INTRODUCTION TO STATISTICS
LADE, LEC
18th June 2009

Problem 1. (2.5 points) A provincial demographic institute wants to analyze the relationship between the distance of a town to the province capital and the average age of the population of that town. After taking a sample of ten towns, the following data is available:

Distance to the capital (km)	Average age of population (years)
53	40.0
15	38.5
87	40.4
65	40.3
48	39.6
7	38.7
29	38.9
114	41.0
94	39.7
72	39.3

So we get:

$$\sum_{i=1}^{10} x_i = 584 \quad \sum_{i=1}^{10} y_i = 396.4 \quad \sum_{i=1}^{10} x_i \cdot y_i = 23367 \quad \sum_{i=1}^{10} x_i^2 = 45038 \quad \sum_{i=1}^{10} y_i^2 = 15719$$

a) (0.5 points) Represent the data on a scatter plot.

b) (0.75 points) Calculate the correlation coefficient between the two variables. Interpret the result.

c) (0.75 points) Calculate the regression line by the least squares method.

d) (0.5 points) Predict the average age of the population of a town which is 35 km from the capital of the province.

Problem 2. (2.5 points) Given the discret random variable X, whose probability function is given by:
$$P(X = x) = kx \text{ for } x = 1, 2, 3, 4, 5.$$

a) (0.75 points) Calculate the value of constant k.

b) (0.75 points) Obtain the distribution function of X.

c) (1 point) Calculate the mean and the variance of X.

Problem 3. **(2.5 points)** A factory produces an electronic device with two different qualities:

- The 60% of the production is of quality A. The life in years of a device of quality A is given by the density function:

$$f_A(x) = \begin{cases} e^{-x} & \text{if } x > 0 \\ 0 & \text{if not} \end{cases}$$

- The 40% of the production is of quality B. The life in years of a device of quality B is given by the density function:

$$f_B(x) = \begin{cases} 2e^{-2x} & \text{if } x > 0 \\ 0 & \text{if not} \end{cases}$$

a) (0.75 points) Calculate the probability that the life of a device of quality A will be longer than one year.

b) (1 point) If we choose randomly a device among all the production of the factory, what is the probability that its life will be longer than one year?

c) (0.75 points) If we choose randomly a device among all the production of the factory, and we observe that its life is longer than one year, what is the probability that this device is of quality A?

Problem 4. **(2.5 points)** The probability that a client that goes into a car dealer end up buying a car has decreased to 2% in the last months. A given week, a car dealer has 20 visitors. Calculate:

a) (0.5 points) The probability that the car dealer doesn't sell any car during that week.

b) (0.5 points) The probability that the car dealer sells three cars during that week.

c) (0.5 points) The probability that the car dealer sells more than one car during that week.

d) (1 point) The expected turnover in a month, if the average price of the cars sold by the car dealer is 18.000 euros.

Vocabulary: car dealer = *concesionario de automóbiles*, turnover = *facturación*.

IMPORTANT:
Duration of the exam: 2 hours and a half.
Complete each exercise in a different booklet.
You must hand in the 4 booklets.