators is more efficient and compute its relative efficiency.

   c) Find another estimator that is more efficient than any of them.

Confidence intervals for the mean - Normal population and known variance

4. One product of a food company is wheat flour boxed in 500 gr. packages. The weight of each package is a random variable that follows a normal distribution with known standard deviation, equal to 8 gr. In a given day a simple random sample of 64 packages has been selected, and their weight has been measured obtaining a mean value of 498 gr.

   a) Compute a confidence interval at the 99 % level for the average weight of the packages boxed in the day.

   b) What would be the confidence interval if the confidence level where selected as 95 %?

c) Assume that the actual value of the standard deviation for the production during the day is 12 gr. Which would be the confidence interval corresponding to this new value?

d) If we increase the sample size while keeping the confidence level of 99 %, and we assume that the average sample weight remains unchanged as 498 gr. (with a standard deviation of 12 gr.), which sample sizes would provide confidence intervals that would not include the advertised value of the weight (500 gr.)?

5. In the selection process to cover a position in a given company, the candidates are requested to complete an aptitude test. The results from the test are assumed to follow a normal distribution with known standard deviation, equal to 1.15. A simple random sample of 16 results from the test has a sample mean value of 4.26.

   You are asked to:

   a) Compute a confidence interval at the 90 % level for the average score of all candidates in the test.

   b) You are given a confidence interval computed by another person for the preceding data, going from 3.86 to 4.66. Which confidence level was used to compute this interval?

6. The weekly sales of cars (measured in euros) in the salesrooms of a given company are assumed to follow a normal distribution with known standard deviation, equal to 21.000 euros. A simple random sample of weekly sales has been collected, obtaining the following values (in thousands of euros):

$$123 \quad 145 \quad 88 \quad 150 \quad 115 \quad 128 \quad 97 \quad 104 \quad 125$$

   You are asked to compute:

   a) A confidence interval for the average of the monthly sales in all salesrooms, at a 95 % level.

   b) The value of this interval, if the confidence level is changed to 99 %.

   c) Which would be the confidence interval at the 95 % level if we use only the first 6 values in the sample?

Confidence intervals for the mean - Large samples

7. A simple random sample of 225 marketing experts were consulted on the level of use of some quantitative techniques in their companies (on a scale from 0 to 5). The results obtained had a sample mean of 3.35 and a sample standard deviation of 0.72.

   a) Compute a confidence interval for the mean value of the use of the techniques in all companies (the population), at a level of 99 %.

   b) Which would be the confidence level corresponding to an interval from 3.25 to 3.45, for these data?

   c) If the values of the sample mean and standard deviation do not change, find the sample sizes for which a confidence interval at the 99 % level would have a size smaller than 0.20.

8. In a simple random sample of 900 consumers it was observed that the average monthly expenditure in transportation was 325 euros, with a sample standard deviation of 112 euros.

   a) Compute a confidence interval at the 90 % level for the mean monthly transportation expenditure in the population.

   b) Compute a confidence interval at the 90 % level for the population mean if the sample standard deviation is 140 euros and the sample size is 400 consumers, while the mean monthly expenditure remains unchanged.

9. You have been given a simple random sample of 100 observations of daily electricity consumption, with the following summary values:

$$\sum_{i=1}^{100} x_i = 1{,}723 \quad \sum_{i=1}^{100} x_i^2 = 45{,}21$$

a) Compute the sample mean and the sample standard deviation. Use an unbiased estimator to obtain an estimate for the population variance.

b) Compute a confidence interval at the 95 % level for the population mean.

Confidence intervals for proportions - Large samples

10. 90 small companies have been asked if they consider that improvements in the quality of products and processes are the most important ideas to improve their competitiveness. 35 of the companies gave an affirmative answer to this question. Assuming that these answers are a simple random sample,

a) Compute a confidence interval at the 99 % level for the proportion in the population that considers these ideas as the most important ones.

b) What would be the value of the interval for a confidence level of 95 %?

11. The mayor of a town is considering improving its public sporting facilities. To fund this improvement a small increase in local taxes is contemplated. He has commisioned a poll among 150 townspeople, and 44 % of the persons in the sample have shown support for this measure.

a) Compute a confidence interval at the 95 % level for the proportion of town dwellers that would support this measure.

b) If the proportion of favorable answers and the confidence level remain unchanged, which is the minimum value of the sample size that would leave out of the confidence interval a proportion of 50 % favorable answers?

12. A survey has been conducted involving 500 voters. They were asked for their voting preferences regarding a political party. 183 of the persons polled showed a high likelihood of voting for the party in the coming elections.

a) Compute a confidence interval at the 99 % level for the voting intentions for the party in the population.

b) Which would be the minimum sample size that would yield an interval size of 1 %, assuming that the proportion of favorable voters and the confidence level remain constant?

c) For the initial data you are given a confidence interval from 30.3 % to 42.9 %. Which confidence level has been used to compute this interval?

Confidence intervals for the mean - Normal data and unknown variance

13. A clinic offers weight reduction treatments. It is assumed that the weight loss observed after a two-month treatment follows a normal distribution. From a simple random sample of 16 patients the following weight reduction values have been observed:

$$12,5 \quad 14,3 \quad 9,8 \quad 15,3 \quad 10,5 \quad 11,8 \quad 9,5 \quad 8,4$$
$$11,3 \quad 8,9 \quad 10,6 \quad 12,0 \quad 14,1 \quad 8,8 \quad 12,1 \quad 9,4$$

a) Compute a confidence interval at the 95 % level for the weight loss of all patients following the treatment.

b) Which would be the value of the interval if the confidence level is selected as 99 %?

14. The speed of 12 drivers in a given road segment has been measured, obtaing the following values:

$$89 \quad 82 \quad 95 \quad 102 \quad 84 \quad 80$$
$$86 \quad 79 \quad 96 \quad 81 \quad 98 \quad 86$$

Assuming that the speed values in that segment follow a normal distribution,

a) Compute a confidence interval at the 99 % level for the average speed of all drivers in the road.

b) Which would be the value of the interval if it is computed using only the first 8 values in the sample?

c) For the complete sample, which confidence levels would leave out of the interval the speed limit (90 Km/h) in that road?

15. A University tracks the salaries of its alumni. For a sample of 20 alumni from a given school it has observed that the sample mean of the monthly salary one year after graduation is 1700 euros, and its standard deviation is 350 euros. We assume that the population follows a normal distribution.

    a) Compute a confidence interval at the 95 % level for the mean monthly salary of all alumni of the school.

    b) Which would be the value of the interval if the confidence level is selected as 99 %?

    c) Which sample sizes would yield intervals of size smaller than 100 euros, if the values of the sample mean and standard deviation remain unchanged, and the confidence level is 95 %?

Confidence intervals for the variance - Normal data

16. A phone company has conducted a study of the time its clients spend talking on their cellular phones each day. For a sample of 25 clients the company has obtained a sample standard deviation of 12.5 minutes. We assume that the talk time follows a normal distribution.

    a) Compute a confidence interval at the 95 % level for the variance of the talk time in the population.

    b) Which is the value of the interval for a confidence level of 99 %?

17. 12 financial analysts have been consulted on the earnings per share they expect a company will report for the coming semester. The resulting values are indicated below:

$$
\begin{array}{cccccc}
2.3 & 1.6 & 0.6 & 1.9 & 1.2 & 3.5 \\
1.6 & 2.0 & 0.8 & 0.0 & 1.1 & 1.6
\end{array}
$$

We assume that these forecasts constitute a simple random sample, and the population values follow a normal distribution.

    a) Compute a confidence interval at the 95 % level for the standard deviation of the population (the forecasts of all analysts).

    b) Which would be the value of the interval if you would compute it using only the first 6 values in the sample?

18. A company applies a quality control procedure to the products sent by a supplier. For one of the attributes of the product that the company wishes to control, a simple random sample of 20 products is selected and measured, and a sample standard deviation equal to 2,3 is obtained. We assume that the values of this attribute follow a normal distribution.

    a) Compute a confidence interval at the 90 % level for the population variance.

    b) Assuming that for another sample of size 40 we obtain the same sample standard deviation value, which would be the value of the variance confidence interval for this sample, using the same confidence level?